Some experiments with the bass drum

Harvey Fletcher and Irvin G. Bassett

Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84602
(Received 6 December 1976; revised 12 July 1978)

Measurements were made of bass drum tones to determine the characteristic properties of the sound. The analysis was performed with computer-modeled band-pass filters with 3-Hz bandwidth. Frequency, peak sound pressure level, and decay rate were determined for each of the major components from 40 Hz up to 1000 Hz. The frequencies of the components were found to change with time, decreasing as amplitude of vibration decreases. Observed frequencies agreed with theoretical values to within 5% in some cases and less than 1% in most. It was found that a blow near the center of the drum puts more energy into the lower-frequency components, while a blow near the edge emphasizes those of higher-frequencies. Drum tones were synthesized by summing the contributions of each of the important components. Recordings of synthesized and real tones were presented randomly in listening tests. Synthesis was viewed as successful when the synthetic tones were judged to be real as often as they were judged to be synthetic. In the listening tests, a jury of 31 people correctly identified the synthetic tones only 51% of the times those tones were presented.

PACS numbers: 43.75.Hi

INTRODUCTION

Not much research has been reported on the bass drum, although considerable theoretical work has been done on the vibrations of membranes. The kettle drum has received some attention,[1,2] since its sound has a more musical quality, and a definite pitch can be associated with it. An Indian drum with harmonic partials, resulting from a loaded membrane, was studied and described by investigators in India.[3-5] In 1935, Obata and Tesima[6] reported research on two Japanese drums that resemble the orchestral bass drum. Sivian, Dunn, and White[7] included a brief analysis of the bass drum in their study of the orchestral instruments reported in 1931. Corrections of the results of that paper were later made by Young and Dunn[8] and by Dunn.[9]

The bass drum sound has elements of both percussive (impulsive) sound and of steady-state sound, and might be treated either way. The vibration patterns of the heads produce discrete frequencies. Some of these frequency components last for several seconds after the blow is struck. This study approaches the analysis from the point of view of a quasi-steady-state signal, considering the sound to be made up of a series of single-frequency, "pseudo-sinusoidal" components having time decaying amplitude. The methods of analysis and synthesis follow the same general lines as those of previous work with piano tones.[10]

The work reported here investigates in some detail the characteristics of the bass drum sound, showing the relationships between the theoretical and observed components. The reliability of the analysis is tested by creating synthetic tones from the data produced in the analysis, and comparing the synthetic with recordings of the real sounds.

I. THEORETICAL CONSIDERATIONS

The bass drum can be described as a short circular tube of rather large diameter which is closed at both ends by stretched membranes. These membranes are set into vibration by striking either one or both of them with suitable drum sticks or padded mallets. The spectrum of the tone emitted changes with time and depends upon the location on the membrane where the blow is struck and the vigor of the blow.

There are three main parts of the drum that are set into vibration, namely, the two membranes and the volume of air between the membranes. A resistive damping load is provided by the external air in contact with the membrane. Lord Rayleigh[1] and others have worked out the mathematical relations that describe the vibration when a single drum membrane is vibrating in the air, provided the following assumptions are made:

(1) The membrane is circular and is clamped firmly at a radius a.

(2) The membrane has a uniform tension T and surface density σ over the entire surface.

(3) The restoring force when the membrane is deflected is due entirely to the tension of the membrane. Effects due to the air compressed on either side of the membrane are assumed to be negligible.

Rayleigh's derivation leads immediately to the Bessel function of the first kind, designated $J_m(x)$, where

$$x = 2\pi f a (\sigma/T)^{1/2}. \qquad (1)$$

Under the conditions stated above, the allowed frequencies of vibration f can be found from the values of x for which the Bessel function equals zero.

For each order m of the Bessel function there are many values of x which make $J_m(x) = 0$. These values of x will be designation x_{mn}. The integers m and n correspond to the number of diametrical and circular nodal lines, respectively.

The possible resonance frequencies of a single circular membrane, that is fastened securely along its circumference, are given by

$$f_{mn} = (1/2\pi a)(T/\sigma)^{1/2} x_{mn}. \qquad (2)$$

The fundamental or lowest resonance frequency (with

 0001-4966/78/6406-1570$00.80

TABLE I. First one hundred theoretical frequencies of the various modes of vibration of a single membrane where the fundamental frequency is 40 Hz. The integers m and n designate the number of diametrical and circular nodes, respectively.

