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Asymptotic Behavior in the Numerical Propagation of Finite-Amplitude Jet Noise

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Abstract. One issue of interest pertaining to the development of a numerical model applicable to the nonlinear propagation of jet noise is the behavior of spectral predictions at large distances. In this study, a recorded noise waveform from a military jet aircraft is numerically propagated via a hybrid time-frequency domain solution to the generalized Burgers equation that incorporates spherical spreading and atmospheric absorption and dispersion. Numerical results show that the spatial rate of change of the difference between the nonlinearly- and linearly-predicted power spectra appears to approach constant nonzero behavior at high frequencies. This asymptotic relationship is analogous to that predicted by analytical theory for initially-sinusoidal plane and spherical waves.

Keywords: jet noise, nonlinear, finite-amplitude, propagation, long-range
PACS: 43.25.Cb

BACKGROUND

As part of the development of a numerical model to predict the noise propagation from high-performance military jet aircraft [1], long-range propagation has been considered to determine the extent of nonlinearity and see if the nonlinear portion of the model can simply turned off at some distance. Previous studies that have treated the asymptotic behavior of nonlinear propagation analytically are now reviewed.

Blackstock [2] studied the long-range decay of planar, initially-sinusoidal waves in a thermoviscous medium with the Fay Fourier-series solution. He found that the *n*th harmonic of the fundamental decays as $e^{-n\alpha_0 x}$, where α_0 is the absorption coefficient of the fundamental and *x* is distance. This represents a slower decay than the expected small-signal solution, $e^{-n^2\alpha_0 x}$, for all of the harmonics except the fundamental (n = 1), signifying that "old-age" is not synonymous with "linear." Webster and Blackstock [3] extended the exact plane-wave solution by the use of a fifth-order accurate perturbation solution and showed that similar behavior holds for spherically-spreading, initially-sinusoidal waves.

The old-age behavior of initially-Gaussian, finite-amplitude noise propagation through thermoviscous media has been examined by Gurbatov and colleagues in a number of publications (see Refs. [4,5] and additional references therein). They have found that a weak-shock (f^{-2}) spectral slope at high frequencies eventually gives way to an exponential law roll-off at sufficiently large distances, due to the absorptive

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properties of the medium. Extension of these thermoviscous-medium results to atmospheric propagation is only qualitative; however, similar behavior is observed in the numerical atmospheric propagation of finite-amplitude jet noise waveforms. Limited results are now shown and discussed.

RESULTS

A waveform recorded at a distance of 61 m from a tied-down F/A-22 Raptor with one engine at afterburner has been numerically propagated according to a generalized Burgers equation that accounts for nonlinearity, spherical spreading, and atmospheric absorption and dispersion. The numerical solution technique, a hybrid time-frequency domain algorithm similar to those of Pestorius [6] and Anderson [7], is documented in Ref. [1]. Nonlinearly- and linearly-predicted third-octave spectra between 61-3048 m (200-10,000 ft) are shown in Fig. 1. Energy transfer from the peak-frequency region (100-300 Hz) to higher frequencies is readily seen. Comparison of the difference between the linear and nonlinear predictions for a given third-octave band demonstrates that this difference apparently continues to grow, particularly at high frequencies, out to the extent of the propagation range.

This behavior is more readily examined in the context of the nonlinear gain, *NG*, which is the difference in decibels between the nonlinearly- and linearly-predicted sound pressure levels (SPL_n and SPL_l). *NG* is written as

$$NG(r, f) = SPL_n(r, f) - SPL_l(r, f) \tag{1}$$

and its spatial partial derivative may be estimated over a range step Δr as

$$\partial NG(r, f)/\partial r \approx [NG(r + \Delta r, f) - NG(r, f)]/\Delta r\,. \tag{2}$$

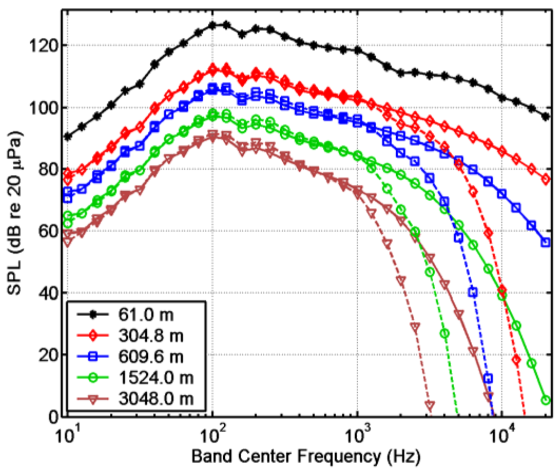

FIGURE 1. Predicted third-octave spectra between 61 and 3048 m. The nonlinear predictions are shown with solid (—) lines and the linear predictions with dashed (---) lines.

