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Minimal Quantum Electrodynamics

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Abstract

A simple and coherent formulation of quantum electrodynamics is obtained within the general framework of the LSZ field theory. The commutation relations for the intereacting fields are obtained rather than being postulated a priori and the current densities fulfill the one particle stability conditions. Thus, the inconsistencies which appear in the canonical formalism are avoided. The resulting spectral representations do not have any ambiguities so that we do not have to introduce the "renormalization" concept.

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1.INTRODUCTION.

The great sucess of quantum electrodynamics (QED) in providing the necessary means for making quantitative predictions of physical data [1], is partially obscured by the ultimate problems QED faces. The Lagrangian approach and the perturbation method, both in the interaction picture [2] and in the Heisenberg picture [3], have run up against difficulties of principle that have been solved, at most, from a practical point of view. The canonical formalism implicitly assumes that all the results in the theory should follow from the postulated Lagrangian and the field (anti-)commutation relations. However, the product of operators at equal spacetime events is ill-defined [4] so that the equal time commutation relations for the interacting fields cannot be assumed in advance in a consistent fashion, thus questioning not just the existence of the interaction picture but also of a well defined Lagrangian in Heisenberg's picture. As a consequence, QED in the canonical formalism cannot be regarded as a conceptually consistent, physically complete relativistic quantum field theory.

Taking that into account, the axiomatic approaches arose not just in the natural search of generality but,most of all, trying to achieve the neccessary logical coherence in quantum field theories. Thus,very general assumptions which seem to be compatible to one another, are made. These include basic properties of the physical state taken as a Hilbert space and transformations therein, causality, locality, uniqueness of the vacuum state and asymptotic completeness. In the LSZ version [5-7], there is a relationship between the interacting field (or interpolating field) and a corresponding free field (in or out) through the asymptotic boundary conditions. These schemes usually leave open the question of how to build a specific theory, e.g. QED, since their interest lies in defining a general framework.

This work is devoted to the construction of a field

theory of QED with a minimun of assumptions-which can be taken essentially as those of the LSZ formulation- and avoids the in compatibilities which appear in the canonical approach.

In section II, the notation is set and the consequences of the general assumptions that give rise to the inconsistencies of the canonical approach are clearly stated. In order to avoid them, we do not take any a priori form of the equal-time commutation relations for the interacting fields. Neither do we take the usual definition [3] for the current density and the interaction term in the equations of motion. In section III, we do show that they both follow in an unique fashion as results from our general postulates and the first order interaction which defines QED. Finally section IV contains a general discussion and comparasion with other approaches.

II. GENERAL FRAMEWORK.

QED describes the interaction of structureless charged particles(e.g. electrons) via photons. We assume the existence of Heisenberg fields for the photon and the electron $A_\mu(x), \Psi(x)$. These fields tend asymptotically to free incoming fields $a_\mu(x)$, $\phi(x)$ when $t \longrightarrow -\infty$ and free outgoing fields $a_\mu^{out}(x), \phi^{out}(x)$ when $t \longrightarrow +\infty$. They both fulfill the sourceless equations:

$$K_x a_\mu(x) \equiv \partial_\lambda \partial^\lambda a_\mu(x) = 0 \qquad (2.1)_a$$

$$\mathcal{D}_x \phi(x) \equiv (i\gamma_\lambda \partial^\lambda - m)\phi(x) = 0 \qquad (2.1)_b$$

in the Lorentz gauge, the usual commutation relations for the photon free field:

$$[a_\mu(x), a_\nu(x)] = -i D_{\mu\nu}(x-x'), \qquad (2.2)_a$$

and anticommutation relations for the electron-positron field:

$$\{\bar{\phi}_\alpha(x), \phi_\beta(x')\} = -i S_{\alpha\beta}(x-x'). \qquad (2.2)_b$$

Here D and S are the Jordan-Pauli functions.

