

Measuring Temperature with a Silicon Diode

Due to the high sensitivity, nearly linear response, and easy availability, we will use a 1N4148 diode for the temperature transducer in our measurements.

1.0 Analysis of the thermal dependence in the diode equation

The current conducted through an ideal diode is described by the Shockley ideal diode equation:

$$I_D = I_S \left[\exp \left(\frac{q V_D}{n k_B T} \right) - 1 \right]$$

where q is the electron charge, k_B is Boltzmann's constant, T is the temperature of the diode junction, and V_D is the voltage across the diode (positive voltage indicates a forward-biased diode). n is a quality factor or emission coefficient that typically lies in the range of 1–2 and is usually assumed to be approximately 2 for a diode.

I_S is the reverse bias saturation current given approximately by

$$I_S = A \exp \left(-\frac{E_g}{2 k_B T} \right)$$

where A depends primarily on the geometry and doping of the junction region and E_g is the semiconductor band gap. The band gap for silicon is usually given as 1.17 eV at 0 K and 1.11 eV at 300 K. For the 1N4148 diode we usually measure a value of ~ 1.20 eV.

Although the equation for I_D , the diode current, appears to be very nonlinear (a product of exponentials appears to dominate the equation), it is possible to find operational parameters that produce a very linear behavior, at least over a reasonable range of temperatures.

We can solve this for the voltage V_D assuming that $n = 2$:

$$V_D = \frac{\ln \left[\frac{I_D}{A} + \exp \left(-\frac{E_g}{2 k_B T} \right) \right] 2 k_B T + E_g}{q}.$$

Since $E_g \sim 1.20$ eV and $k_B T \sim 0.025$ eV at room temperature, $\exp[-E_g/(2k_B T)] \sim 10^{-10}$ and it can probably be neglected. So we arrive at the expression

$$V_D = \frac{\ln\left(\frac{I_D}{A}\right) 2k_B T + E_g}{q}.$$

If the current through the diode is held constant, there is now a simple linear relationship between voltage and temperature. If the diodes we will use were ideal, this expression would be all we need. For real diodes, the functional form is still quite linear, *e.g.*,

$$V = mT + b$$

where m and b are constants. However, b may be slightly different from E_g/q and the parameter A in $\ln(I_D/A)$ is unknown. The constants m and b will need to be determined experimentally for accurate temperature measurement.

2.0 Checking the validity of the linear assumption

It is advisable to verify that our assumption of linearity in the calibration equation is correct. We have found that $A \approx 90$ is a lower limit on the value for a “typical” 1N4148 diode. Any nonlinearity in the equation will result primarily from the logarithmic term in the numerator:

$$\ln\left[\frac{I_D}{A} + \exp\left(-\frac{E_g}{2k_B T}\right)\right].$$

[Figure 1](#) shows a graph of the temperature dependence of this term for four values of I_D assuming a value of $A = 90$ A to help in determining an appropriate diode current for linear behavior.

From this analysis, it is apparent that $I_D = 10 \mu\text{A}$ would be adequate for measurements from less than 70 K up to about 300 K (room temperature). It becomes noticeably nonlinear by the time we get to 370 K (boiling water). $I_D = 100 \mu\text{A}$ will allow the use of the diode up to about 370 K but that is at the start of exponentially increasing error. Since we are interested in temperatures from 75 K (boiling liquid nitrogen) to 370 K, we can reasonably use $I_D = 100 \mu\text{A}$ for our measurements.

