Keller, Seckler, and Lewis[19])

$$U_K^{(\mathrm{III})}(P)=\frac{1}{k}\sum_i\left\{\frac{\sin\varphi}{k[\partial(s+\phi)/\partial l]_+[\partial(s+\phi)/\partial l]_-}\right\}_j \times\left\{\frac{ik}{4\pi}(\hat{s}-\mathbf{p})\cdot\mathbf{n}\frac{A}{s}\exp[ik(s+\phi)]\right\}_j, \quad \text{(D15)}$$

where the subscripts + and − denote limiting values at the two sides of the corner point Q_j. Since $\partial(s+\phi)/\partial l = (\hat{s}+\mathbf{p})\cdot\mathbf{l}$ and $A\exp(ik\phi)=U(Q)$, (D15) may be written as

$$U_K^{(\mathrm{III})}(P)=\frac{1}{ik}\sum_i U(Q_j)\frac{\exp(iks_j)}{4\pi s_j} \times\left\{\frac{\sin\varphi\cdot(\mathbf{p}-\hat{s})\cdot\mathbf{n}}{[(\hat{s}+\mathbf{p})\cdot\mathbf{l}^+][(\hat{s}+\mathbf{p})\cdot\mathbf{l}^-]}\right\}_j. \quad \text{(D16)}$$

Comparison with (5.13) shows that $U_K^{(\mathrm{II})}=U_{(\mathrm{II})}^{(B)}$.

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Test of the Effect of Edge Parameters on Small-Angle Fresnel Diffraction of Light at a Straight Edge

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Precision photometric measurements of the optical Fresnel diffraction pattern produced by a straight edge were made on eight edges of various materials and with various edge cross sections to ascertain the effect of these parameters on the pattern. For unpolarized light at a wavelength of 5461 Å, approximately 600 fringes were detected and measured in each pattern, and 800 fringes were detected in some of the patterns. The theoretical position of the geometric shadow ($I/I_0=0.25$) is verified to an angular accuracy of 1×10^{-4} deg. The light intensity distribution was scanned at a constant speed and detected by a 1P21 photomultiplier tube cooled to dry-ice temperature. The distance between the light source and the photometer was 52 m, and the edge was placed 20 m from the source. A simple technique for automatic compensation for fluctuations of the primary light source is also described. Accuracy of approximately 0.2% in the intensity measurements and of 0.03% in fringe position far from the shadow boundary is claimed. To within the accuracy of the experiment, the intensity distribution for all eight edges agreed throughout the complete pattern, but there is a slight variation in fringe position from the simple scalar theory.

I. INTRODUCTION

In the microwave frequency range of the electromagnetic spectrum, measurable variations have been observed between the diffraction patterns by conducting and nonconducting objects and between diffraction patterns produced by diffracting screens of varying thicknesses.[1,2] These microwave measurements along with the numerous theoretical treatments involving conductivity,[3] thickness, and shape,[4–9] of the diffracting object prompted this precise measurement of the diffraction of visible light at straight edges of various materials and cross sections. Detailed discussions of diffraction theory are available.[10–13] The lack of sufficient numerical evaluations of the more rigorous theories has prevented any detailed prediction of the expected variations. For this reason many workers in the field feel that experimental results are greatly needed to help guide the theory to a simpler formulation involving specific effects. This investigation was an effort to supply some reliable, precise experimental data. The results, however, show that there are no measurable variations due to edge parameters in the region of the geometric shadow at distances far from the edge although the resolution and accuracy are several times that of previous work.

The simplicity of the straight edge, which has allowed the extensive theoretical treatments of edge parameters, also makes the straight edge the ideal object for precise, reliable measurements. Measurements were made only with unpolarized light in the region of the geometric shadow. Study of diffraction at wide angles and with polarized light is now in progress and shows definite measurable effects.

