Equivalent circuit modeling and vibrometry measurements of the Nigerian-origin Udu Utar drum

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The Udu drum, sometimes called the water pot drum, is a traditional Nigerian instrument. Musicians who play the Udu exploit its aerophone and idiophone resonances. This paper will discuss an electrical equivalent circuit model for the Udu Utar, a modern innovation of the traditional Udu, to predict the low frequency aerophone resonances and will also present scanning laser vibrometer measurements to determine the mode shapes of the dominant idiophone resonances. These analyses not only provide an understanding of the unique sound of the Udu instrument but may also be used by instrument designers to create instruments with resonance frequencies at traditional musical intervals for the various tones produced and to create musical harmonic ratios. The information, specifically the laser vibrometry measurements, may also be useful to musicians in knowing the best places to strike the Udu to excite musical tones. © *2013 Acoustical Society of America*.
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I. INTRODUCTION

Originating with the Igbo people of Nigeria, the Udu is a clay water pot constructed for musical use. It is both an idiophone—an instrument using vibration of the instrument's body to produce sound—and an aerophone—an instrument that uses vibrating columns and cavities of air to produce sound. The typical Udu retains the traditional Igbo water pot shape, reminiscent of a gourd, but derives many of its unique aural qualities from the addition of a hole in the side of the pot which enables players to modulate the tones produced by the aerophone. The side-hole water pot drum has uncertain beginnings, and ascertaining the reasons for punching the extra hole in the pot in ancient Igboland will be left to anthropologists. Of its cultural significance, this much is known: until recent decades, only women made and played the Udu, relative to their prominent role in Igbo religion;[1] the deep bass aerophonic resonances of the Udu were and are believed to be the "voices of the ancestors,"[2] and as such the Udu plays an important role in religious ceremonies—in Chinua Achebe's *Things Fall Apart*, arguably the most important and influential Igbo and pan-African piece of literature, the Udu is mentioned alongside other Igbo percussion instruments as forming the basis of the rhythmic support for these ceremonies.[3]

Frank Giorgini, an artisan working in upstate New York, is primarily responsible for bringing the Udu into popularity outside of Nigeria. While studying pottery in Maine, Giorgini came into contact with Abbas Ahuwan, a Nigerian potter, and was introduced to the Udu drum. Giorgini began researching and producing Udus in earnest, eventually traveling to Nigeria to learn from master Igbo Udu maker Ahuwan.[2,4] Artists such as Sting, Miles Davis, and Alex Acuña have utilized Giorgini's Udus in their music, and, recently, mass production of the Udu by percussion companies, including Latin Percussion, Inc., has enabled many musicians outside of Nigeria to learn and experiment with the drums.

The literature currently extant on the Udu is limited to a few articles in various music and instrument magazines, describing the sound, method of playing, and background of the drum. The most complete of these is the article written by Giorgini in 1990,[2] which includes the history of and his history with the drums, a brief description of how Udu are made, Giorgini's experimentation and innovation, brief qualitative description of the link between Helmholtz resonances and the Udu, and Giorgini's goals with Udu production. No in-depth scientific description or modeling of the acoustics of the Udu has yet been published, and the purpose of this paper is to expand the current understanding of these unique instruments by filling this gap.

This paper provides a basis for understanding the sound radiated from an Udu from both idiophonic and aerophonic standpoints by making acoustic measurements of an Udu Utar, one of the mass-produced models from the Giorgini collaboration with Latin Percussion, Inc. An equivalent

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circuit model of the aerophone resonances is developed, and, using scanning laser vibrometry, the modal patterns on the top surface of the Udu Utar at the idiophone resonance frequencies have been measured. This information is useful both to musicians and Udu makers as mastery of the instrument and tighter control of production rely on an understanding of what sounds are possible, how to produce them in playing, and what parameters might be changed when making an Udu to meet any particular musical specification.

II. PLAYING THE UDU

The interest of the Udu drum is primarily in the wide range of sounds that can be coaxed from it. We will qualitatively describe aerophonic resonances and how to obtain them, and then idiophonic resonances and how to obtain them. By combining these techniques, a vast sound palette is achievable. Though the Udu Utar is the specific subject of our research (shown in Fig. 1), the sounds obtained are typical of all Udu drums and similar techniques can be applied. Note that in Sec. I, we refer to the side hole as being a hole cut into the large cavity of the pot. However, from this point on we will refer to the hole in the cavity of the pot as the "top hole" and the hole out of the neck of the pot as the "side hole" with reference to their respective positions in the photo in Fig. 1.

The signature sounds of the Udu are the deep bass aerophonic resonances and terminal octave slide. By using the palms of the hands to strike and cover either of the openings, low bass tones are achieved. The side hole is typically more responsive to this action with a much stronger response obtained. When the hand strikes a hole and immediately releases, the tone quickly slides up nearly an octave before dissipating.

All of the tone gradations between the lowest note and the highest note can be achieved as well. Closing the top hole and striking the side hole achieves the low tone, and as the player raises one side of the palm on the top hole and continues to strike the side hole the tone raises in frequency. As the angle between the palm and the top hole increases to perpendicular, the pitch increases to the tone nearly an octave higher. Acoustically the player is dynamically changing the end correction for the top hole, and therefore the amount of acoustic mass loading at that end of the Udu. When the palm covers the top hole the acoustic mass goes to infinity, but as the palm opens up the top hole, from closed to all the way open, the acoustic mass decreases to the finite value determined by the end correction for the acoustical reactance presented to that hole. This will be further analyzed in Sec. III C.

FIG. 1. (Color online) Photograph of the Udu Utar drum used in this study.

Many other sounds can also be made. The player can choose to only cover part of either hole when striking and thus achieve many different tones; covering the side hole and striking the top hole without completely closing it off can produce very low tones; and a "swoop" sound, similar to a dripping faucet, can be made by covering the side hole and striking the top hole with the palm, completely covering it and immediately releasing.

