

# Physics 441 Formula Sheet (17 May 2024 version)

## ELECTRIC

### Statics

- $q = \int \lambda dl$   
 $q = \int \sigma da$   
 $q = \int \rho d\tau$
- $\mathbf{F} = \frac{qQ}{4\pi\epsilon_0 r^3} \hat{\mathbf{r}}$
- $\mathbf{F} = Q\mathbf{E}$
- $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i \hat{\mathbf{r}}_i}{r_i^2}$   
 $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}') \hat{\mathbf{r}}}{r^3} dl'$   
 $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\mathbf{r}') \hat{\mathbf{r}}}{r^3} da'$   
 $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}') \hat{\mathbf{r}}}{r^3} d\tau'$
- $\boxed{\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}}$
- $\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{a}$
- $\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\epsilon_0}$
- $\nabla \times \mathbf{E} = 0$  (this gets modified below)
- $\mathbf{E} = -\nabla V$  (this gets modified in Phys 442)
- $V(\mathbf{r}) = -\int_{ref}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$
- $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$   
 $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}')}{r} dl'$   
 $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\mathbf{r}')}{r} da'$   
 $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{r} d\tau'$
- $U = \frac{1}{2} \sum_{i,j} \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}}$   
 $U = \frac{1}{2} \sum_i q_i V(\mathbf{r}_i)$   
 $U = \frac{1}{2} \int \rho(\mathbf{r}') V(\mathbf{r}') d\tau'$   
 $U = \frac{\epsilon_0}{2} \int E^2 d\tau$
- $C = \frac{Q}{V}$
- $U = \frac{1}{2} \frac{Q^2}{C}$
- $E_1^\perp - E_2^\perp = \frac{\sigma}{\epsilon_0}$
- $\mathbf{E}_1^\parallel = \mathbf{E}_2^\parallel$
- $V_1 = V_2$
- $\nabla^2 V = -\frac{\rho}{\epsilon_0}$
- $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \times \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta') \rho(\mathbf{r}') d\tau'$
- $\mathbf{p} = \sum_i \mathbf{r}'_i q_i$   
 $\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d\tau'$
- $V_{dip} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$
- $\mathbf{E}_{dip}(r, \theta) = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} (2\cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\boldsymbol{\theta}})$

## MAGNETIC

### Statics

- $I = \int K_\perp dl$   
 $I = \int \mathbf{J} \cdot d\mathbf{a}$
- No easy parallel for magnetic field
- $\mathbf{F} = Q(\mathbf{v} \times \mathbf{B})$
- $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{l}' \times \hat{\mathbf{r}}}{r^3}$   
 $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}') \times \hat{\mathbf{r}}}{r^3} da'$   
 $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I(\mathbf{r}') \times \hat{\mathbf{r}}}{r^3} d\tau'$
- $\boxed{\nabla \cdot \mathbf{B} = 0}$
- $\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{a}$
- $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$
- $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$  (this gets modified below)
- $\mathbf{B} = \nabla \times \mathbf{A}$ ,  $\nabla \cdot \mathbf{A} = 0$  (Coulomb gauge)
- No direct parallel for the magnetic field
- $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{r} d\mathbf{l}'$   
 $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{r} da'$   
 $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I(\mathbf{r}')}{r} d\tau'$
- $L = \frac{\Phi}{I}$
- $U = \frac{1}{2} LI^2$
- $B_1^\perp = B_2^\perp$
- $\mathbf{B}_1^\parallel - \mathbf{B}_2^\parallel = \mu_0 (\mathbf{K} \times \hat{\mathbf{n}})$
- $\mathbf{A}_1 = \mathbf{A}_2$
- $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
- $\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \times \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint (r')^n P_n(\cos\theta') d\mathbf{l}'$
- $\mathbf{m} = I \int d\mathbf{a} = I\mathbf{a}$
- $\mathbf{A}_{dip} = \frac{\mu_0 \mathbf{m} \times \hat{\mathbf{r}}}{4\pi r^2}$
- $\mathbf{B}_{dip}(r, \theta) = \frac{\mu_0 m}{4\pi r^3} (2\cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\boldsymbol{\theta}})$

- $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$  ( $\tau$  = torque here, not volume)
- $\mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} = \nabla(\mathbf{p} \cdot \mathbf{E})$
- $U = -\mathbf{p} \cdot \mathbf{E}$

### Materials

- $\mathbf{P}$  = electric dipole moment per unit volume  
 $\mathbf{p} = \int \mathbf{P}(\mathbf{r}') d\tau'$
- $V_{pol.object} = \frac{1}{4\pi\epsilon_0} \int \frac{\mathbf{P}(\mathbf{r}') \cdot \hat{\mathbf{r}}}{r^3} d\tau'$
- $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$
- $\rho_b = -\nabla \cdot \mathbf{P}$
- $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$
- $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$
- $\boxed{\nabla \cdot \mathbf{D} = \rho_f}$
- $\mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E}$
- $\oint \mathbf{D} \cdot d\mathbf{a} = q_{f,enc}$
- $\epsilon_r = 1 + \chi_e$
- $U = \frac{1}{2} \int \mathbf{D} \cdot \mathbf{E} d\tau$
- $D_{1\perp} - D_{2\perp} = \sigma_{free}$

### Dynamics

$$38. \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

$$39. \boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

$$40. \boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}} \text{ (unchanged for materials)}$$

### Equations that don't fit anywhere else

Stress = force/area on a conductor:  $\mathbf{stress} = \frac{\sigma^2}{2\epsilon_0} \hat{\mathbf{n}}$

$$Q_{ij} = \int \left( \frac{3}{2} r'_i r'_j - \frac{1}{2} r'^2 \delta_{ij} \right) \rho(\mathbf{r}') d\tau'$$

$$V_{quad}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0 r^3} \sum_{i,j} \hat{r}_i \hat{r}_j Q_{ij}$$

$$23. \boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$

$$24. \mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$

$$25. U = -\mathbf{m} \cdot \mathbf{B}$$

### Materials

- $\mathbf{M}$  = magnetic dipole moment per unit vol.  
 $\mathbf{m} = \int \mathbf{M}(\mathbf{r}') d\tau'$
- $\mathbf{A}_{magn.object} = \frac{\mu_0}{4\pi} \times \int \frac{\mathbf{M}(\mathbf{r}') \times \hat{\mathbf{r}}}{r^3} d\tau'$
- $\mathbf{J}_b = \nabla \times \mathbf{M}$
- $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$
- $\mathbf{M} = \chi_m \mathbf{H}$
- $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$
- $\boxed{\nabla \cdot \mathbf{B} = 0}$  (still)
- $\mathbf{H} = \mathbf{B}/\mu = \mathbf{B}/(\mu_0 \mu_r)$
- $\oint \mathbf{H} \cdot d\mathbf{l} = I_{f,enc}$
- $\mu_r = 1 + \chi_m$
- $U = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} d\tau$
- $\mathbf{H}_{1\parallel} - \mathbf{H}_{2\parallel} = \mathbf{K}_{free} \times \hat{\mathbf{n}}$

### Dynamics

38.  $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$  (same equation as in left hand column; connects charge to current)

$$39. \boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}}$$

$$40. \boxed{\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}}$$