Freq. (Hz)	m	n	Freq. (Hz)	m	n	Freq. (Hz)	m	n	Freq. (Hz)	m	n
40.0	0	1	246.1	2	4	345.8	12	2	414.7	7	5
63.7	1	1	246.5	7	2	346.1	9	3	422.8	16	2
85.4	2	1	248.3	0	5	346.4	4	5	422.8	20	1
91.8	0	2	259.3	11	1	350.7	16	1	423.0	5	6
106.1	3	1	261.1	5	3	351.2	2	6	424.3	10	4
116.7	1	2	266.8	8	2	352.8	0	7	427.6	13	3
126.2	4	1	269.8	3	4	360.0	7	4	428.3	3	7
140.0	2	2	274.0	1	5	365.2	13	2	430.9	1	8
143.9	0	3	277.7	12	1	366.7	10	3	436.9	8	5
145.9	5	1	282.8	6	3	368.8	17	1	440.7	21	1
162.4	3	2	286.8	9	2	369.6	5	5	441.8	17	2
165.3	6	1	293.0	4	4	375.6	3	6	445.3	11	4
169.2	1	3	296.1	13	1	378.6	1	7	446.1	6	6
184.0	4	2	298.7	2	5	381.7	8	4	447.6	14	3
184.4	7	1	300.6	0	6	384.5	14	2	452.4	4	7
193.3	2	3	304.2	7	3	386.8	18	1	456.1	2	8
196.1	0	4	306.6	10	2	387.2	11	3	457.3	0	9
203.3	8	1	314.4	14	1	392.3	6	5	458.5	22	1
205.2	5	2	315.7	5	4	399.5	4	6	458.8	9	5
216.5	3	3	322.8	3	5	403.1	9	4	460.7	18	2
221.6	1	4	325.3	8	3	403.7	15	2	466.2	12	4
222.1	9	1	326.3	1	6	403.7	2	7	467.4	15	3
226.0	6	2	326.3	11	2	404.8	19	1	468.9	7	6
239.1	4	3	332.5	15	1	405.1	0	8	476.2	5	7
240.8	10	1	338.0	6	4	407.4	12	3	476.4	23	1

one node at the outer edge) is given by

$$f_{01} = (1/2\pi a)\,(T/\sigma)^{1/2}\,x_{01}. \tag{3}$$

Let the ratio of the frequency f_{mn} to the fundamental be R_{mn}. Then,

$$R_{mn} = f_{mn}/f_{01} = x_{mn}/x_{01}. \tag{4}$$

The fundamental frequency of the bass drum in this experiment was about 40 Hz so that

$$f_{mn} = 40 R_{mn}. \tag{5}$$

The values of x_{mn} can be obtained from a table of Bessel functions and the values of R_{mn} calculated. The values of f_{mn} are then obtained from Eq. (5). When all the possible vibration patterns are taken into account for a membrane whose fundamental frequency is 40 Hz, there are more than 435 resonance frequencies between 40 and 1000 Hz. The first one hundred of these are given in Table I, so that a comparison can be made with the values found in the analysis.

In the experiments described below, the two membranes of the bass drum were tuned so that they had about the same fundamental frequency (40 Hz). For the lowest and most prominent frequencies generated by the drum, the wavelength in the air space between the two membranes is large compared to their distance of separation, and it would be expected that the three vibrating parts would vibrate in phase. In this approximation the values of component frequencies which are observed should be approximately those frequencies calculated from Eq. (5). However, there are a number of perturbations which will tend to make actual modal frequencies differ from the simple theoretical values. Since the membranes are quite closely coupled, a splitting of the resonances could occur and give rise to frequencies other than those predicted for the single membrane case. Further, the calfskin from which the heads are made is not completely uniform in texture. Also the tension applied around the outer edge is likely not uniform.

II. EXPERIMENTAL MEASUREMENTS

The bass drum used in this study had a membrane diameter of 0.90 m. The distance between the membranes was 0.41 m. A measurement of a similar calfskin membrane gave a surface density of $\sigma = 0.412$ kg/m^2.