Examination of $\partial NG(r,f)/\partial r$ as a function of range permits study of the evolution of nonlinear interactions on the spectral level. Note that $\partial NG(r,f)/\partial r = 0$ at ranges and frequencies for which the propagation is linear.

Displayed in Figs. 2 and 3 are $\partial NG(r,f)/\partial r$ calculations between 61-3048 m for the third-octave bands between 63-125 Hz and 6.3-12.5 kHz, respectively. A comparison of these two figures demonstrates very different nonlinear behavior for these different frequency regions. Figure 2 reveals that, in the vicinity of the peak-frequency region of the spectrum, the rate of change between nonlinear and linear theory is very slow and eventually $\partial NG(r,f)/\partial r \to 0$. On the other hand, Fig. 3 demonstrates a very different behavior in that $\partial NG(r,f)/\partial r$ appears to approach non-zero constant behavior by several hundred meters, especially for the 10- and 12.5-kHz third-octave bands. The increase in $\partial NG(r,f)/\partial r$ for the 12.5-kHz band at about 2500 m is caused by the noise floor of the nonlinear algorithm, which, once reached, causes the high-frequency energy to decay more slowly than it otherwise would [1].

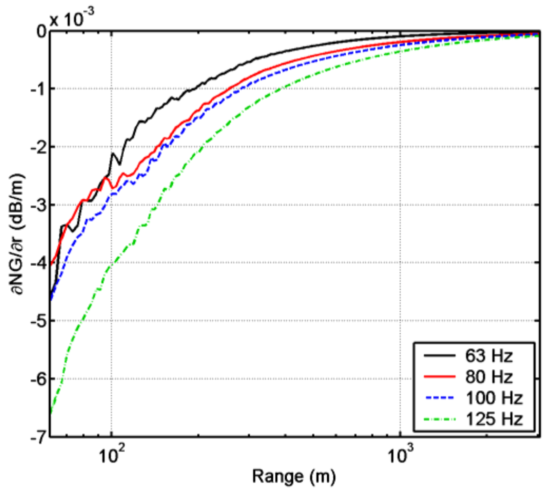

FIGURE 2. Spatial derivative of the nonlinear gain (*NG*) between 61 and 3048 m for four third-octave bands in the vicinity of the peak-frequency region of the spectrum

CONCLUSIONS

These results for the long-range numerical propagation of an F/A-22 afterburner noise waveform suggest asymptotic behavior that is analogous to that discussed by Blackstock [2] and Webster and Blackstock [3] for initially-sinusoidal waveforms. The propagation in the peak-frequency region of the spectrum eventually appears linear, but the high-frequency energy appears to always decay nonlinearly. Additional work is needed, however, to understand these how results may impact the perception of noise at large distances [1].

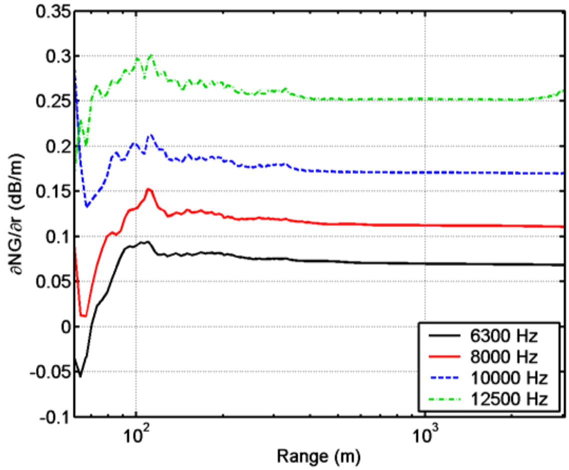

FIGURE 3. Spatial derivative of the nonlinear gain (*NG*) between 61 and 3048 m for the four third-octave bands two decades above those shown in Fig. 2. Note that the anomalous behavior at about 2500 m for the 12.5-kHz curve is attributed to the nonlinear algorithm noise floor.

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