In a previous investigation by one of the authors with McDonald[14] some measurements were made on the straight-edge pattern at optical frequencies, and measurable differences due to edge parameters were reported. The accuracy and resolution of the apparatus used was

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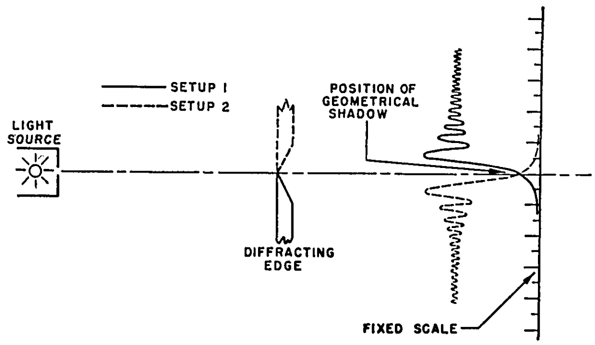

FIG. 1. Precise determination of geometric shadow. A pattern was measured with the diffracting edge in two positions indicated. These two patterns were later aligned on the same scale, and this intersection designates the geometric shadow.

not felt to be completely satisfactory, however. The entire apparatus has therefore been redesigned and constructed with resolution improved by a factor of 4 and with approximately four times the accuracy. In order to obtain this additional resolution and accuracy, it was necessary to lengthen the light path by a factor of 4, and, therefore, a direct comparison with the previous work cannot be made. This improved apparatus allows measurements much farther from the geometric shadow into the diffraction fringes, and thus delves farther into the region where all present theories deviate one from another to a greater extent. In 1937, Hufford[15] was able to detect 40 fringes in the diffraction pattern of a straight edge. This appears to be the largest number of fringes measured previous to this work.

A technique has also been used to measure the precise position of the geometric shadow since this is one region of the pattern where one would expect the parameters of the diffracting edge to have some influence. Previous investigators have assumed the geometric shadow to be at its theoretical position and have then made measurements relative to this point. Although the complete intensity[16] distribution of the diffraction patterns was measured and compared, it is impossible to show this comparison graphically due to close agreement of the various patterns.

II. PRECISE DETERMINATION OF GEOMETRIC SHADOW

Consider the two diffraction patterns produced by a single edge mounted in two different positions such that the line of the edge is precisely in the same position in the two cases as illustrated in Fig. 1. Since the two patterns originate from the same edge, they must be identical patterns mirrored about the line of the geometric shadow. The point where the two patterns have equal relative intensities designates this geometric shadow. Experimentally it is necessary only to identify the position of the two patterns relative to the same fixed scale and later to align the two patterns as if they were produced at the same time. The intersection of the two curves, which can then be ascertained, designates the geometric shadow. The accuracy is limited by (1) how precisely the line of the edge can be returned to the same position in the two cases, and (2) how accurately the two patterns can be positioned relative to the fixed scale. The errors involved are discussed later.

III. APPARATUS

The intensity distribution of the diffraction pattern was measured by a photometric technique. A 1P21 photomultiplier tube cooled to dry-ice temperature detected the light, and a simple scanning mechanism moving at uniform speed transferred the diffraction pattern from a spatial to a time coordinate. After amplification, the signal from the phototube was recorded on a strip-chart recorder. The experiment was performed in a light-tight hallway 180 ft in length as illustrated in Fig. 2. The light source, the edge, and the photometer were mounted on cement piers in order to minimize vibrations. Photometric fluctuations of the mercury H4AB arc lamp used as the light source were monitored, and automatic compensation was effected electronically.

The two-dimensional character of the straight-edge diffraction problem permitted the use of long slits at the source and photometer to increase the total flux incident onto the phototube. Since the accuracy of the experiment was limited by the statistical nature of the emitted light quanta, this increase in flux was of primary importance. Great care was taken to align the two slits properly with the diffracting edge in order to utilize the resolution or coherency available due to the narrow slits. A compromise between resolution and accuracy was necessary when adjusting the slitwidths, and they were finally set at approximately 0.002 in. The source slit was 3 in. long and the photometer slit was approximately 8 in. long.