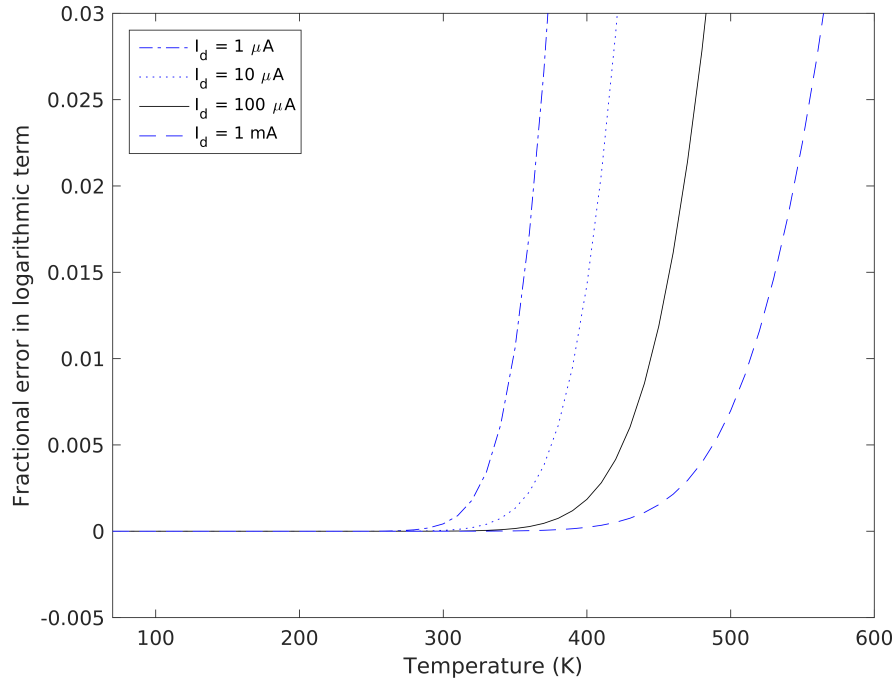


Figure 1: $\ln[I_D/A + \exp(-E_g/(2k_b T))]/\ln[I_D/A] - 1$, the fractional error due to neglecting the exponential function in the logarithmic term of the voltage equation as a function of temperature for several values of the diode current I_D . These curves assume a value of $A = 90 \text{ A}$ and $E_g \sim 1.19 \text{ eV}$.

3.0 Estimating the error in the measured temperature

Any measurement is guaranteed to have some error associated with it (review the “[Uncertainty, Errors, and Noise in Experimental Measurements](#)” handout). The obvious problem is that we can’t measure a voltage with infinite precision. But there are other concerns that may crop up in a careful analysis of the errors.

There are two approaches to determining the calibration constants for the diode.

3.1 Calibrating with $V=mT+b$

For this form of the equation, the constants are directly related to the theoretical equation derived earlier in this handout. Rearranging to get the temperature from the value of the diode voltage, $T = (V_d - b)/m$, the error analysis is slightly complicated.

$$\begin{aligned}
(\Delta T)^2 &= \left(\frac{\partial T}{\partial V_D} \Delta V_D \right)^2 + \left(\frac{\partial T}{\partial b} \Delta b \right)^2 + \left(\frac{\partial T}{\partial m} \Delta m \right)^2 \\
&= \left(\frac{1}{m} \Delta V_D \right)^2 + \left(\frac{-1}{m} \Delta b \right)^2 + \left(\frac{-(V_D - b)}{m^2} \Delta m \right)^2 \\
&= \frac{1}{m^2} (\Delta V_D^2 + \Delta b^2) + \left(\frac{V_D - b}{m^2} \right)^2 \Delta m^2.
\end{aligned}$$

Using the original equation for T and rearranging somewhat we get

$$\begin{aligned}
(\Delta T)^2 &= \left(\frac{T}{V_D - b} \right)^2 (\Delta V_D^2 + \Delta b^2) + \left(\frac{T}{m} \right)^2 \Delta m^2 \\
\left(\frac{\Delta T}{T} \right)^2 &= \frac{\Delta V_D^2 + \Delta b^2}{(V_D - b)^2} + \left(\frac{\Delta m}{m} \right)^2.
\end{aligned}$$

This form of the equation is not useful if you are using temperatures in Celsius because of the problem of dividing by 0. It is useful with Kelvin temperatures. The previous form without the explicit presence of T in the equation can be used with temperatures in Celsius.

As an example, data taken for a calibration in ice water resulted in the values $T = 273.15$ K and $V_D = 0.5586857$ V. For these values 10000 samples were acquired at 100000 samples/s and averaged to arrive at the final value. For each sample, it was found that $\sigma_V = 54 \mu\text{V}$. Since we averaged 10000 samples to arrive at V_D , $\sigma_{V_D} = \sigma_V / \sqrt{10000} = 0.5424 \mu\text{V}$. The *Mathematica* script NLSQFit240 will return estimated uncertainties in the fitting parameters if we include uncertainties in the measurements in the third column of the input file. Doing so with the estimated uncertainties in the measurements resulted in $m = -0.0023776$ V/K, $\sigma_m = 9.8 \times 10^{-7}$ V/K, $b = 1.2053$ V, and $\sigma_b = 2.64 \times 10^{-4}$ V. Using these values in the above equation, we get

$$\begin{aligned}
\left(\frac{\Delta T}{T} \right)^2 &= \frac{(5.424 \times 10^{-7})^2 + (2.64 \times 10^{-4})^2}{(0.5586857 - 1.2053)^2} + \frac{(9.8 \times 10^{-7})^2}{(-2.377 \times 10^{-3})^2} \\
&= 1.667 \times 10^{-7} + 1.748 \times 10^{-7} \\
&= 3.415 \times 10^{-7} \\
\Delta T &= 5.84 \times 10^{-4} T.
\end{aligned}$$

