Thermal Measurements Lab: Heating and Cooling of the Cryostat Assembly

1.0 Expected Learning Outcomes

- Design and fabricate a heating coil for use with the aluminum heater block from the machine shop exercises.
- Build and calibrate a practical temperature sensor system to monitor the aluminum heater block.
- Understand and account for systematic errors in experimental measurements.
- Acquire thermal data using a computer-based data acquisition system.
- Develop a simple model for a thermal system.
- Use the acquired data and the model to determine the constants of the thermal system (heat capacity, rate, and routes of heat loss, and equilibrium temperature).
- Write a technical report describing the thermal measurements and the results of the analysis.

2.0 Introduction

The goal of this experiment is to measure and model the thermal behavior of your cryostat assembly (heater block, cooling rod, and cap) as it is heated and cooled. We are making these measurements in prepartion for the development of a temperature controller in a future lab exercise. Understanding the thermal behavior of the cryostat assembly helps in arriving at a working controller.

There are several steps required to prepare the heater block and the necessary hardware and software before you can begin taking measurements. These steps include building a heater coil on your heater block, developing a method of measuring the temperature of the heater block, and developing a method of recording the temperature of the block as a function of time using the computer. It is then necessary to analyze those measurements to compare them with a simple model of the thermal behavior of the heater block assembly and determine the thermal constants for that system.

Finally, you will write a report describing your experiment and the results.

In preparation for this exercise, you should read Section 8.1 (`THE MEASUREMENT OF TEMPERATURE`) in the text. Pay particular attention to Section 8.1.4 (`Semiconductor Thermometers`).

If you do not have access to the text, you can read the short comparison of thermometers found in "A Brief Introduction to Temperature Transducers" (Learning Suite, `Content ⇒ Thermal Measurements Lab ⇒ Using a Diode as a Thermometer`).

3.0 Equipment

Each group should have one complete cryostat (heater block, cap, cooling rod).

In the storage cabinet in the lab:

Fluke or Cen-Tech digital thermometer	two small screwdrivers
thermocouples for thermometers	desoldering pump
plastic storage box (one/group)	two teasing needles
Cen-Tech multimeters	scissors
cardboard toolkit boxes (one/group)	cardboard boxes of connection wires (shared)
needle-nose pliers	various wire and cables
diagonal cutters	soldering iron (one/group)
wire strippers	various electronic components

In the lab:

5 L LN2 (as needed)	stainless steel cup
Styrofoam insulating containers: octagonal and rectangular	gray/dark brown Tenma or GW Laboratory DC Power Supply, (0-20 V, 0-3 A for 4-lead measurements)
hot plate	

At each station:

brown/tan +5/+15/-15 power supply
gray Kepco ATE36-8M power supply

4.0 Grading

The grading for this portion of the class will be as follows:

- Heating coil built and characterized – 10 pts.
- Build constant current source for diode temperature measurements – 10 pts.
- LabVIEW VI built to measure temperature and diode correctly calibrated – 10 pts.
- Measure heating and cooling curves, determine C, γ, T_{eq} – 20 pts.
- Lab notebook – 40 pts.
- Report on results – 40 pts. broken down as follows

    - Report format followed – 5 pts.
    - Report clearly written (flow, grammar, spelling, etc.) – 15 pts.
    - Content, questions in the handout answered reasonably – 20 pts.

5.0 Summary of steps to carry out the measurements

1. Build a heater coil on your aluminum heater block (page 7).
2. Read the description of how to use a silicon diode to measure temperature (page 8).
3. Build a $100\,\mu A$ constant current source to provide the current for a temperature measurement diode (page 8).
4. Create a VI to acquire the voltage across the temperature measurement diode and to accumulate those voltages in a data file for analysis (page 9).
5. Use ice water, boiling water, and boiling liquid nitrogen to determine a calibration equation to convert the diode voltage to temperature (page 8).
6. With the cryostat fully assembled and the diode in place in the 1/8" hole on the bottom, record the temperature as a function of time as you heat the block to about $120\,°C$ and then allow it to cool for about 20 minutes (page 13).

    - Use the Kepco ATE-36-8M power supply to provide current to your heater coil.
    - Set the supply to provide about 70-90 W to the coil so that it takes about 2 minutes to reach the designated temperature.