The drum was set up in an anechoic chamber with a microphone suspended 1.0 m from the struck side of the drum. This microphone was on an axis perpendicular to the face of the drum. The signal went from the microphone through a preamplifier and an isolation transformer to the analog-to-digital converter of the computer. The signal was digitized at a rate of 10 000 15-bit samples per second. The peak sound pressure level of each blow was measured by a General Radio 1933 sound level meter, located one meter from the struck side of the drum and 0.3 m off axis. The sound level meter was set for peak reading "flat" electrical response (5 Hz–100 kHz). Three blows were recorded and identified as R_1, R_2, and R_3. The peak sound pressure levels were 121, 120, and 112 dB, respectively, for the three blows.

A preliminary analysis of the sound showed that the frequency of the components changed with time, decreasing from the instant the drum was struck. The frequency changed rapidly during the first second or two, and then leveled off as the sound died away. A typical change with time for the fundamental is shown in Fig. 1. The frequency change for several higher components was measured, and found to be proportional to the change of the fundamental. In a study of this phenomenon, Cahoon[11] showed that there was an increase in the effective tension for large deflections of the membrane, and that the restoring force was not directly proportional to the deflection of the membrane. He showed that the fundamental frequency could be expressed by:

$$f_{01} = (2.405/2\pi a)\left[(T_0 + kd^2)/\sigma\right]^{1/2}, \tag{6}$$

where a is the radius of the drum head, T_0 is the tension at equilibrium, σ is the average surface density of the membrane, and k is a constant relating the increase in tension to the deflection amplitude d. For the drum used, $a = 0.450$ m, $T_0 = 9.11\times10^2$ N/m, $k = 4.72\times10^6$ N/m^3, and $\sigma = 0.412$ kg/m^2. If these values are introduced into Eq. (6), the fundamental frequency is

$$f_{01} = 13.25\,(9.11 + 4.72\times10^4 d^2)^{1/2}. \tag{7}$$

For very small displacements ($d \to 0$) Eq. (7) yields $f_{01} = 40$ Hz. A hard blow on the drum will produce dis-

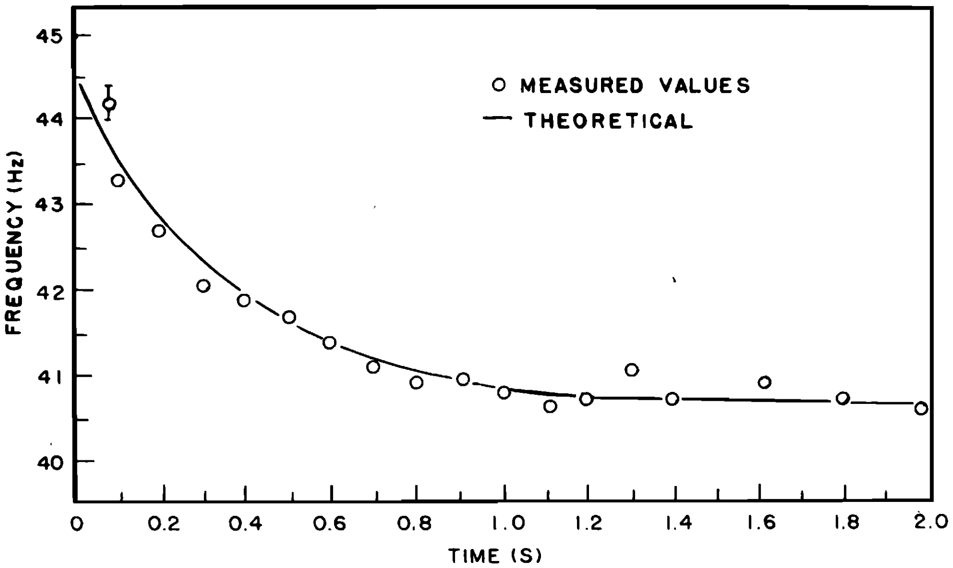

FIG. 1. Change of frequency with time when the drum is struck a hard blow.

placements of at least 6 mm, resulting in a frequency shift of nearly 10%.

III. ANALYSIS OF THE TONES

A compensation was made for the frequency shift described above, so that the components of the tone would have constant frequencies for analysis purposes. The frequency compensation was accomplished by "time stretching" each part of the tone by an amount equal to the frequency shift for that part of the tone as shown in Fig. 1. For example, at 0.4 s into the signal, the frequency is 42 Hz, or 5% above the final value of 40 Hz. A "time stretch" of 5% was applied to this portion of the signal so as to decrease the frequency by 5% and make it equal to the final value of 40 Hz. (The actual "time stretching" was accomplished on the digitized waveform by an interpolation process.) In this way the frequencies of the fundamental and all of the higher components remained at constant values as the signal was passed through the analysis filters.