The edge rather than the photometer was moved to supply the scanning motion since the photometer was found to be sensitive to vibrations; furthermore, spatial variations in the light field incident upon the edge become less significant by an order of magnitude if the pattern is moved across the photometer rather than the photometer moved across the pattern. This reduction in the effect of spatial variation was experimentally investigated and theoretically examined for a very simple variation. Since the theory for comparison assumes a spatially uniform light field, a movement of the edge better fits the theoretical requirements of the experiment. If one assumes a uniform light field and straight-line scanning motion rather than the more

[15] M. E. Hufford, J. Opt. Soc. Am. 27, 408–410 (1937).

[16] As in much of the literature on diffraction, intensity of an element of a diffraction pattern means here the flux passing through unit area of the element. It should not be confused with intensity in its specialized photometric sense of flux per unit solid angle from a source essentially confined to a point.

rigorous circular scan, the position of the edge and the photometer are completely relative, and either can be moved. A fixed scale was mounted to indicate the position of the edge to an accuracy of 0.001 cm. The scale was compared with a calibrated precision Gaertner optical microscope for standardization. The scale was transferred to the strip-chart recorder by manually flashing a light as the moving vernier passed a specific scale mark. The light flash was picked up by the phototube and marked on the chart. Such a marking is shown at left of Fig. 5. The scanning speed was approximately 0.001 cm/sec, which made this method of scale transfer sufficiently accurate.

The two patterns indicated in Fig. 1, which were necessary in order to determine the position of the geometric shadow, were obtained as follows: A pattern was recorded with the edge in position (1); the cross hairs of two cathetometers were focused on the edge, and the edge was then removed and placed in position (2) such that the edge was aligned with the cross hairs of the cathetometer. The precision to which the edge could be replaced was of the order of 0.002 cm and was the limiting factor in the accuracy of the shadow boundary determination. This reversal of the edge was made without moving the scanning mechanism, and thus the same scanning scale applies to both patterns as required by the discussion of Sec. II.

The General Electric H4AB, high-pressure mercury-arc lamp was used as a source with an appropriate optical system for utilizing as much light as possible. The H4AB lamp has an active volume only 1 in. long. The optical system simply collected light emanating from the source in three different directions and formed three images of the 1-in.-long source vertically above one another to form a single 3-in. source with effectively the same luminance. This is possible since resolution in only one direction is needed, and a larger slit gives added flux with no loss in resolution. The monochromator was a simple Littrow-type mounting with a 4-in. grating which would accommodate the 3-in.-high source slit. The 5461 Å line of mercury was used rather than the 4385 Å line because of increased output of the phototube for the 5461 Å line when source intensity and tube sensitivity are combined and because the requirements on resolution for the larger wavelength is slightly less critical since the pattern is more spread out.

Stability in the photometric intensity of the primary light source was of fundamental importance. An arc lamp such as the H4AB is by nature very unstable. Operation of the H4AB from a regulated ac voltage supply improved the stability to a very great extent, but was not sufficient. A relatively simple system was developed to detect continuously and compensate for intensity fluctuations in the primary light source. This system is discussed below.

The photometer was of standard construction with a cylindrical aspheric plastic lens behind the slit to focus

