**Spring 2016** barcode here

**Physics 441**

**Exam 3**

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**No time limit. Student calculators are allowed. One page of notes allowed (front & back). Books not allowed. HANDOUT WITH FRONT AND BACK INSIDE COVERS OF GRIFFITHS TEXTBOOK SHOULD BE PROVIDED.**

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Instructions:* Please label & circle/box your answers. **Show your work**, where appropriate! And remember: **in any problems involving Ampere’s Law, you should explicitly show your Amperian loop**. There are 100 total points.

You shouldn’t need any special integrals or derivatives.

(20 pts) Problem 1: Multiple choice, 2 pts each. Circle the correct answer.



* 1. A proton (positive charge) is released from rest in uniform E and B fields. E points up, B points into the page, as shown. Which of the paths will the proton initially follow?





* 1. A wire loop in a B field has a current *I*. The magnetic field is localized; it only exists in the hatched region, and is essentially zero everywhere else. Which way must *I* be flowing to hold the mass in place as shown (gravity points down)?
1. Clockwise
2. Counter-clockwise
3. You cannot “levitate” a mass like this
	1. A "ribbon" (width *a*, length *b*, and infinitely thin in the third dimension) with a uniform surface current density K to the right is in a uniform magnetic field Bext, oriented as shown. What is the magnitude of the force on the ribbon?
4. *KBext*
5. *aKBext*
6. *abKBext*
7. *bKBext/a*
8. *KBext/(ab)*



* 1. To find the magnetic field B at the point P due to a current-carrying wire we use the Biot-Savart law. What is the direction of the infinitesimal contribution dB created by current in the section of wire, dl, that is shown?
1. Upwards
2. Up and to the left
3. Up and to the right
4. Downwards
5. Down and to the left
6. Down and to the right
7. Into the page
8. Out of the page
	1. A wire makes a 90° bend as shown. The current flows to the left, then downwards. What is the magnitude of the magnetic field created by the wire, at the point shown a distance *s* above the bend?

1. μ0*I*/(π*s*)
2. μ0*I*/(2π*s*)
3. μ0*I*/(4π*s*)
4. μ0*I*/(8π*s*)
5. None of the above



* 1. A sheet of current with surface current density K in the y-direction as shown serves as a boundary between two regions of space. If $B=B\_{0}\hat{x}$ just to the RIGHT of the sheet, in what direction will B be just to the LEFT of the sheet? (The magnetic fields are not solely due to the sheet itself; they are produced in part by additional currents not shown.)
1. +x
2. –x
3. +z
4. -z
5. +y
6. –y
7. None of the above
	1. The vector potential A due to a long straight wire with current *I* along the *z*-axis, will be in which direction? (Assume Coulomb gauge, and don’t worry about the sign.)
8. $\hat{z}$
9. $\hat{ϕ}$
10. $\hat{s}$
	1. With regards to the vector potential A (in general), which of the following is continuous as you move past a boundary?
11. A
12. Not all of A, just the perpendicular component
13. Not all of A, just the parallel component
14. Nothing is guaranteed to be continuous regarding A
	1. A very long aluminum (paramagnetic!) rod carries a uniformly distributed current *I* along the +zdirection. What is the direction of the bound surface current? Hint: the current will set up a B field inside the rod (as well as outside); that B field will induce a magnetization.
15. It points parallel to I
16. It points antiparallel to I
17. It points clockwise (viewed from above)
18. It points counter-clockwise (viewed from above)
19. It is zero
	1. Same situation. What is the direction of the bound volume current?
20. It points parallel to I
21. It points antiparallel to I
22. It points clockwise (viewed from above)
23. It points counter-clockwise (viewed from above)
24. It is zero

(18 pts) **Problem 2**. An infinitely long, thick cylindrical shell of inner radius *a* and outer radius *b* carries a current given by the combined effects of a volume current density, $J=J\_{0}\frac{s^{2}}{b^{2}}\hat{z}$ that exists between *a* and *b*, and a surface current density $K=K\_{0}\hat{z}$, that exists at *s* = *b*. (There is no current inside *s* = *a*.) Find ***B*** everywhere due to these current densities, in terms of the given quantities.

**J**

**K**

*a*

*b*

(16 pts) **Problem 3**. Suppose you have some vector potential, $A=k\hat{ϕ}$, in a region of space.

(a) Find the magnetic field corresponding to that vector potential.

(b) Find the current density corresponding to that vector potential.

(16 pts) **Problem 4**. A cube of side *a* is centered on the origin. It is made out of a ferromagnetic material with a permanent magnetization, $M=M\_{0}\hat{z}$. (a) What bound currents are implied by this magnetization? Make a sketch and calculate the magnitude(s). (b) Approximately what is the magnetic field produced by this cube at a point, (*x*, *y*, *z*), a large distance away from the cube? You can give your answer in terms of the usual spherical coordinates *r*, *θ*, and *ϕ*.

• (*x*, *y*, *z*)

**M**

(16 pts) **Problem 5**. Consider a large compound slab (dimensions *w* × $l$ × 2*b*, with *w* and $l$ both much greater than *b*) consisting of an inner conductor of thickness 2*a* carrying a uniform current (current density *J* = *I*/2$l$*a*) in the +y direction, and outer conducting shells located at *z* = ±*b* carrying uniform currents (surface current densities *K* = *I*/2$l$) in the –y direction. That is, the total current flowing in the +y direction, *I*, is equal and opposite to the total current flowing in the –y direction. The space between the conductors (*a* < *z* < *b* and *–b* < *z* < *–a*). is filled with a linear paramagnetic material with magnetic susceptibility *χm*.

Determine **H**, **B**,and **M** (magnitude and direction)in the region between the inner and upper conductors, i.e. for *a* < *z* < *b*. Since the slab is very very thin, you may use the “infinite slab” assumption.

*z*

$$K=-I/2l \hat{y} $$

at *z* = +*b*

$J=I/2la \hat{y}$

between *z* = –*a* and *z = +a*

$$l$$

*w*

*z = b*

*z = a*

*y*

*x*

material with *χm*

$K=-I/2l \hat{y} $

at *z* = –*b*

(14 pts) **Problem 6**. A square loop of wire, with sides of length *a*, lies in the first quadrant of the x-y plane as shown. (The z-direction is out of the page.) In this region there is a non-uniform time-dependent magnetic field $B\left(x,t\right)=kx^{2}t^{3}\hat{z}$ (where *k* is a positive constant).

*x*

*y*

(a) Find the magnitude of the EMF induced in the loop.

(b) In what direction will the induced current flow (CW vs. CCW)? Justify your answer.