The unfiltered peak sound pressure level of the digitized version of each tone was arbitrarily adjusted to about 60 dB above the background noise level of the analysis system. This was done so that comparisons could easily be made of the relative levels for each of the various components.

Band-pass filters with 3-Hz bandwidth were modeled in a computer. The sound was analyzed in 1-Hz steps from 30 to 1000 Hz. (It was found that there was little energy in the region from 1000 to 5000 Hz, so no analyses were performed there.) At each frequency, a plot was made of the sound pressure level as a function of time. Two such plots are shown in Fig. 2. (The rise time of the envelope is an indication of the response time of the filter. By comparison, the unfiltered signal rises from zero to its peak in 5 ms.) The highest value of each plot was noted. These values were plotted as a function of frequency, and the relative maxima from among them were chosen as the major components of the sound. The decay rate for each component was determined from the general slope of its sound level versus time plot.

Several problems made the locating of the major components and the determination of their decay rates difficult. First, many occur very close together in frequency and even with a bandwidth as narrow as 3 Hz, it was not always possible to isolate a component in a filter. This interference between components in the filter produced decay curves that were far from monotonically decreasing. Figure 2(b) is a typical example. This type of interference might also be coming from coupling or from beats between the two heads, which could be slightly out of tune with each other. These problems could also contribute to less accurate values of the decay rates. Another problem arises for components that have fast decay rates. The ringing of the filter itself dies away at about 40 dB/s, so components which decay faster than this could not be measured accurately.

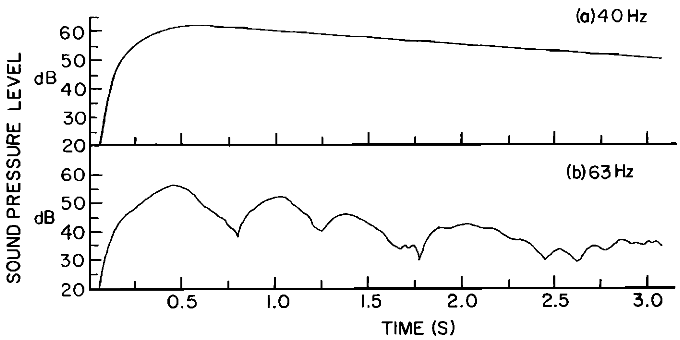

FIG. 2. Decay patterns of two components of the drum sound R_2. The tone was picked up on a microphone one meter away from the struck side of the drum, and passed through the analog to digital converter of a computer. The digitized signal was passed through 3-Hz bandwidth filters (in the computer). The initial buildup of each curve indicates the response of the filter. For the fundamental component (a) the decay is smooth. The lower curve (b) is typical of the higher frequency components. The fluctuations in level produce strong beating in the sound from interaction between neighboring modes of vibration.

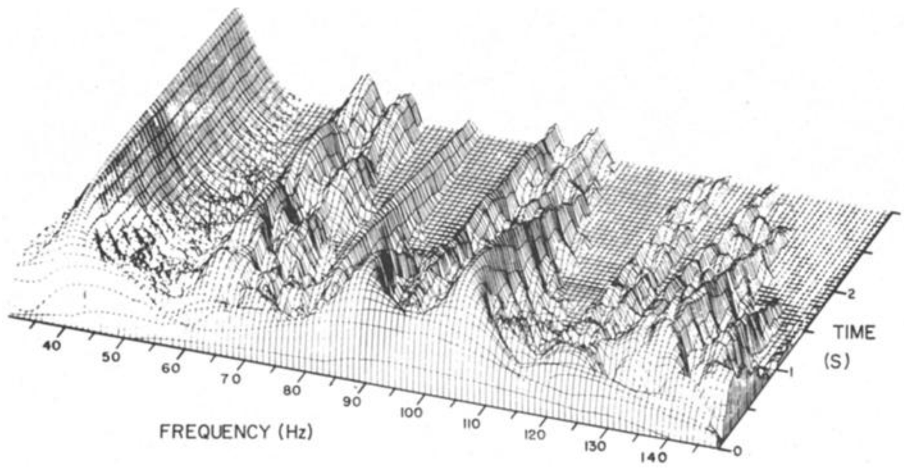

FIG. 3. Perspective plot of part of the total filtered signal of drum sound R_2. The vertical scale is logarithmic and represents the sound pressure level. The figure is a composite of the outputs of the 3-Hz bandwidth filters centered at each frequency.