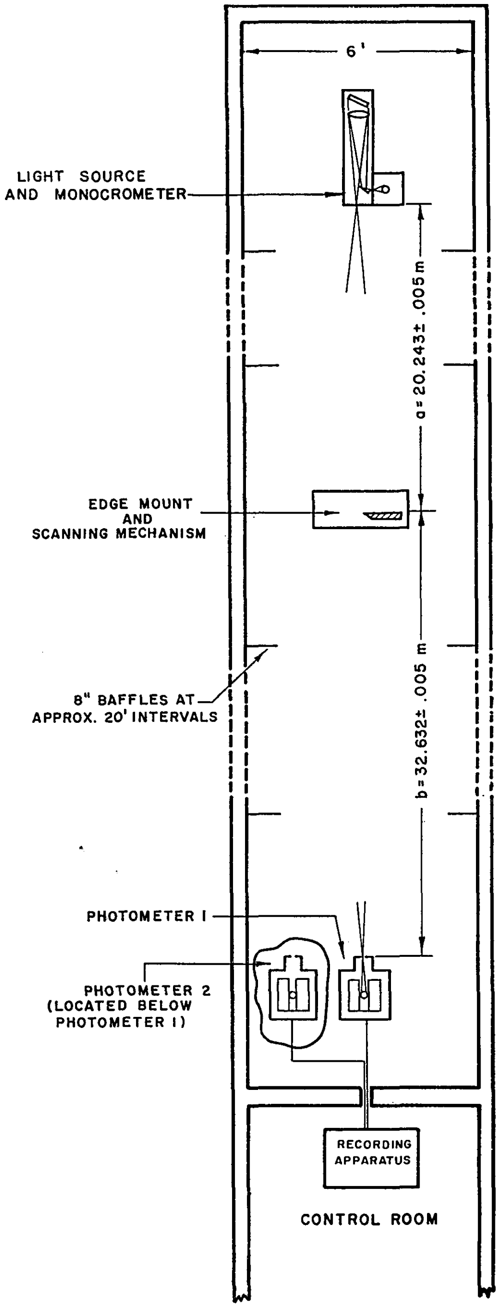

FIG. 2. Instrumental layout of light-tight hallway.

the light onto the phototube. The high-voltage supply for the phototubes was constructed after Fellgett,[17] and the amplifier for each phototube was, with minor modifications, that used by Harris and McDonald in the previous work.

A compensating circuit, which simply performs the operation of division, is a simple modification of a strip-chart recorder to serve as an analog divider. The compensating system can best be understood by analyzing

[17] P. Fellgett, J. Sci. Instr. 31, 217 (1954).

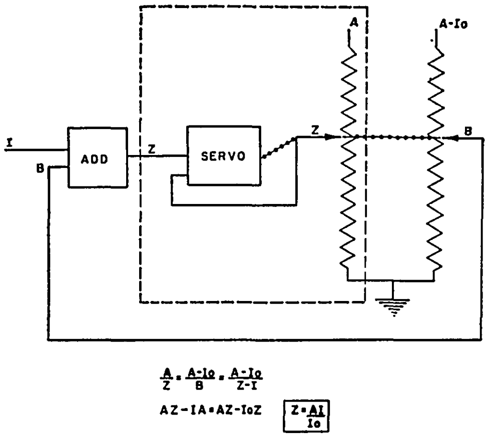

FIG. 3. Analog dividing circuit. (a) Note that all components within the dashed box are precisely the components found in the strip-chart recorder. The dotted lines represent mechanical connections.

the analog computer circuit shown in Fig. 3 in a manner standard to analog-computer theory. Two voltage signals, I and I_0, are applied to the system along with the standard voltage A. The two rheostats are linear and are mechanically tied together. The following relationships hold:

$$A/Z=(A-I_0)/B, \quad B+I=Z,$$

where all symbols indicate voltages in the circuit which vary in time. Eliminating B, we have

$$Z=A(I/I_0).$$

Now we note that the components within the dotted box are precisely the components within a strip-chart recorder. Moreover, the recorder continuously records the value $Z=A(I/I_0)$ on the chart. The voltage I_0-A is obtained by applying a dc bias to the compensating signal by use of the balancing system built into the amplifier. To fabricate the divider circuit it was necessary only to attach mechanically a second potentiometer to the servomotor shaft in the recorder and then to make the necessary electrical connections. Since the electronic system of the recorder is completely isolated from ground, the subtraction operation could be accomplished by a simple wire connection.

With proper adjustment the recorder showed less than 0.5% change for source-intensity changes as high as 200%. In order to eliminate transient effects from very rapid intensity fluctuations, the RC time constant in the two photometers must be accurately equal. In practice, the accuracy and stability of the compensating system were dependent upon the accuracy and stability of the two photometers themselves. Long-term instability in the system was attributed to this cause. One could improve the system by using a split-beam apparatus which utilized a single photometer.