The various idiophonic resonances of the Udu are excited by tapping different parts of the body of the Udu yielding different dominant tones. These tones are higher in frequency than the aerophonic tones and often ring for some time. The duration of these tones depends on the quality of the Udu (the Utar is a fired clay instrument giving its walls a high degree of stiffness and thus a high quality factor) and the method of holding the Udu. An Udu typically rests in the player's lap, but this tends to dampen the vibration of the clay. To raise the Udu off of the player's lap, straw rings that the Udu rests on are traditionally used, and mounts of foam-covered tubular steel are also in use. Both of these methods allow the Udu to vibrate with less damping and longer duration for the idiophonic resonances.

Any type of tap, slap, or slide that the player can think of is considered appropriate technique, but typical playing method consists of tapping with the pads and inner joints of one's fingers. Using knuckles, nails, and full hand slaps is also common, and persons who have played conga or tabla generally have success transferring the techniques they already know. Non-traditional tools such as mallets are also used on occasion, depending on the musical situation.

Finally, it should be mentioned that Udu drums are sometimes filled partially with water for playing, which opens many more possibilities for aural effects and seems to improve the sound projection in some Udu. These effects will partly be considered in Sec. III E.

III. AEROPHONE RESONANCES

A. Acoustic measurements

There are three main aerophone, or acoustical, resonances that can be easily made on the Udu Utar drum, though these three resonances may be excited in five principal ways. Three of the excitations can be made by striking the side hole of the drum and two more can be made by striking the top hole. The first acoustical resonance can be excited with the side closed and the top open (SCTO) by striking the side hole and leaving the hand there to cover the hole, all the while with the top hole remaining open. The second acoustical resonance can be excited with the side open and the top closed (SOTC) by striking the side hole and releasing the hand quickly to leave the side hole open, all the while with the top hole being closed by the player's other hand. The third acoustical resonance can be excited with the side open and the top open (SOTO) by striking the side hole and releasing the hand quickly to leave the side hole open, all the while with the top hole remaining open. The second acoustical resonance can also be excited with the top closed and the side open (TCSO) by striking the top hole and leaving the hand there to cover the hole, all the while with the side hole remaining open. The third acoustical resonance can also be excited with the top open and the side open (TOSO) by striking the top hole and releasing the hand quickly to leave the top hole open, all the while with the side hole remaining open. A significantly audible response at the first acoustical resonance frequency cannot be made by striking the top hole and releasing the hand quickly to leave the top hole open, all the while with the side hole being closed by the player's other hand.

The five ways to excite the resonances of the Udu Utar drum were individually recorded in an anechoic chamber. A type-1, calibrated, free-field microphone was placed 1 m from the Udu (PCB 377A02 microphone and Larson Davis PRM426 preamplifier) to record the spectra in conjunction with a Hewlett Packard 35670 dynamic signal analyzer. The resonances were produced 3 or 4 times for each recording and the analyzer averaged the results. The resulting spectra are displayed in Fig. 2. Note that the resonances for TOSO and SOTO are at the same frequency as should be expected since the boundary conditions are the same once the hand has excited the instrument and subsequently been removed from the instrument. Note also that the resonances for SOTC and TCSO are also at the same frequency since the boundary conditions are again the same after the initial excitation. The three aerophone resonance frequencies are 73, 113, and 137 Hz, and essentially correspond to the musical notes D_2, A_2, and $C_3^{\#}$.

As observed from Fig. 2, the Udu Utar essentially has three distinct aerophone resonances. The frequencies of these resonances are nearly spaced according the musical intervals of a fifth and a seventh relative to the lowest resonance frequency. Perhaps one of the more interesting acoustic features of the sounds produced by the Udu Utar is the varying tone produced by adjusting the opening of one hole while striking the other. This feature is produced by dynamically modifying the degree to which one hand covers the top hole as the side hole is struck and released (ranging from SOTC 73 Hz to SOTO 137 Hz). The musician is thus dynamically changing the end correction (the inertia and resistance at the opening of the top hole) of the top hole. Figure 3 displays a spectrogram from a recording of a player dynamically varying the top hole opening while striking and releasing the side hole opening. This figure will be discussed further later on.

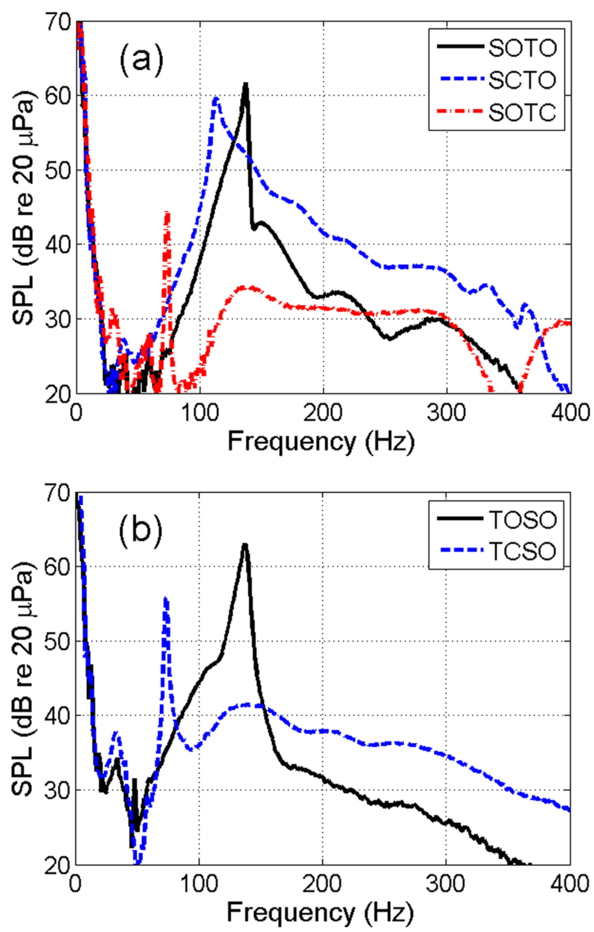

FIG. 2. (Color online) Frequency spectra of the sound produced by acoustic resonances of the Udu Utar drum. (a) Side hole excitations. (b) Top hole excitations.