The in fields are unitarily related to the out fields by the S-matrix

$$S^{-1} a_\mu(x) S = a_\mu^{out}(x), \qquad (2.3)_a$$

$$S^{-1} \phi(x) S = \phi^{out}(x) \qquad (2.3)_b$$

which, assuming asymptotic completeness in the usual sense [5-9], can be expanded in terms of normal ordered products of the in fields

$$S = 1 + \sum_{m,n=1}^{\infty} \frac{i^n(-)^m}{n!(m!)^2} \int C_{m,n}(x_1,\dots,x_m;y_1,\dots,y_m;z_1,\dots,z_n) : \bar{\Psi}(x_1)\cdots\bar{\Psi}(x_m)\Psi(y_1)\cdots\Psi(y_m)a_{\mu_1}(z_1)\cdots a_{\mu_n}(z_n) : \quad (2.4)$$

The expansion coefficients are given by the reduction formulae [5,10,11] in terms of the 2m+n-point propagator

$$C_{m,n}(x;y;z) = \mathcal{D}_1\cdots\mathcal{D}_m K_1\cdots K_n \langle T\bar{\Psi}(x_1)\cdots\Psi(y_1)\cdots A_{\mu_1}(z_1)\cdots\rangle \overleftarrow{\mathcal{D}}_1\cdots\overleftarrow{\mathcal{D}}_m . \quad (2.5)$$

It can be shown [8,9] that defining the current densities $j_\mu(x)$ and $f(x)$ from the S matrix as

$$j^\mu(x) = i\left[\delta S/\delta a^{out}_\mu(x)\right] S^+, \quad (2.6)_a$$

$$f(x) = i\left[\delta S/\delta \bar{\phi}^{out}(x)\right] S^+ \quad (2.6)_b$$

the Heisenberg fields $A_\mu(x)$, $\Psi(x)$ obey the equations of motion

$$K_x A_\mu(x) = e\, j_\mu(x), \quad (2.7)_a$$

$$\mathcal{D}_x \Psi(x) = e\, f(x) \quad (2.7)_b$$

in the Lorentz gauge. The integral form of these equations in terms of the in fields is [1,2]

$$A_\mu(x) = a_\mu(x) - e\int D^{ret}_{\mu\nu}(x-x') j^\nu(x')dx', \quad (2.8)_a$$

$$\Psi(x) = \phi(x) + e\int S^{ret}(x-x') f(x')dx'. \quad (2.8)_b$$

Now, taking into account the relativistic covariance of the theory as well as the existence of a unique vacuum, it can be shown [4,9] that the current densities must fulfill the stability conditions

$$\langle 0|\, j_\mu(x)\,|1\ ph.\rangle = 0 \quad (2.9)_a$$

$$\langle 0|\, f(x)\,|1\ el.\rangle = 0 \quad (2.9)_b$$

Notice that these conditions are not fulfilled by the canonical current densities. We may also observe that a similar relationship, related to the vacuum stability, gives rise to the need of introducing normal ordering in the definition of the free electron current . In fact the free currents

$$e\, j_\mu^{(\omega)}(x) = e : \bar{\phi}(x) \gamma_\mu \phi(x) : , \qquad (2.10)_a$$

$$e\, f^{(\omega)}(x) = e\, A(x) \phi(x) \qquad (2.10)_b$$

satisfy the identities

$$\langle 0 | j_\mu^{(\omega)}(x) | 0 \rangle = 0 \quad ; \quad \langle 0 | j_\mu^{(\omega)}(x) | 1ph \rangle = 0 , \qquad (2.11)_a$$

$$\langle 0 | f^{(\omega)}(x) | 0 \rangle = 0 \quad ; \quad \langle 0 | f^{(\omega)}(x) | 1el. \rangle = 0 \qquad (2.11)_b$$

so that in a coupling constant expansion of the ineracting currents, they can be taken to be the lowest order terms. Equations (2.10) then correspond to chosing the elementary vertex to be such that

$$S^{(\omega)} = -ie \int : \bar{\phi}(x) \gamma_\mu \phi(x) a^\mu(x) : dx \qquad (2.12)$$

This is, of course, the "minimal coupling" interaction. It is local and yields a gauge inavariant theory. As it is well known, it is also C, P and T invariant.

We have then found that the canonical currents contradict our general postulates. What about the equal time commutation relations? That they are ill-defined is a well known fact[6,9]. In particular Haag's theorem[9,13] shows that they cannot be assumed to be the canonical ones. These apparently negative result may be seen from a different point of view. We may ask how much the general postulates restrict the commutation rules. Or even better, given the elementary interaction and the general assumptions, can the commutation rules be uniquely determined? As already anticipated in the introduction, the answer is affirmative and part of purpose of the next section is precisely to show it.