The edges used in this experiment were not intended to be of optical quality. This refinement would be very

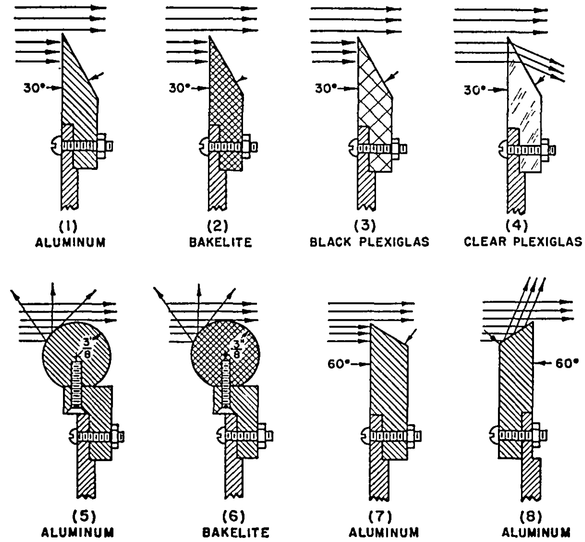

FIG. 4. Cross section of the eight-edges studies. The edges are shown mounted with respect to the incident light.

desirable but would also be extremely difficult and expensive to attain. The edges were machined to an accuracy of the order of 0.001 in. They were 12 in. in length, and the cross sections of the various edges measured are given to scale in Fig. 4. The material of each edge is printed below the edge, and the edges are shown as they were mounted on the edge mount relative to the incident light. The clear plastic edge was studied in order to eliminate the argument that at optical frequencies the dye used in Bakelite of black Plexiglas represented a conducting surface.

IV. ANALYSIS

As previously mentioned, the theories involving the material and shape of the diffracting edge are very difficult to evaluate in the region of the shadow boundary. Furthermore, these theories indicate that the material and shape of the edge give only slight corrections to Fresnel's simplified formulation. For this reason the results of this experiment were compared with Fresnel's theory in which the intensity distribution is given by[18]

$$\left|\frac{U}{U_0}\right|^2=\frac{I}{I_0}=\left|\frac{1-i}{2}\int_{-\infty}^{v} e^{\frac{1}{2}i\pi\tau^2}\right|^2. \tag{1}$$

The variable v is proportional to the distance from the geometric shadow to the point P of the pattern in question and is conventionally defined by

$$v=\left[\frac{2}{\lambda}\left(\frac{1}{a}+\frac{1}{b}\right)\right]^{\frac{1}{2}}S,$$

where a and b are the distances indicated in Fig. 2; λ is

[18] A. Sommerfeld, *Optics* (Academic Press Inc., New York, 1954), p. 259.

the wavelength of the incident light, and S is the distance from the edge to the straight line between the point P and the source. Equation (1) is generally written in terms of the familiar Fresnel integrals $S(v)$ and $C(v)$ as[19]

$$I/I_0=\tfrac{1}{2}[\tfrac{1}{2}+S(v)]^2+\tfrac{1}{2}[\tfrac{1}{2}+C(v)]^2. \qquad (2)$$

For large values of v one can obtain a simplified expression by applying Cauchy's expansion of the Fresnel integrals to Eq. (2) and simplifying. One obtains a solution which is accurate to 0.1% for $v>10$ (i.e., beyond the 25th fringe). This expression is

$$I/I_0=1+(2/\pi v)\sin(\tfrac{1}{2}\pi v^2+\tfrac{1}{4}\pi). \qquad (3)$$

Measurements beyond the 25th fringe were compared with the amplitude and fringe position given by this expression.