7. Fit the equation from the model for the cryostat assembly to the temperature vs. time data. Determine the values of C (heat capacity), γ(heat loss coefficient), and T_{eq} (the equilibrium temperature) (page 15).
8. Write the technical report describing your measurements (page 4).

6.0 Lab notebooks

This is the first exercise in Physics 240 that requires the use of a lab notebook. It is **strongly** recommended that you review the materials in "Your Lab Notebook" (Learning Suite, `Content` ⇒ `Your Lab Notebook`). The lab notebook is a significant portion of your final grade since it is a critical part of physics research.

7.0 Your technical report on the experiment

After you complete your measurements and analysis, you will turn in a typed, double-spaced description of the temperature measurement experiment. It is unlikely that you will be able to write an adequate report that isn't *at least* two pages long.

Please remember: ***Most writing assignments in Physics 240 are done individually - there is only 1 group writing assignment and this one is not it.***

This report is based on a simplified technical report of the type usually used for internal communications or communication with a contractor. It is often not intended for general public consumption, so it is possible to make some assumptions regarding your audience.

In the case of this report, you can assume that the reader has some acquaintance with the overall project but not with your specific measurements.

Most descriptions of technical report contents and formats are for the engineering disciplines, but they apply equally well to such reports in the physical sciences. You can search for descriptions of technical reports (search term "Technical Report"). Some potentially useful descriptions would include

- Engineering Technical Reports, Colorado State University (`http://writing.colostate.edu/guides/pdfs/guide88.pdf`) (24 October 2015).
- How to Write a Technical Report by Alan Smithee (`http://www.physics.byu.edu/faculty/petersonb/Phys240/HowToWriteATechnicalReport-AlanSmithee.pdf`) (9 January 2017). Note: I'm not sure where this came from. It is found on many websites, and I haven't found one with any attribution.
- Boris Korsunsky, "The Results Were the Results of Errors, Promoting Good Writing by Bad Example," The Physics Teacher, vol. 48, pp. 10-11, `https://doi.org/10.1119/1.3274349`.

We are simplifying this report somewhat by not requiring an Executive Summary, Table of Contents, List of Figures, or List of Tables. The report does need to be complete and divided into the following sections as appropriate for your content.

1. **Title Page** (The title of your report and who you are – this is a separate page with no other content except possibly your abstract)

2. **Abstract** (*One* paragraph describing what was done, why it was done, and the key results.)

3. **Introduction** (What is this measurement about?)

4. **Experimental Methods** (What did you use to make the measurement? How did you make the measurement?)

5. **Experimental Results** (This is your data and a description of your data.)

6. **Analysis and Discussion** (This is a determination of what your data means.)

Some important additional reminders:

- Use units with all data (we really can't say this too often – it is very important).

  There are two online documents that are very valuable regarding the use of SI units. The NIST Guide for the use of the International System of Units (`https://www.nist.gov/physical-measurement-laboratory/special-publication-811`) and Writing with Metric Units (`https://www.nist.gov/pml/weights-and-measures/writing-metric-units`)

- **Use labels for graph axes**. These labels should include both what is represented by the axis and the associated units if applicable. Use arrows and describe what is significant on the graphs.

  "Give every figure [graphs, pictures, etc.] a caption, complete and intelligible in itself without reference to the text."[1]

- Describe any analysis models and approximations completely.

- Indicate uncertainty in your data and subsequent results as accurately as possible.

- Answer all questions that are found in this handout. It is not expected that the answers be given explicitly (*i.e.*, "the answer to question xxx is yyyy."). It is expected that the answers will at least show up somewhere in the discussion.

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[1]AIP Publication Board, *AIP Style Manual, Fourth Edition*, American Institute of Physics, New York, 1997, p. 2. Available on the web at `https://www.physics.byu.edu/faculty/petersonb/Phys240/AIP_Style_4thed.pdf`.

8.0 Building Models in experimental physics

Anytime you interact with the physical world you are building models to help you analyze and understand those observations. These models may be mental, mathematical, diagrams, or some combination of these. These models can help to identify underlying assumptions or limitations in your understanding of the observations.