B. Advanced equivalent circuit modeling

The aerophone resonances of the Udu Utar drum are low enough in frequency such that their wavelengths are large compared to the internal dimensions of the drum. For example, the highest aerophone resonance (at present consideration) of 137 Hz corresponds to a wavelength of 2.50 m, whereas the largest internal dimension is approximately 30 cm. For reference, the length of the neck is 14.6 cm and the cavity volume is 9.25 L ($9.25 \times 10^{-3}\,m^3$). Large wavelengths compared to the size of the resonator allow the aerophone resonances of the instrument to be modeled with one-dimensional, electrical equivalent circuit modeling techniques. The advantage of these techniques is that the complicated curvature of the instrument need not be known

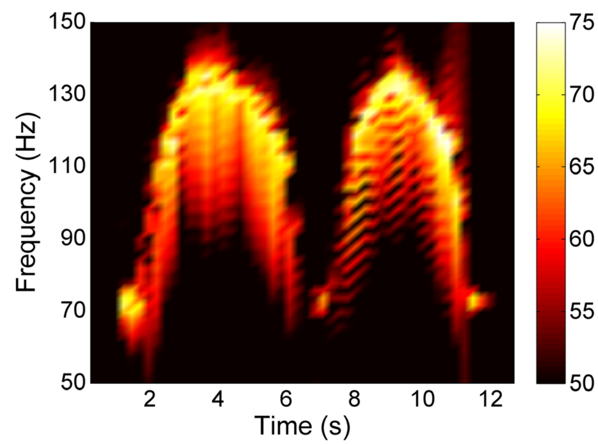

FIG. 3. (Color online) Spectrogram of the sound produced by the Udu Utar drum as the instrument is struck and released at the side hole multiple times while the alternate hand is used to dynamically change the end condition on the top hole from a closed condition to an open condition, back to a closed condition (this cycle is repeated twice).

exactly. Using the rules of constructing equivalent circuits, one can then create an electrical circuit diagram that may be solved for impedance quantities, which yield the resonance frequencies of the Udu.

Figure 4(a) displays the simplified-geometry drawing of the Udu Utar used in the equivalent circuit modeling (with the side hole on the left). The various parts of the instrument are then represented as electrical components in the so-called acoustic impedance domain equivalent circuit. The large cavity, labeled "C" in the drawing, of the Utar is modeled using a two-port T-network (the expressions used in T-networks are given by Mason in Ref. 5). The length for the network modeling the large cavity was determined from a measurement of the distance between the center of the top hole and the center of the junction between the neck and the cavity. The surface area of the large cavity was determined by dividing the volume of the cavity (measured by filling the Udu cavity up with water and then measuring the volume of the water) by this measured cavity length. The neck of the instrument, labeled "$N1 - N4$," whose geometry flares out towards the side hole and then abruptly constricts to the side hole opening, is modeled with four T-networks in series, using four segments of the neck length and using the average surface area of those segments. The junction between the neck and the cavity is modeled as a mass, $M_{AJ} = 8\rho_0/[3\pi^2 r_{N4} H(b/a)]$ (see Table I for definitions of the symbols used here and throughout the paper), according to Karal's Eq. (52), with $H(b/a) = 0.7$.[6] The interface between the cavity and the top hole is modeled as an orifice with a resistance, $R_{AOT} = \ln(4r_T/t)\sqrt{2\rho_0\omega\mu}/(4S_T)$, and a mass, $M_{AOT} = \rho_0 r_T/4$, according to the expressions given by Morse and Ingard[7] [Eq. (9.1.23)] and Pierce[8] [Eq. (7-5.10)], respectively. The interface between the side hole and the neck is also modeled as an orifice with a resistance, $R_{AOS} = \ln(4r_S/t)\sqrt{2\rho_0\omega\mu}/(4S_S)$, and a mass, $M_{AOS} = \rho_0 r_S/4$, using those same equations. The radiation impedance loading at the outlet of the top hole is modeled with resistance, $R_{ART} = \rho_0 c(kr_T)^2/(2S_T)$, and mass loading (related to the reactance), $M_{ART} = 8\rho_0 r_T/(3\pi S_T)$, values by approximating this opening as baffled (or flanged) using Eq. (10.2.10) of Kinsler *et al.*[9] The radiation impedance loading

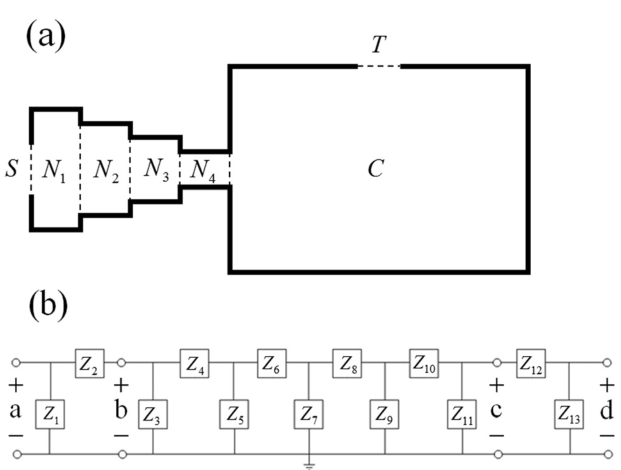

FIG. 4. Acoustical domain equivalent circuit model for the acoustic resonances of the Udu Utar drum. (a) Drawing of acoustical model. (b) The equivalent electrical circuit.