The following known instrumental effects were evaluated and allowance made for these effects: (1) the finite slitwidths of the source and the photometer, (2) the spectral intensity distribution of the light source, and, (3) the integrating effect of the *RC* circuit attached at the output of the phototubes. In addition to these corrections allowance was made for any possible error in vertical alignment of the source and photometer slit from the edge and for the lack of perfectly precise straight slits and edges. Only rough estimates of these latter effects could be made, and it is these estimates which limit the over-all accuracy of the experiment. All of the instrumental effects mentioned are by nature averaging or integrating effects and thus reduce the amplitude of the fringes at distances far from the shadow boundary. These same effects are almost negligible and of minor importance for the first few fringes. The fringe amplitude was also slightly decreased by the so-called "dead band" of the recorder which results because the recorder does not respond to very small signal variations.

In evaluating the experimental data the diffraction pattern was divided into three regions for study: (1) the region away from the geometric shadow (fringes 7 to 800), (2) the region of very small diffraction angles (fringes 1 to 5), and (3) the shadow region including the precise determination of the geometric shadow.

In the normalization of the pattern the local intensity distribution was assumed to oscillate around the value of the unperturbed incident light intensity. Theoretically this assumption is true to within 0.1% for values of $v>7$. For $v<7$, a normalization factor was determined by obtaining the best fit of the first three maxima and the first three minima. Before each pattern was recorded, the gain of amplifier (1) was always adjusted to give a nearly normalized trace. A further adjustment was often made as the recorder approached the first few fringes in order to simplify the normalization. A typical adjustment is shown in Fig. 5.

[19] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, New York, 1959), p. 433.

To evaluate the accuracy in determining the fringe position, standardized measurements were made of all apparatus dimensions and scales. A simple error analysis indicated a probable error of 0.025% or 0.005 cm, whichever is greater. A study of the reproducibility of a trace indicated a probable error slightly less than this amount.

The accuracy in the intensity measurements depended upon all the factors for which corrections were estimated above, as well as statistical variations, amplifier linearity, and uniformity of the light field. Some of these effects are time dependent while others are not. All the errors which have a time dependence are manifest in a stability test, and can be considered collectively. The magnitude of the error due to this instability was estimated at 0.1%. Treating all time-dependent errors as statistical errors, a total net accuracy of approximately 0.2% in the experimental values of I/I_0 for the first few fringes was estimated. In the region far from the geometrical shadow the accuracy of the fringe heights becomes increasingly poor as the fringes become smaller since the 0.2% error is relative to I_0 rather than to fringe height. The accuracy in the determination of the geometric shadow is dependent almost entirely upon the error of replacing the edge to the proper position as discussed in Sec. II. This error is of the order of 0.001 in.

V. RESULTS

In the initial analysis a pattern was normalized and then superimposed upon an uncorrected theoretical curve using a translucent tracing box. Except for small statistical fluctuations and a slight damping of the fringe heights, the two curves agreed to within the width of the recorder line for the first 10 to 15 fringes. At higher diffraction angles the experimental pattern was decidedly damped, as would be expected, due to instrumental effects. The recorder linewidth represented approximately 0.2% of I_0; thus, most values of the pattern agreed to within approximately 0.2%.

A typical experimental curve is shown in Fig. 5 alongside the uncorrected theoretical curve. These curves are shown to give the reader an appreciation of the quality of the patterns obtained. The reproduced scale is too small to permit a detailed comparison by the reader. The accuracy given by the authors can only be obtained from the original strip charts. Scales are shown which indicate the actual size of the pattern in space at the photometer and the displacement of the edge required to produce this pattern. It is interesting to note that the direct line of sight from the source to the position of the 800th fringe misses the edge by about 10 cm and the total measured pattern at the photometer covered approximately 25 cm. To the ordinary worker with light diffraction, these distances are impressive.