Without some model to use in interpreting your observations, it is not possible to draw meaningful conclusions regarding the system you are observing.

As you progress through measurement and analysis of a system or experiment, it is often necessary to go back and tweak your models as you learn more about the system. You may also find that your models will guide you in revising either the measurement system or the physical apparatus to improve your results. This iterative process is common in any experimental project.

You will find that this is also true in both computational and theoretical projects. It is often necessary to go back and modify computer codes or assumptions in your theory (both of which can be described as models) to achieve the desired results.

8.1 Modeling the measurement tools

In physics, mental models are often inadequate for a complete understanding. For example, if we wish to make a measurement using a computer and some transducer, we have to understand how the transducer responds to the phenomenon of interest as well as the way the computer interface and acquisition system respond to the signals from the transponder.

In this lab exercise, we will use a transducer to measure the temperature of the heater block and record it in a file so that the results can be further analyzed. We will develop a model describing how the output of the transducer is related to the temperature of the block. We may also have to include the behavior of the transducer if the supply voltage or current varies.

For example, using a silicon diode to measure the temperature requires some understanding of how the diode responds to varying temperature. The development of a model for the temperature dependence of a diode's characteristics is described in the handout "Using a Semiconductor Diode As a Temperature Sensing Device", also found under `Content` $\Rightarrow$ `Thermal Measurements Lab` $\Rightarrow$ `Using a Diode as a Thermometer` in Learning Suite, This handout also includes some information on estimating the uncertainty in the measured temperature that you will find useful.

To complete the model, you will follow the instructions in "Calibrating a Semiconductor Diode for Use as a Temperature Sensing Device" (`Content` $\Rightarrow$ `Thermal`

`Measurements Lab` $\Rightarrow$ `Using a Diode as a Thermometer` in Learning Suite). This will allow you to quantify the model for the diode so that you can use the voltage across the diode to determine the temperature of the heater block.

We also have to consider how our data acquisition hardware affects the measurements. Considerations may include sample rates, input range, resolution (minimum resolvable voltage change), intrinsic errors in the electronics, the presence of electronic noise, and filtering (both intentional and unintentional). Some of these are easily quantified and should be. Some will remain uncertain and difficult to characterize, but it is important to identify their effect on your results as carefully as possible.

8.2 Modeling the physical apparatus

It is also necessary to model the physical apparatus with which you are working. This will allow you to relate the measured parameters to any outside influences or fundamental characteristics of the apparatus. These outside influences may be intentional, such as driving an oscillator, or out of your control, such as air currents due to building ventilation or vibrations from external forces. In any case, the more complete your model, the better you will be able to understand the measurements.

In the case of our experiment, we need to have a model for the way the temperature of the heater block/heater coil/cooling rod/ice water/brass cap system changes when we change the power in the heating coil. We will develop a simple model for this system on page 14 (Section 11.2) later in this handout.

It is important to look carefully at this model, so you understand the limitations in its application to our system.

9.0 Constructing a heater coil on your aluminum heater block

The first step in carrying out this lab is to build and characterize a heating coil on your heater block. The instructions for building the coil are in "Heater Construction Handout" (also found on Learning Suite in `Content` $\Rightarrow$ `Thermal Measurements Lab`.

To characterize the heater coil, you need to know (and record in your lab notebook) the *final* dimensions and resistance of your coil. It usually doesn't work very well to measure the resistance of your coil with an ohmmeter. You should read "Four-Lead Measurement of Resistance" (and in `Content` $\Rightarrow$ `Thermal Measurements Lab` $\Rightarrow$ `Making a 4-Lead Resistance Measurement` on Learning Suite) for the proper way to accurately determine the resistance. It is often necessary to use the Four-Lead technique for measuring resistance if you have a very small resistance (like your

heater coil) or if the connections to the device being measured are resistive ("poor connections").

Do NOT use the beige/black $+15/-15/+5\,$V supply to provide the current for the four-lead measurement – these supplies dislike being attached to a low-resistance load. It is best to use a supply that can be adjusted to provide a constant current. A supply with a current control knob will be available in the lab for this purpose.

Note that you are measuring the resistance of a heater. Running a current through it is supposed to change the temperature. Changing the temperature of the material can change its resistivity.