A graphic picture of the total filtered signal, illustrating its complexity is given in Fig. 3. This figure is a perspective plot of part of a spectrum made from the individual filter-output curves set side by side. The result is a view of the variation of frequency and sound pressure level versus time. For the signal depicted in Fig. 3, no compensation was made for the frequency shift described above. The changing frequency is apparent in the initial portion of the tone. This plot was not used in the actual analysis, but is included here as an aid in visualizing the filtered sound pattern.

IV. MEASUREMENTS OF DECAY RATES

From the frequency, sound pressure level, and decay rate data, a preliminary tone (S_1) was synthesized. (See Sec. V for details on the synthesis.) Careful listening through headphones and via loudspeakers indicated that the most significant difference between the synthesized and the real tones was in the values used for the decay rates. Therefore, a new method was developed to get a more accurate determination of the decay rates for the major components. The drum was set in the anechoic chamber with a 15-in. loudspeaker at a distance of one meter on one side and the microphone one meter away on the other. The microphone signal was sent to a B & K type 2305 level recorder. The loudspeaker signal was used to drive the drum, scanning through the frequency range until a resonance was found. With the drum resonating at a single frequency, the loudspeaker would be shut off suddenly, leaving the drum sound to decay alone. (The decay rate of the loudspeaker was much faster than that of the drum.) The decay rate of the drum tone was then plotted by the level recorder. This method provided new values for decay rates of all of the frequencies at which the drum could be made to resonate, but not all of the components responded with sufficient energy to be detected. Those components that could be made to resonate matched closely in frequency with those of the previous analysis.

The new decay rates were used to synthesize a second drum tone. There was considerable improvement, but the tone still did not sound the same as the original. The problem here may be due to the different modes being not completely independent of each other. A point on the membrane that would be an antinode for one mode might be a node for another, and considerable interplay between partials would occur when the complete tone was sounded. The single frequency resonance data provided additional information but did not yield the complete picture.

Further refinement was accomplished by playing recordings in which real tones were followed immediately by the synthetic. These repetitions were passed through an octave-band filter, and carefully examined with loudspeaker and headphone. The octave-band analysis provided much better time resolution than the original 3-Hz bandwidth analysis, providing a more realistic determination of the faster decay rates. Through this procedure, the decay rates of the higher frequency components were modified to give more rapid decay. Using these values, another synthesis was done which produced more realistic tones. The values of frequency, relative sound pressure level, and decay rate used in the final synthesis of tones S_2 and S_3 are given in Tables II and III, respectively. Table II shows values for a sound produced when the drum is struck near the center of the drum head (tones R_2, S_2). The values in Table III are for a drum struck near the edge (tones R_3, S_3). As one might expect, the major difference between the two tones is that there is more energy in the higher frequencies for the blow struck near the edge of the drum head.

For each of the frequencies in Table II, a corresponding frequency can be found in Table I that is within a few percent of the same value. If 179.0 Hz is matched with 169.2, the difference is 5.8%. Comparing with 184 Hz, the difference is only 3%. However, there is an observed value at 183.4 Hz which is most likely the component that matches the 184.0 Hz theoretical value. The components at 95.8, 110.0, and 131.3 Hz are within 4% of the theoretical values of 91.8, 106.1, and 126.2, respectively. For the remainder, the differences are less than 1%.

The Table III data show five components (55.0, 70.0, 79.0, and 102.0) that do not have a counterpart among the theoretical values of Table I. The origin of these

TABLE II. Observed values of frequency, relative sound pressure level, and decay rate for a hard blow near the center of the drum head. The sound pressure level in each case is the maximum output of the 3-Hz bandwidth filter at that frequency.