III. CONSTRUCTION OF QED.

Given the specific form of the elementary interaction, we can already calculate tree diagrams. It is just necesssary to calculate the appropiate functional derivatives[11,14,15]. The derivative with respect to a_μ, e.g., is obtained by introducing an infinitesimal change $a_\mu \longrightarrow a_\mu + \delta a_\mu$, taking δa_μ as an external field.

Consider then the equation of motion $(2.8)_b$ to lowest order we then get[3]

$$\mathcal{D}_x \, \delta S^c(x,y) = e \int \delta \mathcal{A}(\xi) S^c(\xi,y) \, d\xi \,, \tag{3.1}$$

hence the first functional derivative yields

$$\frac{\delta S^c(x,y)}{\delta a_\mu(\xi)} = e \, S^c(x,\xi) \gamma^\mu S^c(\xi,y). \tag{3.2}$$

This gives an extra elementary vertex with a factor e; functional differentiation increases in one order a perturbative term. Hence, the second derivative will give the lowest order Compton scattering term:

$$\frac{\delta^2 S^c(x,y)}{\delta a_\mu(\xi) \delta a_\nu(\eta)} = e^2 \left[S^c(x,\xi) \gamma^\nu S^c(\xi,\eta) \gamma^\mu S^c(\eta,y) + S^c(x,\eta) \gamma^\mu S^c(\eta,\xi) \gamma^\nu S^c(\xi,y) \right]; \tag{3.3}$$

setting $\delta a_\mu = 0$, and using the reduction formula $(2.5)_b$ together with equation (3.1) we get

$$S^{(2)}_{\text{Compton}} = -\frac{ie^2}{2} \int\int \left[\bar{\phi}(\xi) \mathcal{A}(\xi) S^c(\xi-\eta) \mathcal{A}(\eta) \phi(\eta) : - : \mathcal{A}(\xi) \phi(\xi) S^c(\eta-\xi) \bar{\phi}(\eta) \mathcal{A}(\eta) : \right] d\xi \, d\eta \tag{3.4}$$

The other tree diagrams can be obtained in a similar way. Additional functional derivatives give rise to higher order terms with additional vertices. Extra electron terms are obtained by taking functional derivatives with respect to ϕ and $\bar{\phi}$, in a well known procedure.

However, since functional differentiation does not create any closed loop, we cannot obtain by this single procedure any radiative corrections. We have thus to study the general behaviour of the two electron propagators, where the loops first appear.

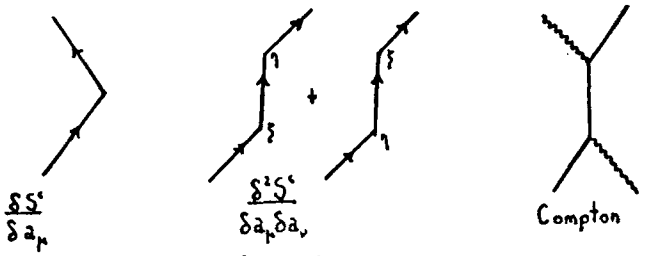


Figure 1

Two point functions.- The spectral representation for the photon and electron propagators can be found [16,17], with the aid of the integral equations (2.8) and the stability condition (2.9). We shall explicitly do it for the photon propagator and, at the end, it will be self evident that an entirely similar result holds for the exact electron propagator .

The exact photon propagator is defined as

$$\mathcal{D}^c_{\mu\nu}(x_1-x_2) = i\langle TA_\mu(x_1)A_\nu(x_2)\rangle. \tag{3.5}$$

In order to have a correct relativistic definition of the T product we introduce the scalar function:

$$\mathcal{D}^c(x_1-x_2) = \tfrac{i}{3}\langle TA_\mu(x_1)A^\mu(x_2)\rangle \tag{3.6}$$

and use the transversality property[10,18]:

$$\mathcal{D}^c_{\mu\nu}(k) - D^c_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)[\mathcal{D}^c(k) - D^c(k)] \tag{3.7}$$

in momentum space. Substituting now Eq.$(2.8)_a$ into Eq.(3.6)