10.0 Measuring the temperature of the heater block

You should review Section 8.1.4 (`Semiconductor Thermometers`) in the text. Pay particular attention to the relative advantages of semiconductor devices for temperature measurement in terms of sensitivity and linearity.

Because of the simplicity and easy availability of a 1N4148 silicon diode, we will be using this diode for our temperature measurements. The small off-center hole in the aluminum heater block is specifically provided to hold a diode for recording the temperature of the block.

- Consult the handout "Using a Semiconductor Diode As a Temperature Sensing Device", also found under `Content` $\Rightarrow$ `Thermal Measurements Lab` $\Rightarrow$ `Using a Diode as a Thermometer` in Learning Suite, to understand why a diode thermometer works and some of the limitations of that device. This handout also includes some information on estimating the uncertainty in the measured temperature that you will probably find useful.
- For the diode to reliably measure the temperature, it is necessary to drive it with a constant current. This current should be near $100\,\mu$A (in the range of roughly $90\,\mu$A to $110\,\mu$A) for the range of temperatures in which we are interested. The instructions for building a reliable constant current supply for operation in this range are given in "Building a Constant Current Source" (in Learning Suite under `Content` $\Rightarrow$ `Thermal Measurements Lab` $\Rightarrow$ `Building a Constant Current Source`).
- For detailed instructions on calibrating your diode to make reasonably accurate temperature measurements see "Calibrating a Semiconductor Diode for Use as a Temperature Sensing Device" (`Content` $\Rightarrow$ `Thermal Measurements Lab` $\Rightarrow$ `Using a Diode as a Thermometer` in Learning Suite).

- Thin Teflon spaghetti tubing is available to insulate the leads on the diode so it can be inserted into the aluminum heater block. When you bend one of the leads back so they both point in the same direction, be careful that you don't bend the lead right at the plastic surface or kink it. As sharp bend can weaken the lead sufficiently that it will break.

11.0 Data collection using LabVIEW

Most of the data for this experiment will be collected using LabVIEW so you can more easily analyze the results. You will need to build a data acquisition VI for this purpose using (at a minimum) an input VI, a "while loop," a time delay, a chart to display the voltage measurements, and a function to save the results to a file.

Please note the terminology here. You are writing a VI that includes a sub-VI that is called the "DAQ Assistant." This phrase refers to a software "wizard" that simplifies the connection between the computer and the external hardware to allow you to acquire signals from the external hardware.

The piece of external hardware that connects between the computer and whatever signals you wish to acquire is known as a data acquisition module or data acquisition board. The abbreviation for this object may be a DAQ module or a DAQ board.

The DAQ board has a model name/number. ***When you are describing your experiment in a document or publication, you should refer to it by manufacturer and model name or number, not as just "the DAQ board" and especially not as the "DAQ Assistant" (remember that the "DAQ Assistant" is the sub-VI "wizard").***

- Considerations for your analog input VI:
  1. The analog input devices that we use can only directly measure voltages. When you configure your `DAQ Assistant` Express VI it will give you the option of measuring several different types of signals including current, resistance, and temperature.
     If you have configured it to measure anything but voltage, it will be necessary further to configure the series resistance (for current measurement), the current (for resistance measurement), or the calibration equation (for temperature measurement). ***If you select anything but voltage measurement, it will be necessary to <u>verify every class period</u> that the configuration is the same as you used in previous measurements***.
  2. Determine the input range you need for your analog input VI to accurately measure the signals you are going to acquire.

- For DC signals, a voltmeter is adequate to determine the range. Measure the maximum and minimum signals for a range of experimental conditions and use those to set the input range (allow a little extra just to be sure).
- For AC signals, you may need to temporarily set the input range to some large value (like $\pm 10\,\text{V}$), acquire the signal under several conditions. You can then use these signals to determine the appropriate input range.

3. Right-click on the VI to get the configuration panel (if you don't do this in the initial configuration) and set the input limits to something that is appropriate for the data you will be acquiring.

   If you choose a range that is too small, it will clip your data. If you choose a range that is too large, it will lead to poor results due to digitization noise.