TABLE I. Symbol explanation. The angular frequency and acoustic wavenumber vary according to the frequency range desired. The other quantities listed are measured (from length measurements of the Udu Utar) or estimated from location specific information such as altitude and temperature (as in the case of the speed of sound, atmospheric density, and coefficient of viscosity).

Symbol	Unit	Description
j	—	imaginary number, $\sqrt{-1}$
ω	rad/s	angular frequency
c	m/s	speed of sound in air = 340 m/s in Provo, UT
k	1/m	acoustic wavenumber
ρ_0	kg/m^3	atmospheric density = 1.01 kg/m^3 in Provo, UT
μ	Pa·s	coefficient of viscosity = 1.85×10^{-5} Pa s
t	m	thickness of Udu Utar wall at hole exits = 0.006 m
l_{NX}	m	Nth length of neck segment = $[4.1, 3.1, 3.6, 2.0] \times 10^{-2}$ m
r_{NX}	m	Nth radius of neck segment = $[4.7, 4.0, 3.3, 2.9] \times 10^{-2}$ m
S_{NX}	m^2	Nth surface area of neck segment = $[7.1, 4.9, 3.3, 2.5] \times 10^{-2}$ m^2
r_S	m	radius of side hole opening = 0.035 m
S_S	m^2	surface area of side hole opening = 0.0038 m^2
r_T	m	radius of top hole opening = 0.025 m
S_T	m^2	surface area of top hole opening = 0.0020 m^2
l_C	m	measured length of large cavity, from neck to top hole = 0.229 m
S_C	m^2	effective surface area of large cavity = 0.041 m^2

at the outlet of the side hole is also modeled with resistance, $R_{ARS} = \rho_0 c(kr_S)^2/(2S_S)$, and mass loading, $M_{ARS} = 8\rho_0 r_S/(3\pi S_S)$, values by also approximating this opening as being baffled. Figure 4(b) shows the equivalent circuit for the Udu Utar. The impedances represented in Fig. 4(b) are

$$Z_1 = R_{ARS} + R_{AOS} + j\omega(M_{ARS} + M_{AOS}), \tag{1}$$

$$Z_2 = j\frac{\rho_0 c}{S_{N1}}\tan\left(\frac{kl_1}{2}\right), \tag{2}$$

$$Z_3 = -j\frac{\rho_0 c}{S_{N1}}\csc(kl_1), \tag{3}$$

$$Z_4 = j\frac{\rho_0 c}{S_{N1}}\tan\left(\frac{kl_1}{2}\right) + j\frac{\rho_0 c}{S_{N2}}\tan\left(\frac{kl_2}{2}\right), \tag{4}$$

$$Z_5 = -j\frac{\rho_0 c}{S_{N2}}\csc(kl_2), \tag{5}$$

$$Z_6 = j\frac{\rho_0 c}{S_{N2}}\tan\left(\frac{kl_2}{2}\right) + j\frac{\rho_0 c}{S_{N3}}\tan\left(\frac{kl_3}{2}\right), \tag{6}$$

$$Z_7 = -j\frac{\rho_0 c}{S_{N3}}\csc(kl_3), \tag{7}$$

$$Z_8 = j\frac{\rho_0 c}{S_{N3}}\tan\left(\frac{kl_3}{2}\right) + j\frac{\rho_0 c}{S_{N4}}\tan\left(\frac{kl_4}{2}\right), \tag{8}$$

$$Z_9 = -j\frac{\rho_0 c}{S_{N4}}\csc(kl_4), \tag{9}$$

$$Z_{10} = j\frac{\rho_0 c}{S_{N4}}\tan\left(\frac{kl_4}{2}\right) + j\omega M_{AJ} + j\frac{\rho_0 c}{S_C}\tan\left(\frac{kl_C}{2}\right), \tag{10}$$

$$Z_{11} = -j\frac{\rho_0 c}{S_C}\csc(kl_C), \tag{11}$$

$$Z_{12} = j\frac{\rho_0 c}{S_C}\tan\left(\frac{kl_C}{2}\right), \tag{12}$$

$$Z_{13} = R_{ART} + R_{AOT} + j\omega(M_{ART} + M_{AOT}). \tag{13}$$

The SOTO resonance, caused when the hand strikes the side hole and releases quickly, is found by determining the impedance at the terminals labeled "a" in Fig. 4(b), then finding the point at which the imaginary part of this impedance (the reactance) goes to zero. The striking excitation creates a pulse of energy that travels down the neck (represented by impedances Z_2 through the first term of Z_{10}) and reaches the junction (represented by the second term in Z_{10}) at the acoustic cavity. Some of the energy resonates within the cavity (represented by the third term in Z_{10} through Z_{12}) and some escapes through the top hole (represented by Z_{13}). Some of the initial energy also escapes out the side hole (represented by Z_1) without traveling down the neck. The SOTO reactance goes to zero at $f_{\mathrm{SOTO}} = 141.3$ Hz and represents a 3.1% error from the measured value of 137 Hz [see Fig. 2(a)].

The SOTC resonance is found by setting $Z_{12} = Z_{13} = 0$ and determining the reactance presented to the terminals labeled "a" in Fig. 4(b). This resonance is similar to that of SOTO except that energy is prevented from escaping from the top hole. The SOTC reactance goes to zero at $f_{\mathrm{SOTC}} = 75.8$ Hz and represents a 3.8% error from the measured value of 73 Hz [see Fig. 2(a)].

The SCTO resonance is found by setting $Z_1 = Z_2 = 0$ and determining the reactance presented to the terminals labeled "b" in Fig. 4(b). The SCTO resonance prevents energy from radiating out of the side hole. The SCTO reactance goes to zero at $f_{\mathrm{SCTO}} = 113.8$ Hz and represents a 0.7% error from the measured value of 113 Hz [see Fig. 2(a)].