$$\mathcal{D}^c(x_1-x_2) = D^c(x_1-x_2) + \frac{ie^2}{3}\left\{\theta(z)\int D^{ret}(x_1-y_1)D^{ret}(x_2-y_2)\langle j_\mu(y_1)j^\mu(y_2)\rangle + (1\leftrightarrow 2)\right\}; \tag{3.8}$$

$$z = t_1 - t_2 .$$

There are several points worth remarking in this expression. The first one is that the cross terms vanish, in view of the one photon stability property $(2.9)_a$. The second one is that the causal character of the propagator follows directly from its definition in terms of the T-product, so it becomes irrelevant whether we use retarded functions in the integrand, or any other photon free propagator. Thirdly, we notice that the argument in the step functions refers to the original variables, and not to the integration variables, appearing as arguments of the current densities. Finally, the current-current correlation function is a well defined function (more precisely a tempered distribution[7,19]) with well known spectral properties[10] . Its Fourier transform has the form:

$$-\tfrac{1}{3}\int\langle j_\mu(y_1)j^\mu(y_2)\rangle e^{iky}dy = J(k^2)\theta(k^0) \quad , \quad y = y_1 - y_2 \quad , \tag{3.9}$$

where J depends on k^2 only. It can be obviously calculated knowing the current density $j_\mu(y)$.

The last step to obtain the spectral representation of $\mathcal{D}^c(k)$ is to take the Fourier transform of Eq. (3.4), yielding

a convolution in ω, the conjugate variable of the time difference τ using the representation of θ :

$$\theta(\pm\tau) = \frac{i}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-i\omega\tau}}{\pm\omega + i\varepsilon}\, . \tag{3.10}$$

So we finally have:

$$\mathcal{D}^c(k) = D^c(k) - \frac{e^2}{2\pi}\int_{-\infty}^{\infty} d\omega\, \frac{J(q^2)}{q^4}\left[\frac{\theta(\omega)}{k^0-\omega+i\varepsilon} + \frac{\theta(-\omega)}{-k^0+\omega+i\varepsilon}\right] \tag{3.11}$$

where

$$q^\mu = (\omega, \vec{k})\,. \tag{3.12}$$

As we mentioned above, we see that we do not need to specify the character of the Green function $1/q^2$.

If we now change the integration variable in Eq. (3.11) to the scalar

$$\lambda \equiv q^2 = k^2 - k^{o2} + \omega^2 \tag{3.13}$$

we get

$$\mathcal{D}^c(k) = D^c(k) + \frac{e^2}{2\pi}\int_0^{\infty} \frac{J(\lambda)}{\lambda^2}\, \frac{d\lambda}{\lambda - k^2 - i\varepsilon}\, . \tag{3.14}$$

This spectral representation shows that the correct spectral density for the interacting part of the photon propagator is $J(\lambda)/\lambda^2$.

The photon proper energy function is defined through the relation [18] :

$$\mathcal{D}^c = D^c + D^c \Pi D^c \tag{3.15}_a$$

$$\Pi(k^2) = k^2\left[\mathcal{D}^c(k) - D^c(k)\right]k^2 \tag{3.15}_b$$

and the transverse projection gives the gauge invariant function:

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Pi(k^2). \tag{3.16}$$

It then follows immediately, from Eq. (3.14) and (3.15),that

$$\Pi(k^2) = \frac{e^2}{2\pi}\, k^2 \int_0^{\infty} \frac{J(\lambda)}{\lambda^2}\, \frac{1}{\lambda - k^2 - i\varepsilon}\, d\lambda\,. \tag{3.17}$$

A similar procedure for the electron propagator yields[16]

$$\mathcal{G}^c(p) = S^c(p) - \frac{e^2}{2\pi}\int_0^{\infty} \frac{\Sigma_1(\lambda) + \not{p}\,\Sigma_2(\lambda)}{(\lambda - m^2)^2(\lambda - p^2 - i\varepsilon)}\, d\lambda\,, \tag{3.18}$$

in terms of the functions

$$\Sigma_1(\lambda) = 2\lambda m\, R_1(\lambda) + (\lambda + m^2) R_2(\lambda) \tag{3.19}_a$$

$$\Sigma_2(\lambda) = (\lambda + m^2) R_1(\lambda) + 2m\, R_2(\lambda) \tag{3.19}_b$$

with

$$\mathcal{T}\langle f(y_1) f(y_2)\rangle = \theta(p^2)\left[\not{p}\, R_1(p^2) + R_2(p^2)\right]. \tag{3.20}$$