4. The connectors on the USB-6221-BNC and the PCI-6040E with the BNC-2120 are configured for *differential* mode only. Your analog input VI must also be configured for differential mode. You should verify that the `Terminal Configuration` option in the lower-right of the `DAQ Assistant` Express VI configuration panel is set to `differential`.

5. If you don't need to read rapid changes in T, you can average over many points (maybe for a second or so depending on the rate your temperature is changing) before displaying T. Averaging many samples is especially useful when calibrating the temperature sensor to reduce the noise in your readings (when you are acquiring calibration data the temperature should no longer be changing). Set the `Acquisition Mode` to "N Samples" and the `Number of Samples` and `Sample Rate` to appropriate values for your experiment.

- Since one of the items included in the instructions for the paper is to include estimated uncertainties in your results, use the `Std Deviation and Variance` VI to get both the mean value and the standard deviation of the acquired voltages. For help on properly handling the uncertainties and propagating them through any calculations, you should see "Measurements and Errors" (`Content` $\Rightarrow$ `LabVIEW Basics Course` $\Rightarrow$ `Measurements, Uncertainties, and Noise` in Learning Suite). Especially note what needs to be done if you average N values to get the Mean from this VI.

- If you wish to wait for a specified time between temperature readings you have a couple of options. One is the `Wait Until Next ms Multiple` that waits the specified number of milliseconds. Another is the `Time Delay` express VI for which you specify the number of seconds to wait. In both cases, you just add the timing VI inside your while loop. The VI does not need to connect to any other VI in the loop. The loop will not continue until all code contained within the loop is completed; you will wait *at least* the specified time per iteration of

the loop (you may wait longer if the other code in the loop takes longer to finish than the timing VI).

- Add a `Write Delimited Spreadsheet` VI to save your data in a file. To properly set up this VI

  1. Increase the number of significant figures for data saved by right-clicking on the `format` terminal (on the top of the VI) and creating a constant with the value "%.6f" (fixed-point notation with six digits to the right of the decimal) or "%.6e" (scientific notation with six digits to the right of the decimal – this one is preferred). This is critical when you are taking small signal data as you eventually will do in the superconductivity measurements.

     Keeping more digits than the measurement justifies is not a problem – you can always throw away the digits that are beyond the actual precision of the measurement. You can't recover significant digits that you didn't save.
  2. The default column delimiter is a tab character. If you wish to use a different delimiter you can change it with the "delimiter" input along the bottom of the VI. A reasonable alternative is to use the comma (",") character as the delimiter.

     Excel uses the ".txt" extension for tab-delimited files and the ".csv" extension for comma-delimited files.
  3. If you do not include an output file name (upper left-hand corner), you will be prompted for a file name every time the spreadsheet VI is called. If you put this VI inside a while loop, provide it with a file name, or it will prompt for a name on every iteration of the loop.
  4. Set the "append to file" input to an appropriate value. If this is set to false, the VI will create a new file (and delete the old copy) every time it is called. If your spreadsheet VI is inside a loop, set this input to true.
  5. The files are most useful if the data are in column format. The `transpose` input on the bottom of the VI will allow you to change the file format if the data are not in the desired column format.

- Some of the tutorials have you use the `Write to Measurement File` VI for saving data. The difficulty of using this arises from the available output file formats. These formats are not a portable as the tab-delimited or comma-delimited files.

  - `Text File (LVM)` – create a text-based "LabVIEW Measurement File." The data are in columns with significant text headers in the file. With a proper choice of parameters it is possible to get something that Logger Pro, MATLAB, or *Mathematica* can read if you are willing to modify the file some.

    The format is discussed in http://www.ni.com/tutorial/4139/en/.

- Binary (TDM) –a binary LabVIEW Measurement File containing the same information as an LVM file with an XML header.
- Binary (TDMS) – a binary file in LabVIEW "TDM Streaming" format. This is a modification of the TDM file.

  The format is discussed in http://www.ni.com/white-paper/5696/en/.
- Excel (xlsx) – a binary file in Excel format. It can be read by Excel and converted to other format that may be more appropriate for a particular analysis program.