In Fig. 5, note the scale marks on the experimental curve which positioned the pattern with respect to the

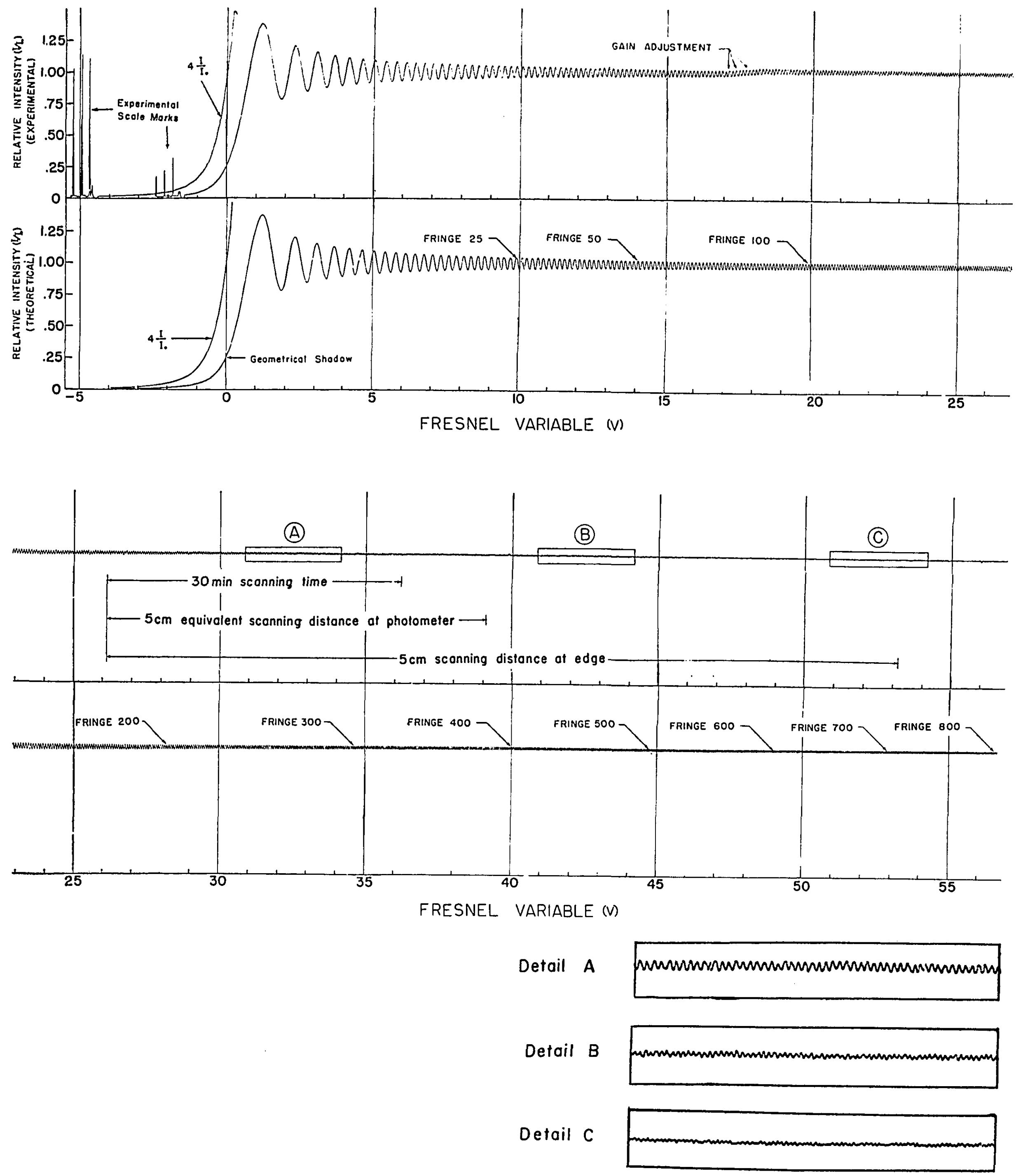

FIG. 5. Typical experimental straight-edge diffraction pattern trace compared with Fresnel theory. The enlarged detailed sections indicate the quality of the strip-chart recording in three regions of the pattern.

fixed scale on the scan. A pattern of the shadow region with the amplifier set at approximately four times the regular gain is also shown superimposed over this region of the regular pattern. In the region beyond the 5th fringe the fringe amplitude and the distance from the fringe maximum to the theoretical geometric shadow boundary were measured for each edge for comparison.