The electron proper energy

$$\Sigma(p) = (\not{p}-m)\left[G^c(p) - S^c(p)\right](\not{p}-m), \tag{3.21}$$

is hence given by :

$$\Sigma(p) = -\frac{e^2}{2\pi}(\not{p}-m)^2 \int_0^\infty \frac{\Sigma_1(\lambda) + \not{p}\,\Sigma_2(\lambda)}{(\lambda-m^2)^2(\lambda-p^2-i\varepsilon)}\, d\lambda . \tag{3.22}$$

The contribution of the two-point propagators to the S-matrix obtains from the reduction formulae, Eq. (2.5)

$$C_{0,2}(z_1,z_2) = -i\,\Pi(z_1-z_2) \tag{3.23}_a$$

and

$$C_{1,0}(x_1;y_1) = i\,\Sigma(x_1-y_1) . \tag{3.23}_b$$

These expressions <u>automatically</u> fulfill the usually <u>imposed</u> "renormalization conditions"[3,10,18],

$$\Pi(\lambda)\Big|_{\lambda=0} = \Pi'(\lambda)\Big|_{\lambda=0} = 0, \tag{3.24}_a$$

$$\Sigma(p)\Big|_{p=m} = \Sigma'(p)\Big|_{p=m} = 0. \tag{3.24}_b$$

The origin of subtraction terms in Eqs. (3.16) and (3.22) is the appearence of the inverses of K and $\mathcal{D}$ respectively when solving the equations of motion (2.7). This is in agreement with the fact that the interaction implies necessarily subs<u>tracted</u> dispersion relations [19] .

Now, we observe that, due to the e^2 factor in the spectral representations, we can calculate the lowest order radiative corrections for the two point propagators from Eqs. (2.4-5). In the case of vacuum polarization, we need to compute the j-j correlation function. To lowest order the current is

$$j_\mu^{(0)} = :\bar{\phi}\gamma_\mu\phi: \tag{2.10}_a$$

so that

$$e^2 \langle j_\mu(x_1) j^\mu(x_2) \rangle^{(2)} = e^2 \langle : \bar{\psi}(x_1) \gamma_\mu \psi(x_1) :: \bar{\psi}(x_2) \gamma^\mu \psi(x_2) : \rangle \qquad (3.25)$$

and the function J in Eq. (3.9) is , to this order[3] ,

$$e^2 J^{(2)}(\lambda) = -\frac{e^2}{3(2\pi)} \theta(\lambda - 4m^2) \sqrt{\frac{\lambda - 4m^2}{\lambda}} \left(\lambda + 2m^2\right) \qquad (3.26)$$

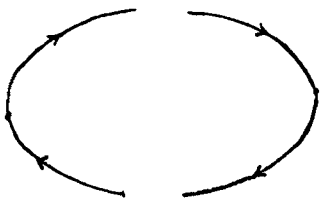
and hence:

$$\pi^{(2)}(k^2) = -\frac{e^2}{12\pi^2} k^4 \int_{4m^2}^{\infty} \frac{d\lambda}{\lambda^2} \sqrt{\frac{\lambda - 4m^2}{\lambda}} \frac{\lambda + 2m^2}{\lambda - k^2 - i\varepsilon} \; ; \qquad (3.27)$$

for small k

$$\pi^{(2)}(k^2) \xrightarrow{k^2 \to 0} - \frac{\alpha}{15\pi m^2} \qquad (3.28)$$

where $\alpha = e^2/4\pi$ is the fine structure constant.



$e^2 \quad < jj >$

Figure 2

Thus, a loop is formed through the correlation function. However, since there is no multiplication by step functions, The product in Eq. (3.25) is well defined as a product of distributions[7] . The diagram in Fig. 2 is then similar to the one used in S-matrix theory[20] . However, in the present work there are no assumptions about the analytic properties, and the substractions in the dispersion relations , Eq(3.17) are derived, instead of being imposed

A similar calculation gives the radiative correction to the electron propagator[3, 21] . Again a loop is formed in the computation of the f-f correlation function. In general, all the new loops result from the contractions appearing in the spectral densities $J(\lambda), \Sigma_1(\lambda), \Sigma_2(\lambda)$. These are not time ordered functions and are well behaved.