- Since you are recording data as a function of time, it is useful to record the elapsed time in one column of your spreadsheet file. You do this by combining the time and temperature into an array using the `Build Array` VI. You can expand the `Build Array` VI to have more inputs if you wish to include the standard deviation of the value.

11.1 Diode calibration: analysis questions and requests

1. How much does the constant b differ from E_g/q? E_g is the band-gap energy of the silicon used in the diode. It is approximately 1.19 eV. q is the charge on the electron. The constant b is from the model equation $V = m\,T + b$ with T in Kelvins. (Refer to Section 1 of the handout `Using a semiconductor diode as a temperature sensing device` found on Learning Suite in `Content ⇒ Thermal Measurements Lab ⇒ Using a Diode as a Thermometer` for the derivation of this equation.) If your equation is in a different form or uses different units, you will have to rearrange your equation in this form to determine b from your fitting constants.

2. How far off is the calibrated diode temperature measurement at each of the three calibration points? Give your response in degrees. This is found by looking at the temperatures predicted from the measured voltages by your calibration equation and comparing them to the corresponding measured temperatures.

3. Is this error consistent with the estimated uncertainty of your calibration constants? (see the temperature uncertainty section of the "Using a Semiconductor Diode As a Temperature Sensing Device" (`Content ⇒ Thermal Measurements Lab ⇒ Using a Diode as a Thermometer` in Learning Suite). You may also want to look at the *Mathematica* notebook SuperconductorParametersAndUncertainty.nb in `Content ⇒ Superconductivity Measurements Lab` on Learning Suite or at https://www.physics.byu.edu/courses/experimental/docs/Physics240/SuperconductorParametersAndUncertainty.nb. for help in finding the uncertainty in your temperature measurements.

   Look at the second section in the notebook: "Calculate the uncertainty in a temperature determined from a diode"; you will need to enter appropriate values for the parameters and data if you use this notebook.

4. How would you improve your calibration?

5. Include a plot of your calibration data and fit – *including the calibration constants* – in your notebook and in your report.

12.0 Measuring the thermal parameters of the cryostat assembly

12.1 Determining the heating and cooling rate of the cryostat assembly

Eventually, you will be asked to design and build a temperature controller for your cryostat assembly using LabVIEW. To design a controller, it is useful to know how the heater block assembly behaves in cooling and heating. Since every assembly is different, it is necessary to measure the behavior of the block assembly you will use in the controller experiments.

You should have already constructed a heater on your heater block, built a VI to record the temperature of the block as a function of time, and calibrated a 1N4148 diode for temperature measurements.

- Support the heating block with the cooling rod in ice water in an insulated container. Be careful that the diode and its leads are not immersed in the water as that will modify the apparent temperature of the block by allowing some of the current to flow through the water instead of flowing through the diode.

- (***Do not use your analog output on the analog I/O module to drive the heater - it can't deliver the necessary current***).

- Connect your heater to the large Kepco ATE36-8 power supply.

  Your goal is to heat the cryostat to about $120\,°C$. The applied power should be low enough to take several minutes for the cryostat to reach this temperature and high enough to get to the desired temperature. About 70-90 W is a good starting point.

  Determine the settings for voltage or current that will apply a known power to the heater. You can set either the current or voltage to be the controlling value in the power supply. The other output parameter will be determined by the resistance of the heater and Ohm's Law. The knob for the other output parameter will be set to some large value, so it won't control the output.

- Simultaneously measure the current and voltage to calculate the power. ***Do not trust the meters on a power supply until you have verified that they are accurate.***
- While recording the temperature of the aluminum heater block vs. time (take at least one temperature reading every 3 seconds), apply a known power (in other words, a power that you have set and recorded) to heat the heater block assembly
- Take heating data until you reach about 120 °C. Turn off the supply and take about 20 minutes of cooling data.
- Save your heating and cooling data to disk and print plots of $T(t)$ for heating and cooling (these will be useful items to put in your lab notebook).
- Remember to report your data in the technical report on the experiment.

12.2 A model for analyzing the heating/cooling data

As noted above, we are interested in estimating the heat capacity, cooling coefficient, and equilibrium temperature for the heater block assembly. We need a mathematical model that will relate the measured behavior of the heater block to the desired parameters.