Freq. (Hz)	Relative sound pressure level (dB)	Decay rate (dB/s)	Freq. (Hz)	Relative sound pressure level (dB)	Decay rate (dB/s)
40.0	63	4	542.0	32	39
61.9	56	7	550.5	36	19
84.4	49	64	565.4	36	50
95.8	58	24	570.6	35	34
110.0	50	23	575.0	30	29
131.3	59	38	583.0	31	50
164.0	50	28	593.8	29	49
179.0	53	24	604.0	22	22
183.4	52	20	611.2	25	60
210.0	48	17	621.5	33	40
225.0	46	16	629.5	28	29
230.0	45	21	635.7	27	47
233.6	45	26	654.7	27	96
239.8	45	10	670.4	22	37
252.0	48	37	676.0	23	48
273.2	40	32	686.1	28	74
285.0	40	22	699.7	31	70
295.0	43	40	723.0	29	30
306.0	38	19	732.8	28	54
311.0	38	12	746.0	28	50
319.2	37	41	758.0	29	27
326.0	40	21	768.0	19	45
337.0	33	21	788.0	17	43
349.2	36	32	799.0	28	86
362.2	35	49	817.0	30	47
369.6	37	18	830.0	28	50
382.0	37	11	840.0	25	33
395.2	37	26	844.0	25	78
403.0	40	19	851.0	25	155
412.0	40	25	858.0	29	115
437.0	34	49	880.0	31	150
449.8	40	22	901.1	28	74
457.6	37	53	922.0	25	195
467.8	39	52	937.0	30	250
479.5	34	32	954.1	29	145
489.6	31	31	987.0	27	150
493.9	32	40	995.0	26	145
506.3	36	32	1033.0	30	150
521.1	33	22	1056.0	24	150
526.0	33	24	1061.0	30	150
534.0	27	55	1092.0	32	150

tones is not known on a theoretical basis at this time. Since they were quite prominent in the analysis, they were included in the synthesis of tone S_3. All of the other components can be matched to corresponding values from Table I to within 1%, except 132 and 152 Hz which are nearer 5%.

Many of the same components appear in both analyses, but there are several that appear in only one or the other. Correspondingly, the two tones sound basically alike but easily recognizable differences can be heard when they are compared.

Frequencies higher than those shown in Table I are so close together that a theoretical value can always be found to match very closely each observed value.

V. SYNTHESIS

Synthesis of the drum tones was carried out on a computer. For each component a decaying sinusoid was generated whose frequency, beginning sound pressure level and decay rate were as shown in Tables II and III. The contribution of all components at each point in time

TABLE III. Observed values of frequency, relative maximum sound pressure level, and decay rate for a hard blow near the edge of the drum head.

Freq. (Hz)	Relative sound pressure level (dB)	Decay rate (dB/s)	Freq. (Hz)	Relative sound pressure level (dB)	Decay rate (dB/s)
40.0	63	4	508.5	50	32
55.0	49	6	519.0	51	22
64.0	51	7	536.0	43	55
70.0	50	20	549.0	50	39
79.0	50	25	560.0	44	50
85.5	53	32	571.0	44	34
91.0	54	23	577.0	47	29
94.0	52	24	591.0	55	49
102.0	59	25	597.5	42	35
106.0	58	23	603.0	44	22
116.5	58	20	609.7	46	60
132.0	56	28	618.0	41	40
152.0	59	23	624.6	47	35
172.0	55	20	633.0	43	29
183.0	47	15	638.0	46	47
190.0	48	15	652.7	49	96
199.0	43	15	664.6	45	60
207.0	52	13	673.0	41	37
218.0	50	13	681.0	41	48
224.0	50	13	689.0	43	70
230.0	48	16	696.0	43	70
238.0	51	18	702.0	46	70
246.0	48	25	709.0	42	54
262.0	52	22	717.0	43	30
271.0	52	20	735.0	40	54
283.0	41	22	745.0	42	50
290.0	50	21	749.0	42	39
297.0	47	26	757.4	44	27
301.0	44	15	770.0	45	45
312.0	42	12	779.3	44	44
319.0	45	26	789.0	46	43
326.0	31	16	813.4	38	47
332.0	46	16	824.0	37	50
337.0	45	16	843.4	43	78
348.0	39	22	859.0	37	115
354.0	39	26	866.0	29	135
358.0	41	26	882.8	39	150
363.0	44	33	893.0	37	100
366.0	46	20	902.0	35	74
374.0	42	18	911.3	41	150
379.0	44	11	925.0	40	195
394.0	51	26	934.0	33	250
402.0	49	19	940.0	34	200
412.0	42	25	952.0	35	145
422.0	50	35	958.0	36	145
431.0	45	49	969.0	32	150
444.0	44	22	980.0	35	150
461.0	48	53	996.0	38	145
470.0	43	52	1000.0	40	145
484.0	54	32	1012.0	37	150
494.0	48	40	1026.0	32	150
503.0	46	36	1044.0	33	150

TABLE IV. Results of listening tests for real (R) and synthetic (S) drum sounds.