$e^2 \langle f\bar{f} \rangle$

Figure 3

Iteration.- We have thus shown that the second order terms can be computed just on the basis of our general assumptions and the elementary vertex. The higher order terms can be computed by iteration: from the two and four-point propagators to second order, we construct the S-matrix, and hence the currents to the same order, using Eqs. (2.4-6). At the same time, we can compute the third order contributions to the vertex by taking the functional derivative $\delta\Sigma/\delta a_\mu$, which yields the S-matrix to third order. Substituing these expressions for the currents in the spectral representatoions for the photon and electron proper energies, Eqs (3.17) and (3.22), gives their correct expression to fourth order, and the process can be iterated once the other n-propagators have been calculated to that order by functional differentiation. The result obtained in this way concide with the usual perturbation expansion, once the latter has been properly renormalized.

Commutation relations.- By the iterative procedure we have just described, QED can be constructed. Thus, the commutation relations can be calculated to the desired order. However, the procedure applied for obtaining the spectral representations of the two-point propagators can be applied to obtain similar relations for the vacuum expectation value of the (anti-)commutation relations[21]. Starting from Eq. (3.14) in configuration space

$$\mathcal{D}'(x_1-x_2) = D'(x_1-x_2) - \frac{e^2}{2\pi}\int_0^\infty \frac{J(\lambda)}{\lambda^2} D'(x_1-x_2;\lambda)\,d\lambda \qquad (3.29)$$

with

$$(\partial^2 + \lambda) D'(x;\lambda) = -\delta(x). \tag{3.30}$$

Taking the even part of Eq.(3.5), and using Eqs. (3.7) and (3.29), we get

$$\langle [A_\mu(x_1), A_\nu(x_2)] \rangle = -i \left[D_{\mu\nu}(x_1 - x_2) - \frac{e^2}{2\pi} \int_0^\infty \frac{J(\lambda)}{\lambda^2} D_{\mu\nu}(x_1 - x_2; \lambda) d\lambda \right]. \tag{3.31}$$

Taking the time derivative of Eq. (3.31) and evaluating it at equal times $t_1 = t_2$, we get

$$\langle [\partial_t A_\mu(x), A_\nu(x')] \rangle \Big|_{t=t'} = -i \delta(\vec{x} - \vec{x}') g_{\mu\nu} \left[1 - \frac{e^2}{2\pi} \int_0^\infty \frac{J(\lambda)}{\lambda^2} d\lambda \right] \tag{3.32}$$

in Loretz gauge.

Similarly, for the electron anticommutation relation, we get

$$\langle \{ \Psi_\alpha(x), \bar{\Psi}_\beta(x') \} \rangle \Big|_{t=t'} = \gamma^0_{\alpha\beta} \delta(\vec{x} - \vec{x}') \left[1 + \frac{e^2}{2\pi} \int_0^\infty \frac{\sqrt{\lambda}\, \Sigma_1(\lambda) + \Sigma_2(\lambda)}{(\lambda - m^2)^2} d\lambda \right] \tag{3.33}$$

Just as expected, the commutation relations depend on the interaction. Furthermore, since the delta functions factor out, we are left with a divergent constant multiplying the delta function, in both cases.

IV. CONCLUSION.

A coherent and systematic picture of QED has been obtained by defining current densities which obey the one-particle stability condition, and taking the integral form of the equations of motion as a starting point. The commutation relations for the interacting fields are computed a posteriori, instead of being assumed, as it is done in canonical theory. In this way we work with "renormalized" fields from the very beginning. The resulting expressions for the two-point propagators turn out to be the correct ones. The rest of the n-point functions are computed by taking the appropiate functional derivatives.

In our procedure, there is no need to impose any additional conditions, either on the mass shell [27], or for large momenta [23]. The electron and photon propagators fulfill automatically the "renormalization conditions [2,10], while the other propagators inherit

the correct boundary conditions by functional differentiation, and no quasilocal operators remain undetermined

It is also very clear that the starting point for the chain of functional derivatives should not be the vacuum to vacuum amplitude. Instead, we must start from the two-point propagators. The reason is apparently, that in this way we defined the interaction in a unique way[19] , and compute the modification suffered by the free propagators due to the interaction.

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