The basic ideas you will use to model the assembly are Newton's law of cooling and the concept of heat capacity. For simplicity, we will assume this is a one-dimensional system. Since our system is a rather complex three-dimensional one, this is a dubious assumption, but a full model is beyond the scope of this class.

Newton's law of cooling: The flow of heat out of or into a body is given by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\gamma(T - T_{\mathrm{eq}}) \ .$$

$\mathrm{d}Q/\mathrm{d}t$ is energy flow. γ, the cooling coefficient, is a constant related to the cross-sectional area, the thickness, and the thermal conductivity of the material between the hot and cold regions. T is the temperature of the body and T_{eq} is the temperature when the body is in equilibrium with the surroundings.

Heating and cooling: If you are also providing heat to the body, which you will do with your heater, you will need to add that term to the equation and the above equation becomes

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = P - \gamma(T - T_{\mathrm{eq}})$$

where P is the *power* being put into the body. Note that T_{eq} is the equilibrium temperature when $P = 0$ so it would have the same value irrespective of the value of P.

The definition of heat capacity, C:

$$\mathrm{d}Q = C\,\mathrm{d}T\;.$$

Recall that the heat capacity depends on the mass of the object and the specific heat(s) of the material(s) of which it is made.

Combining these equations you get

$$C\frac{\mathrm{d}T}{\mathrm{d}t} = P - \gamma(T - T_{eq})\;.$$

Using your data from heating and cooling the cryostat you can fit the **solution** of this differential equation to your data and determine the constants C, γ, and T_{eq}. Note that you have to have both the heating and the cooling data to be able to uniquely determine C and γ.

12.3 Fitting a theoretical model to your experimental results

The values of the constants C, γ, and T_{eq} for your heater block assembly will be determined by fitting the model from the previous section to the measured behavior of the heater block.

A warning regarding the process of fitting parameters: if you have parameters that show up only in simple algebraic relationships involving only those parameters and constants, the fitting process can choose any values that provide the same result from the algebraic relationship.

For instance, you will find the ratio γ/C with no other occurrence of either of the parameters γ or C in one of the model equations. The fitting process can arrive at any values of γ and C as long as the ratio has the correct value. You *can not* get any information about the individual parameters in this case.

Instructions for fitting your data to the model equation are given in the `Content` $\Rightarrow$ `Thermal Measurements Lab` $\Rightarrow$ `Fitting Curves to Experimental Data` section in Learning Suite or on the class web page. There are instructions for using four different software packages that provide nonlinear fitting capabilities (the model equation is very definitely nonlinear). Logger Pro, *Mathematica*, MATLAB, and Python all

provide reasonable capabilities for nonlinear equations and for estimating the uncertainties in the fitting parameters. EXCEL can model the cooling curve but will not be able to model the heating curve without significant manual help. In addition EXCEL does not provide estimated uncertainties in the fitting parameters.

12.4 Thermal parameters: analysis questions and requests

1. What is the solution to the equation that governs $T(t)$ with both the heating and the cooling terms present?

2. What are the values you obtained for C, γ, and T_{eq} for the curves? What are the estimated uncertainties in each of these values?

3. Do the constants agree well for the heating and cooling curves? Is that expected?

4. How good are the fits and how should you quantify the goodness of the fits?

   Strong Hint: a reasonable way to at least subjectively show the quality of your fit is to have a graph of the equation for the fit on the same plot with the experimental data so that it is easy to see if the line does or does not pass through the data in a reasonable way.

5. From your data, try to estimate the thermal conductivity (represented by either κ or k) of the copper rod, from $\gamma = \kappa\, A/L$ where A is the cross-sectional area of the rod and L is roughly the length of the rod between the bottom of the heater block and the surface of the water. If needed, review an introductory physics text or the hyperphysics website http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html on heat conduction (Heat and Thermodynamics $\Rightarrow$ Conduction).

6. How does κ compare with values of some materials in the accompanying table?

Material	Typical thermal conductivity (W/(m $\cdot^\circ$ C))
Copper	401
Aluminum	237
Stainless steel	50
Steel (common)	20
Glass	0.8
Fiberglass batt	0.04
Wood	0.1
Styrofoam	0.03

[Modified: February 12, 2019]