The TOSO resonance is found by determining the reactance presented to the terminals labeled "d" in Fig. 4(b). The TOSO excitation should present the same resonator as the SOTO excitation. Indeed the TOSO reactance goes to zero at $f_{\mathrm{TOSO}} = 141.2$ Hz (nearly identical to that found for f_{SOTO}) and represents a 3.0% error from the measured value of 137 Hz [see Fig. 2(b)].

The TCSO resonance is found by setting $Z_{12} = Z_{13} = 0$ and determining the reactance presented to the terminals labeled "c" in Fig. 4(b). The TCSO excitation should present the same resonator as the SOTC excitation. Indeed the TCSO reactance goes to zero at $f_{\mathrm{TCSO}} = 75.8$ Hz (identical to that found for f_{SOTC}) and represents a 3.8% error from the measured value of 74 Hz [see Fig. 2(b)].

C. Discussion

A preliminary simplified circuit model, which did not divide up the neck into segments or use T-network circuits anywhere in the model, constructed by the authors, yielded resonance frequencies that were within 9.3% error.[10] With the equivalent circuit model considered here the resonance frequencies are predicted to within 3.8% error (average of 2.9% error). A circuit model with further complexities, such as dividing up the neck into smaller sections and dividing the large cavity up into sections could result in even higher accuracy. Perhaps the principal advantage of using the equivalent circuit models is that one may adjust an existing geometrical design of an instrument to optimally produce desired musical intervals, or one may design an entirely new type of Udu drum with desired musical intervals. For example, one could adjust the surface areas of the holes, length of the neck, and/or cavity volume of the Udu Utar in an attempt to adjust the musical intervals to the desired ones. With an accuracy of 3.8%, one may not be able to construct a new design with confidence that it will have the exact musical ratios desired. However, if one models an existing design and wishes to tweak the musical ratios, then the model may be used to determine relative changes in lengths, areas, or volumes required to change the musical ratios by the desired amounts.

As discussed in Sec. II, when one seals off the top hole with one hand and strikes (and releases) the side hole, the deep $f_{\mathrm{SOTC}} = 73$ Hz resonance is excited. However, as one slowly uncovers the top hole while simultaneously continuing to strike the side hole, the pitch increases. When the hand on the top hole is completely removed from covering the hole, a strike at the side hole produces a pitch of $f_{\mathrm{SOTO}} = 137$ Hz. A spectrogram of a recorded demonstration of a couple full cycles of this dynamic pitch variation may be found in Fig. 3. A similar type of dynamic pitch variation may also be demonstrated using an pipe where one repeatedly strikes one end of the pipe and varies the degree to which the other end is sealed off, which is akin to exciting the open-closed fundamental resonance of the pipe up to the open-open resonance of the pipe. In the case of the Udu, the M_{ART} and M_{AOT} values increase from an open top hole condition to infinity when the top hole is closed, which results in $Z_{13} \to \infty$ and hence prevents current from flowing into the $Z_{12},\ Z_{13}$ branch of the circuit.

D. Simplified dual opening Helmholtz resonator

A simplified model of the Udu is dual opening Helmholtz resonator represented by an acoustic compliance, $C = V/[\rho_0 c^2]$, where C is the volume of the cavity, representing the cavity with two acoustic masses, $M_1 = \rho_0 l_1/S_1$ and $M_2 = \rho_0 l_2/S_2$, where l and S represent the length and surface area of the "pipe" opening, respectively, representing each of the two openings as shown in Fig. 5(a). For a basic two-opening Helmholtz resonator one would expect the Pythagorean relationship for the resonance with both openings exposed, $f_{12} = (1/2\pi)\sqrt{(M_1 + M_2)/(M_1 M_2 C)}$, to the two resonance frequencies produced when each of the two openings are individually sealed off, $f_1 = (1/2\pi)\sqrt{1/(M_1 C)}$ and $f_2 = (1/2\pi)\sqrt{1/(M_2 C)}$, such that $f_{12} = \sqrt{f_1^2 + f_2^2}$. The f_{12} resonance is found by determining the impedance at the terminals labeled "a" in Fig. 5(b) with the terminals labeled "b" connected (short circuited) and by determining the frequency at which the reactance of this impedance equals zero. The f_1 resonance is found by determining the impedance at the terminals labeled "a" in Fig. 5(b) with the terminals labeled "b" left open and by determining the frequency at which the reactance of this impedance equals zero. The f_2 resonance is found by determining the impedance at the terminals labeled "b" in Fig. 5(b) with the terminals labeled "a" left open and

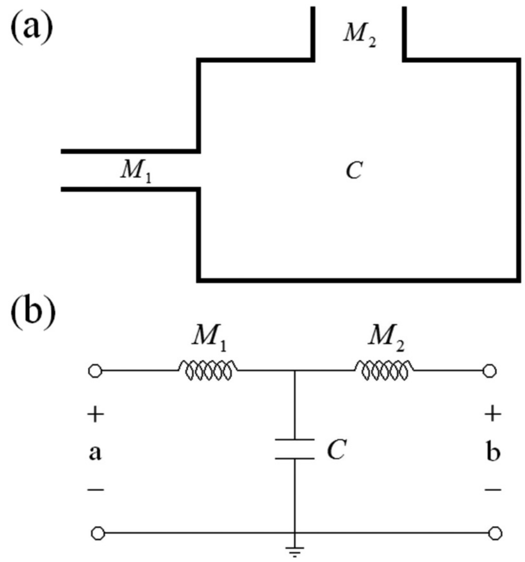

FIG. 5. Simplified Utar acoustical domain equivalent circuit model for the acoustic resonances of a dual port Helmholtz resonator. (a) Drawing of the simplified Utar. (b) The equivalent electrical circuit.

by determining the frequency at which the reactance of this impedance equals zero.