At $T = 273.15$ K we get $\Delta T = 0.160$ K. If we use the measured V_D in the fitting equation we get

$$\begin{aligned}
T &= \frac{0.5586857 - 1.2053}{-0.002377} \\
&= 272.03 \text{ K.}
\end{aligned}$$

This is not within the expected error, so the fit appears to be marginal (at least for ice water).

3.2 Calibrating with $T = m'V_D + b'$

If you have fit your data using the equation $T = m'V_D + b'$ the error analysis is significantly easier:

$$\begin{aligned}(\Delta T)^2 &= \left(\frac{\partial T}{\partial V_D}\Delta V_D\right)^2 + \left(\frac{\partial T}{\partial b'}\Delta b'\right)^2 + \left(\frac{\partial T}{\partial m'}\Delta m'\right)^2 \\ &= (m'\Delta V_D)^2 + (\Delta b')^2 + (V_D\Delta m')^2.\end{aligned}$$

In this format the fitting parameters are $m' = -420.6 \text{ K/V}$, $\sigma_{m'} = 0.17 \text{ K/V}$, $b' = 506.9 \text{ K}$, and $\sigma_{b'} = 0.12 \text{ K}$. Using the measured values at 273.15 K with $V_D = 0.5586857 \text{ V}$ and $\sigma_{V_D} = 5.424 \times 10^{-7} \text{ V}$ we find

$$\begin{aligned}\Delta T &= \sqrt{(-420.6 \times 5.424 \times 10^{-7})^2 + 0.121853^2 + (0.5586857 \times 0.174364)^2} \\ &= 0.16 \text{ K}\end{aligned}$$

and the temperature from the measured voltage and fitting equation is

$$\begin{aligned}T &= -420.6 \times 0.5586857 + 506.9 \\ &= 271.92 \text{ K}.\end{aligned}$$

The estimated error in the temperature is comparable for this method, but the difference between the measured temperature and the calculated temperature is larger than the first method.

4.0 Estimating the effect of varying diode current on the measured temperature

The diode current, I_D will only affect the value of the constant m . Note that this is using the value of m from fitting the equation $V_D = mT + b$. Recall that

$$m = \frac{2k_B}{q} \ln\left(\frac{I_D}{A}\right)$$

(ignoring the small exponential term) and that A is a geometry and composition dependent constant of the diode (q and k_B are still constants of the universe), so we only have I_D dependence

$$\begin{aligned} (\Delta m)^2 &= \left(\frac{\partial m}{\partial I_D} \Delta I_D \right)^2 \\ &= \left(\frac{2k_B}{q I_D} \Delta I_D \right)^2. \end{aligned}$$

Using the original equation for m and rearranging some we get

$$\begin{aligned} (\Delta m)^2 &= \left[\frac{m}{I_D \ln(I_D/A)} \Delta I_D \right]^2 \\ \left(\frac{\Delta m}{m} \right)^2 &= \left[\frac{\Delta I_D}{I_D \ln(I_D/A)} \right]^2. \end{aligned}$$

With the target value of $I_D = 100 \mu\text{A}$, the equation for m and the fitting parameters we can estimate the value of A . Solving for A we get $A = I_D \exp[-m q / (2 k_B)] = 98.3$ for the value of m given above.

Using these values, we get

$$\begin{aligned} \left| \frac{\Delta m}{m} \right| &= \left| \frac{\Delta I_D}{I_D \ln(I_D/A)} \right| \\ &= \frac{\Delta I_D}{1.38 \times 10^{-3}}. \end{aligned}$$

For a change in I_D of $0.5 \mu\text{A}$ we get (approximately)

$$\begin{aligned} \left| \frac{\Delta m}{m} \right| &= 3.62 \times 10^{-4} \\ \Delta m &= 8.60 \times 10^{-7}. \end{aligned}$$

This value for Δm is an order of magnitude smaller than the value found from the calibration of the diode. It will not significantly affect the uncertainty in the temperature. If we do include it, the uncertainty in the temperature at 273 K will increase by roughly 0.1 K. This gives us a rough measurement of the accuracy with which we must regulate the diode current.

[Modified: January 16, 2019]