Sound	No. of correct responses	No. of incorrect responses	Percent correct
R_1	119	36	77
S_1	103	52	67
R_2	83	72	54
S_2	69	86	45
R_3	81	74	52
S_3	63	92	41
Overall	518	412	56

was summed. If each sine wave were started at zero phase and increased in the positive direction, the result would be a large amplitude spike at the beginning of the sound. To prevent this spike from occurring, the phase was set at zero for the fundamental frequency, and shifted by 180° for each succeeding component. A "time compression" was imposed on the waveform at the beginning of each tone so that the frequency change shown in Fig. 1 was included in the synthesis. This "time compression" of the first part of the signal was simply the reverse of the "time stretching" used in the analysis.

A ten-second tone was computed for each synthesis. The digital representation of each tone was digital-to-analog converted (at a rate of 10 000 samples per second) and recorded on audio tape. Three synthetic tones labeled S_1, S_2, and S_3 were computed and recorded. The data used in S_2 and S_3 are shown in Tables II and III, respectively. Tone S_1 was taken from the Table II data before the final refinements were made in the decay rates. The recording level was adjusted to give maximum amplitude on the tape without distortion.

VI. LISTENING TESTS

The sound from an actual bass drum is different from the sound of the same drum played back through a tape recorder. Even the best loudspeakers have less than perfect frequency response characteristics and become directional at higher frequencies. The use of headphones instead of loudspeakers as a transducer makes the recorded sound more realistic. Since the synthetic tones needed to be presented to the listeners via headphones, it was necessary to use recordings of the real drum, rather than the actual drum tones for comparison in the tests.

Three synthetic tones and the three real tones (from which the synthetics were modeled) were recorded on audio tape from a digital-to-analog converter. A tape was made so that the six different tones were presented five times each, making a total of thirty tones in the test. The order of presentation of the tones was randomized. The tape was played back through a tape recorder, amplifier, and KOSS PRO 4 AA headphones. The output level was adjusted to give a maximum fast sound pressure level under the headphones of 112 dB for the lightest blow and 126 dB for the loudest.

The jury included 31 adults, ranging from 18 to 91 years of age including trained musicians, physicists, students, secretaries, and housewives. Although no hearing evaluation was performed, all indicated that they had normal hearing. The tests were performed following the same general guidelines as with the previous experiments.[10] The jurors were told that some of the tones were recordings of real tones and some were synthetic. Their task was to judge each tone on its own merits and decide whether it sounded real or synthetic to them. The results of the tests are shown in Table IV.

If a juror simply guessed all of the time, statistically he should achieve 50% correct responses. Tone R_1 was quite easily recognized as a real tone with 77% correct responses. Real tones R_2 and R_3 were not so easy to tell from their synthetic counterparts. They were correctly called real on 54% and 52% of the responses, respectively, just a little above the 50% guessing level. Synthetic tone S_1 was the preliminary attempt at synthesis. It lacked the proper decay rates, and was correctly picked as synthetic two-thirds of the time by the jurors. Tones S_2 and S_3 were synthesized after the best data had been obtained for the decay rates. These synthetic tones were called real more times than they were called synthetic. A decription of a tone may be considered successful when synthetic tones and real tones each produce about equal numbers of "real" and "synthetic" judgments. The overall correct response of all six tones is 56%. Averaging the correct responses of just the three synthetic tones gives a result of 51%. This is only slightly above the level of guessing.

Table V gives the confusion matrices and information theory statistics[12] for the three pairs of sounds. With an uncertainty of 1.0000 bit in the actual sound, 0.1411, 0.0003, and 0.0034 bits were transmitted in the three situations, respectively. The coefficient of constraint, $C_{x \cdot y}$ represents the proportion of the average information (uncertainty) in the judging task which is transmitted to, or picked up by the judges. In other words, only about 14% of the average information (uncertainty) associated with the first pair of tones (as to which is synthesized and which is real) is transmitted to the judges. There is less than 1% transmitted for each of the other two pairs.