If we define a quantity $\alpha = M_1/M_2 = (l_1 S_2)/(l_2 S_1)$ we can plot the frequency ratios $f_2/f_1 = \sqrt{\alpha}$ and $f_{12}/f_1 = \sqrt{\alpha + 1}$ so that we can design a simplified Udu to have musical intervals for these frequency ratios. Note that if $\alpha = 0$ then $f_{12} = f_1$, if $\alpha = 1$ then $f_2 = f_1$, and if $\alpha = \infty$ then $f_{12} = f_2$, none of which are musically useful utilizations of both openings. Here we assume that $M_1 > M_2$ and that one would start by choosing a fundamental frequency f_1 from which to design an Udu cavity and one hole opening. One then might wish to view the relationships of the frequency ratios defined above in order to design the second hole opening as shown in Fig. 6. Note that if one selects $\alpha = 1.25$ the frequency ratios result in musical intervals of a second and a fifth relative to f_1, or if one selects $\alpha = 1.78$ the frequency ratios result in musical intervals of a fourth and a sixth. Further note that these frequency ratios are independent of the cavity volume, though the values for the frequencies themselves depend on the cavity

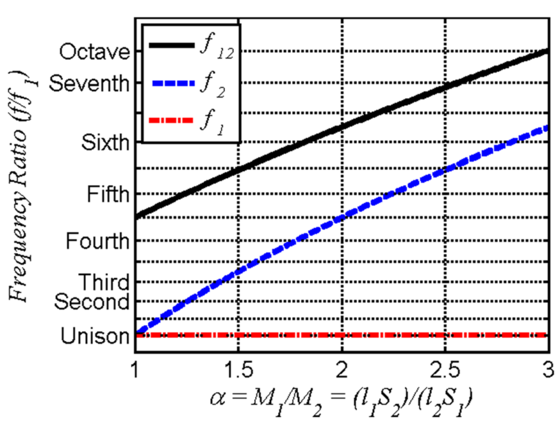

FIG. 6. (Color online) Plot of the resonance frequency ratio f/f_1 versus the ratio of the acoustical masses of the two ports, $\alpha = M_1/M_2$, for a simplified model of an Utar (a dual Helmholtz resonator). This plot may be used to create an initial design for an Udu like instrument.

volume. Figure 6 may be used to obtain a quick, rough estimate of the resonance frequencies that will result from a new type of Udu design based on the choice of lengths and surface areas of the ports and the volume of the cavity.

The Udu Utar resonance frequencies do not exactly agree with the Pythagorean relationship of the simplified model with, $f_{\mathrm{SOTO}} = 137\,\mathrm{Hz}$ whereas $\sqrt{f_{\mathrm{SOTC}}^2 + f_{\mathrm{SCTO}}^2} = \sqrt{73^2 + 113^2}\,\mathrm{Hz} = 134.5\,\mathrm{Hz}$. This -1.8% error departure of this relationship for the Udu Utar is not large but the authors feel that this small departure is perhaps by chance. There are competing complexities in the differences between an idealized dual port Helmholtz resonator and the Utar that one encounters as one improves the equivalent circuit model shown in Fig. 6(b). Some of these complexities include the acoustic resistances in the advanced equivalent circuit model (which do in fact alter the resonance frequency values), the radiation mass loadings, and also the effects of orifice type impedance loadings presented by the relatively small hole openings on the Utar. The reader should note that despite the decent agreement in the relationship of the resonance frequencies of the Utar with the simplified model, the dual port Helmholtz resonator does not predict the actual resonance frequency values of the Utar to even within 100% error accuracy, with predicted values of $f_1 = 93\,\mathrm{Hz}$, $f_2 = 243\,\mathrm{Hz}$, and $f_{12} = 268\,\mathrm{Hz}$ (compared to values of 73, 113, and 137 Hz), underscoring the need for the advanced circuit model.

E. Adjusting the model

In order to test the ability of the equivalent circuit model to predict changes to an Udu design we add water to the large cavity of the Utar as is sometimes done by musicians. When 1.00 L of water is added, we measure two of the resonance frequencies to shift up to values of $f_{\mathrm{SCTO}} = 116.5\,\mathrm{Hz}$ and $f_{\mathrm{SOTO}} = 143.5\,\mathrm{Hz}$. The effective length between the neck junction and the top hole, l_C, remains the same when water is added but we must adjust the effective surface area of the cavity. Since the volume of the cavity, $V_C = S_C l_C$, we change S_C from a value of $0.041\,\mathrm{m}^2$ to a value of $0.036\,\mathrm{m}^2$ to account for the decrease in volume of the cavity by 1.00 L. The equivalent circuit model then predicts that $f_{\mathrm{SCTO}} = 119.3\,\mathrm{Hz}$ and $f_{\mathrm{SOTO}} = 149.1\,\mathrm{Hz}$ resulting in percent errors of 2.4% and 3.9%, respectively.

When 2.00 L of water are added, we shift S_C to a value of $0.032\,\mathrm{m}^2$. The measured resonance frequencies with 2.00 L of water added shift upward further to values of $f_{\mathrm{SCTO}} = 123.5\,\mathrm{Hz}$ and $f_{\mathrm{SOTO}} = 151.5\,\mathrm{Hz}$. We then predict resonance frequencies of $f_{\mathrm{SCTO}} = 125.6\,\mathrm{Hz}$ and $f_{\mathrm{SOTO}} = 158.3\,\mathrm{Hz}$ with the equivalent circuit model. This model is off by 1.7 and 4.5 %, respectively.