There was no significant difference between various edges since the variations in the data were well within the estimated probable error. There appeared, however, to be definite expansion of the experimental patterns relative to the theory of approximately 0.1%. This variation is of the order of three times the estimated probable error and is felt to be significant.

The data for fringe amplitude for $v>7$ again indicated that there are no significant differences among the edges, and in this case, there is agreement with theory within the limits of accuracy of the experiment.

The effective fringe height of the second maximum and second minimum appears, from the data, to be consistently small. Although fringe one shows no decrease in amplitude, there appears to be some tendency for fringes two, three, and four to be damped from the expected value. This discrepancy is very small, but it is the only intensity measurement where the reproducibility appears to be better than the variation from theory. The apparent difference here is on the limit of detection of the apparatus, and a conclusive statement of its validity cannot be made. The difference is, however, quite consistent in comparison with the other variations throughout the data. The possibility of a systematic error which has not been accounted for is not out of reason, and one would desire a more precise evaluation of systematic errors.

The close agreement of every detail of the patterns produced by each edge indicates very strongly the lack of edge-parameter effects. The slight expansion of all the patterns as compared to the scalar theory seems to indicate a variation due to the inadequacies of the scalar theory. Extensive numerical tables of readings taken from the original strip charts have been compiled, and the above results simply give a summary of these tables.[20]

ACKNOWLEDGMENTS

The authors are indebted to Claude Hooper for help in the design and construction of the apparatus and to the University of Utah Research Committee for financial assistance.

[20] J. D. Barnett, "Effect of edge parameters on Fresnel diffraction of light at a straight edge," dissertation, University of Utah (1959).

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Far-Ultraviolet Auroral Spectra

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Further studies of the spectral data obtained in the February 1960 rocket experiment [Ann. Geophys. **17**, 109 (1961)] have confirmed much of the preliminary analysis. New bands of the Lyman–Birge–Hopfield system in the 1700–2200 Å region have been identified. More precise wavelength determinations have been made, and previously indicated bands of N_2 have been more positively identified. The Lyman α line of hydrogen is just detectable. The intensity distribution among the various bands during weak auroral conditions is shown to differ markedly from more intense displays. Laboratory data obtained from a microwave discharge in very pure N_2 show good correlation with some features of the rocket spectra.

INTRODUCTION

A SCANNING photoelectric Ebert spectrometer was carried to a height of 125 km by an Aerobee Hi rocket at Fort Churchill on February 27, 1960. This experiment was described at the Copenhagen Aeronomy Symposium in July, 1960[1]. A preliminary analysis showed several bands of the second-positive system of N_2, a line of O I at 2972 Å, several bands of the Vegard–Kaplan system, and several weak features in the vacuum ultraviolet region which were tentatively identified as Lyman–Birge–Hopfield bands.

The rocket flight, which was the first of a series that are planned, was primarily an engineering test flight of the spectrometric equipment and a survey of the short-wavelength region for guidance in planning future rocket experiments. The work was carried out on an expedited basis, and only crude wavelength and detector sensitivity calibrations were obtained prior to the flight. However, very careful analysis of the flight data, calibration of the belatedly recovered flight instrument, and subsequent studies of laboratory discharges have permitted very significant extension of the previous analysis.

As a result of this further work, the following new information from the February, 1960, flight is presented in the present paper:

(1) Identification of LBH bands between 1800 and 2200 Å;

(2) Identification of Goldstein–Kaplan bands at 2863 and 3005 Å;

(3) Comparison of Vegard–Kaplan and second-positive band intensities in newly-composed traces of subauroral rocket spectra in the region 2200 to 3500 Å;

(4) An estimate of the relative intensities of the unresolved line of atomic oxygen and the second-positive band of molecular nitrogen at 2972 Å;

[1] W. G. Fastie, H. M. Crosswhite, and T. P. Markham, Ann. Geophys. **17**, 109 (1961).