VII. CONCLUSIONS

The sound from a bass drum can be considered to be made up of a series of "pseudo sinusoids" of different amplitudes, frequencies, and decay rates. Because frequency and amplitude are both changing rapidly with time, the components are not true sinusoids. However, using sine functions to generate a series of components and changing the frequency and amplitude with time it is possible to synthesize a signal that closely resembles recordings of real tones. The results of this study support the following conclusions:

(1) Many of the natural modes of oscillation are stimulated when the drum is struck. All do not respond at the same amplitude. Some are so weak that they are

TABLE V. Information theory statistics and confusion matrix comparison of the judgments of real drum tones with their synthetic counterparts.

R_1, S_1

		Judged (Y) R	Judged (Y) S	
Actual (X)	R	119	36	155
	S	52	103	155
Total:		171	139	310

$U(X)=1.000$ $T(X,Y)=0.1411$
$U(Y)=0.9923$ $C_{x\cdot y}=0.1411$

R_2, S_2

		Judged (Y) R	Judged (Y) S	
Actual (X)	R	83	72	155
	S	86	69	155
Total:		169	141	310

$U(X)=1.000$ $T(X,Y)=0.0003$
$U(Y)=0.9941$ $C_{x\cdot y}=0.0003$

R_3, S_3

		Judged (Y) R	Judged (Y) S	
Actual (X)	R	81	74	155
	S	92	63	155
Total:		173	137	310

$U(X)=1.000$ $T(X,Y)=0.0034$
$U(Y)=0.9903$ $C_{x\cdot y}=0.0034$

lost in the background. Some decay so rapidly that they are not detectable. Many components of appreciable amplitude are transmitted to the surrounding air and reach the listening ear.

(2) For a hard blow (large amplitude of vibration) the frequency starts higher and then decreases rapidly as the amplitude decreases. The relative (percentage) frequency change was found to be the same for all the components.

(3) The amplitude of each of the components (and thus the timbre of the sound) changes depending on the strength of the blow and the position on the membrane where the blow is applied.

(4) Decay rates seem to depend more on the characteristics of the drum than on how or where the drum is struck.

Agreement between theoretical and observed frequencies was quite good, but not perfect. The perturbations described under Sec. I. may explain the discrepancies. However, they have not been investigated quantitatively in this study.

The results of the listening tests show that the synthetic tones sounded at least as drumlike as the recording of the real ones. Bass drum sounds, either real or synthetic, played back on a tape recorder do not sound the same as the sounds one hears from the actual drum. A careful observer hears a different sound from the drum depending on where he stands with respect to the drum.

[1]Lord Rayleigh, *The Theory of Sound* (Dover, New York, 1945), Vol. 1, Chap. 9.
[2]A. H. Benade, *Fundamentals of Musical Acoustics* (Oxford U. P., New York, 1976), pp. 141–145.
[3]B. S. Ramakrishna and Man Mohan Sondhi, "Vibrations of indian musical drums regarded as composite membranes," J. Acoust. Soc. Am. **26**, 523 (1954).
[4]B. S. Ramakrishna, "Modes of vibration of the indian drum dugga or left-hand thabala," J. Acoust. Soc. Am. **29**, 234 (1957).
[5]T. Sarojini and A. Rahman, "Variational method for the vibrations of the indian drums," J. Acoust. Soc. Am. **30**, 191 (1958).
[6]J. Obata and T. Tesima, "Experimental studies on the sound and vibration of drums," J. Acoust. Soc. Am. **6**, 267–74 (1935).
[7]L. J. Sivian, H. K. Dunn, and S. D. White, "Absolute amplitudes and spectra of certain musical instruments and orchestras," J. Acoust. Soc. Am. **2**, 330–371 (1931).
[8]R. W. Young and H. K. Dunn, "On the interpretation of certain sound spectra of musical instruments," J. Acoust. Soc. Am. **29**, 1070–1073 (1957).
[9]H. K. Dunn, "Absolute amplitudes and spectra of certain musical instruments and orchestras," I. R. E. Trans. Audio **AU-7**, 47–75 (1959).
[10]H. Fletcher, E. D. Blackham, and R. Stratton, "Quality of piano tones," J. Acoust. Soc. Am. **34**, 749–761 (1962).
[11]D. E. Cahoon, "Frequency-time analysis of the bass drum sound," Masters degree thesis, Brigham Young U., Provo, UT (1970) (unpublished).
[12]C. H. Coombs, R. M. Dawes, and A. Rversky, *Mathematical Psychology* (Prentice-Hall, Englewood Cliffs, 1970), Chap. 10.