Thus the model's predicted resonance frequencies with added water in the cavity are not much larger than the model without the water added. While we may not be able to predict the actual frequency values with accuracy better than 4%, we can use the model to predict the relative resulting change. One way to analyze the model's ability to make relative predictions is to determine the measured shifts in resonance frequencies as a ratio when water is added and compare the measured shifts to the predicted shifts we calculate using the model when water is added. The shifts in the measured resonance frequencies when the 1.00 L of water is added are ratios of $f^{1\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.03$ and $f^{1\,\mathrm{L}}_{\mathrm{SOTO}}/f_{\mathrm{SOTO}} = 1.04$, and when 2.00 L of water is added the measured ratios are $f^{2\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.09$ and $f^{2\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.11$. The corresponding shifts predicted by the model for 1.00 L of added water are $f^{1\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.05$ and $f^{1\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.06$, and are $f^{1\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.10$ and $f^{1\,\mathrm{L}}_{\mathrm{SCTO}}/f_{\mathrm{SCTO}} = 1.12$ when 2.00 L of water is added. Thus the average measured shifts are 1.04 for 1 L and 1.10 for 2 L while the average calculated shifts are 1.05 for 1 L and 1.11 for 2 L, which is good agreement.

IV. IDIOPHONE RESONANCES

A. Acoustic measurements

As discussed in Sec. II, the Udu is not just an aerophone instrument but is also an idiophone instrument. A musician may take advantage of the rich amount of idiophone sounds by striking the Udu's surface in various places to excite different relative amounts of various structural resonances of the instrument's body. One of the more pronounced and musical idiophone sounds of the Udu Utar is made by striking the lip of the end of the instrument where the side hole is located. The resulting tone is similar to that of a cowbell tone. Acoustic recordings of the idiophone resonances produced by the Udu Utar were made in the anechoic chamber using the same equipment described in Sec. III.

Figure 7 displays measured spectra produced when the Utar is struck in three different locations: at the cowbell location described above, a couple centimeters west (left) of the top hole, and a couple centimeters north (above) of the top hole. Note that in the spectrum for the cowbell tone that the lowest dominant partial is at 556 Hz (approximately $C_5^{\#}$) and there exists nearly harmonic partials at 1108 Hz (approximately $C_6^{\#}$) and at 1644 Hz (approximately $G_6^{\#}$). These nearly harmonic partials represent ratios of 1.99 and 2.96, respectively, and thus are slightly flat relative to perfect harmonic ratios. Note also the partial at 970 Hz (closet to B_5), which is excited strongly by striking near the top hole (with a stronger response when striking to the west of the top hole). This partial has a near harmonic located at 1898 Hz (a ratio of 1.96 relative to the fundamental, nearly an octave above 970 Hz). Thus when one needs more cowbell and

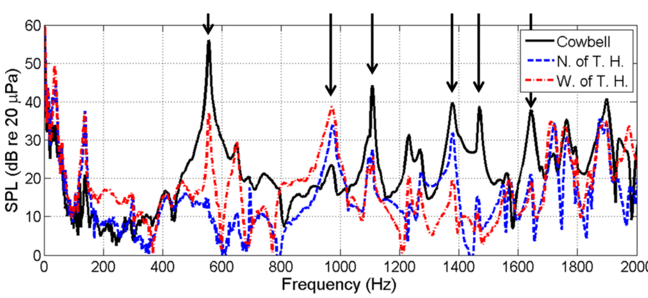

FIG. 7. (Color online) Frequency spectra of the sound produced by idiophone resonances of the Udu Utar drum: (a) when struck at the cowbell, (b) when struck north of the top hole, and (c) when struck west of the top hole (north and west striking locations are identified in Fig. 1).

strikes the lip of the side hole, a nearly harmonic tone is produced with a fundamental frequency of 556 Hz, while the response at 970 Hz is greatly reduced. When one strikes near the top hole a higher pitched tone results with a fundamental frequency of 970 Hz, while the response at 556 Hz is greatly reduced. Hence the striking location greatly influences the pitch and tonal quality of the idiophone resonances.

B. Scanning laser vibrometer measurements

In order to better understand the structural modes of the Udu Utar that contribute to the tones described in Sec. IV A, a Polytec Scanning Laser Doppler Vibrometer (SLDV) PSV-400 was employed. A LDS Shaker V203, with the shaker's stinger affixed with bees wax to the underneath side of the side hole lip, was used to drive the Udu at selected resonance frequencies determined from Fig. 7 with a peak to peak voltage amplitude of 500 mV. The sensitivity of the SLDV was adjusted as appropriate to optimize the digitization of the acquired signals. A 3200 Hz bandwidth with 1600 lines of resolution was used along with five complex FFT averages, and a 20 Hz high pass filter. A rectangular grid of scan points was made up of 391 total points (23 by 17) with 218 of those points on the surface of the Utar. Small pieces of reflective tape were placed at each of the scan point locations to optimize the reflectivity of the incident laser signal, and to account for the surface contour variations away from normal incidence. The Udu was driven at frequencies of 556 ($C_5^{\#}$), 970 (B_5), 1108 ($C_6^{\#}$), 1378 (F_6), 1470 ($F_6^{\#}$), and 1644 ($G_6^{\#}$) Hz corresponding to most of the dominant resonance frequency peaks in Fig. 7 (the peaks denoted with arrows in Fig. 7).

Figure 8 displays images obtained from the six SLDV measurements. These images represent snapshots in time of the spatial distribution of the surface velocity when the mode reached a maximum in amplitude. Note that while considered not important here (we are interested in the modal shapes only), the velocities displayed in Fig. 8 are only a component of the total out of plane surface velocity at each scan point (a component in the direction of the laser), since not all scan points were on surfaces normal to the laser's incidence angle. Lines have been drawn onto these modal images to aid visualization of the nodal lines, along with + and − symbols to denote antinodes of relative phases of positive and negative, respectively. The instantaneous velocities are given in units of mm/s. Note that the mode shapes indicate why different modes are more or less strongly excited when the Udu is struck at different locations. For example, the 566 Hz mode [Fig. 8(a)], is strongly excited when struck at the side hole lip due to the presence of the antinode, while it is minimally excited when struck just west of the top hole and hardly excited when struck just north of the top hole, due to the proximity of these striking locations to a node (refer to Fig. 7 to verify the relative strengths of exciting the 566 Hz mode at these striking locations). A similar visual analysis can be made for each of the modes displayed in Fig. 8, and their corresponding levels in the sound spectra displayed in Fig. 7, to note their dependencies on striking location and the proximity of the striking location to the nodes.

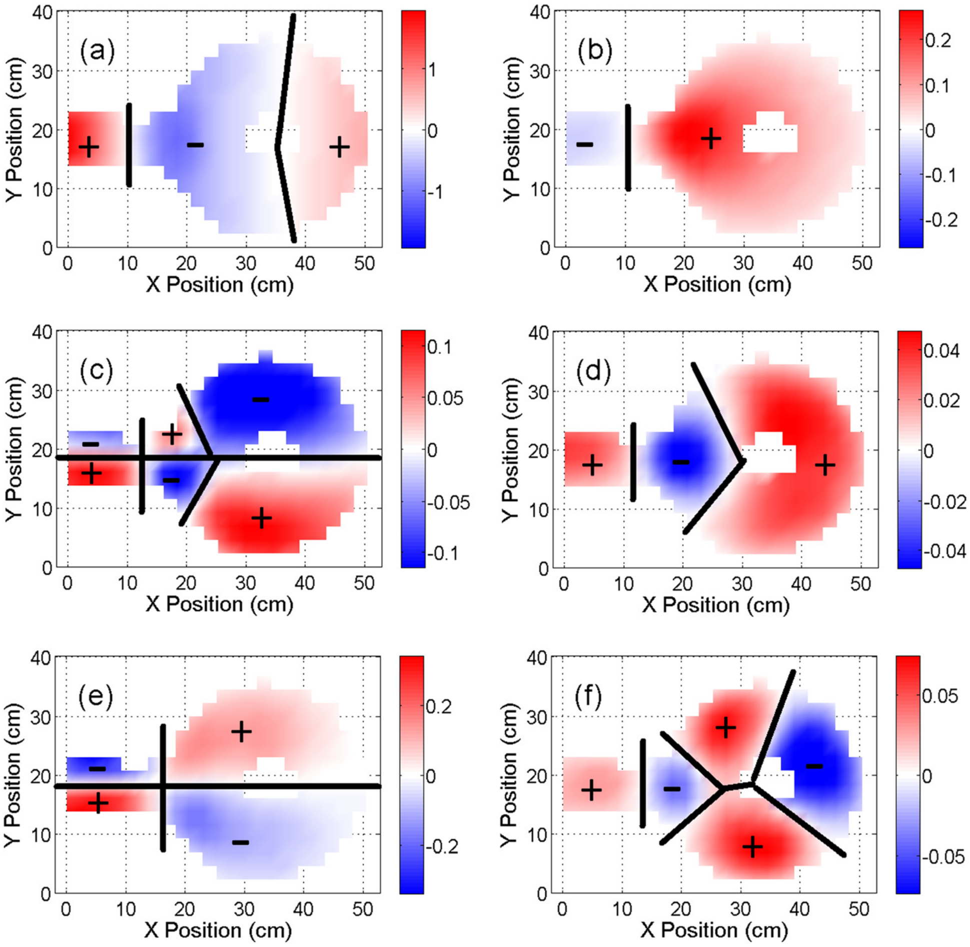

FIG. 8. (Color online) Modal distributions of six selected idiophone modes at (a) 556, (b) 970, (c) 1108, (d) 1378, (e) 1470, and (f) 1644 Hz of the Udu Utar drum. Amplitudes are displayed on a gray scale (or color online) and are expressed in units of mm/s.

C. Discussion

Knowledge of the modal patterns of the Udu Utar with their corresponding resonance frequencies can assist the musician in learning where to strike the instrument to achieve tones with harmonic overtones. A player typically must find sweet spots, or striking locations that produce musical tones, on the instrument through practice.

The designer of this particular Udu Utar drum spent 2–4 years designing and developing new Udu designs, one of which was the Utar, that "sounded right" to him based on years of experience in both playing and designing Udu drums. Designers do not currently utilize models or measurements to shape the drum and instead rely on their experience with various designs. Designers of the instrument can also benefit from this modal analysis information as they can potentially modify future designs by adding or removing some of the wall material in certain locations in order to tune the instrument as is commonly done for xylophones and marimbas when their bars are undercut to achieve nearly harmonic tuning of upper partial frequencies. It is interesting to note that the designer of the Udu Utar was able to produce a handmade instrument that possesses musically spaced resonance frequencies without the use of a model.

V. CONCLUSIONS

It has been shown that the deep aerophone resonances of the Udu Utar drum may be predicted using electrical equivalent circuit modeling techniques. This may aid an instrument designer in adjusting the design of an Udu drum to result in desired musical intervals for its principal aerophone resonances. The unique ability to produce the full spectrum of tones between the lowest and highest principal resonances by varying the end condition of one of the holes has also been explained. A simplified model of the Utar has been shown, which may be used as a rough starting point for designing a newly shaped Udu. Finally, an example has been given when the model may be adjusted to predict relative changes in the Udu's tones when the instrument's geometry is changed.

The idiophone resonance frequencies of the Udu Utar have been found to occur nearly at musical intervals. The modes that correspond to these resonance frequencies have been measured with a scanning laser vibrometer. Visual analysis of the modal patterns allows one to determine the strength of these resonance frequencies in the radiation spectra. Knowledge of these modal patterns may aid a musician in learning how to produce musical tones with harmonic upper partials, and they may aid an instrument designer in determining how the thickness of the Udu walls may be slightly modified to fine tune these resonance frequencies.

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