the manuscript.

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[1] In principle, there exists a second invariant in the fourth-order derivatives of M. However, in a system with translational invariance this term is identical with that given in (1).

[2] See K. G. Wilson and J. Kogut, Phys. Rep. 12C, 75 (1974); S.-k. Ma, Rev. Mod. Phys. 45, 589 (1973); M. E. Fisher, Rev. Mod. Phys. 46, 597 (1974); and A. Aharony, in "Phase Transitions and Critical Phenomena," edited by C. Domb and M. S. Green (Academic, New York, to be published), Vol. 6, for excellent general introductions to the renormalization group and its application to critical phenomena.

[3] This point divides the λ line into two segments (see Fig. 1). The Lifshitz condition restricts the representations to which the order parameter may belong on only one of these segments. See E. M. Lifshitz, J. Phys. (Moscow) 6, 61 (1942); L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, New York, 1968), 2nd ed., Chap. XIV; I. E. Dzyaloshinski, Zh. Eksp. Teor. Fiz. 46, 1420 (1964) [Sov. Phys. JETP 19, 960 (1964)]; S. Goshen, D. Mukamel, and S. Shtrikman, Int. J. Magn. 6, 221 (1974).

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[10] The system anisotropy will, in general, affect other terms in (2) [and also in (5)]. This will be discussed elsewhere.

[11] The classical correlation-function exponents for a Lifshitz point are $\nu_{l4}=\frac{1}{4}$, $\nu_{l2}=\frac{1}{2}$, $\eta_{l4}=\eta_{l2}=0$.

[12] Alternatively, Eq. (4) can be obtained by examining the divergence of the lowest-order diagrams contributing to the four-point function.

[13] To first order in ϵ_l the critical exponent for a uniaxial Ising-type Lifshitz point (i.e., $d=3$, $m=n=1$) are $\eta_{l4}=0$, $\nu_{l4}=\nu_{l2}/2=0.31$, $\alpha_l=\beta_l=0.25$, $\gamma_l=1.25$, $\delta_l=5.0$.

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Muon-Pair Separation Measurements and Comparison with Transverse-Momentum Models*

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Rates of high-energy cosmic-ray muon pairs have been measured for separations up to 70 m. Detailed calculations imply that the mean transverse momentum $\langle p_T\rangle$ of mesons with $x > 0.01$ is 0.66 ± 0.10 GeV/c at laboratory energies of $\gtrsim 10\,000$ GeV. We find that the high-p_T muons result mostly from decay of abundantly produced particles with lifetimes $\gtrsim 10^{-8}$ sec, such as pions and kaons.

We report here measurements of pair separation distributions (decoherence curves) for deep underground muons using a main detector and auxiliary "outrigger" detectors. These data are compared to predictions of Feynman scaling and several p_T models. Previous Utah decoherence data from Coats *et al.*[1] (analyzed by Adcock *et al.*[2] with a different interaction model than used here) had a significant systematic error because of the loss of about a 20% contribution to main-detector decoherence curves due to events with too large a number of muons in the main detector. An expanded buffer memory removed this problem.

The decoherence curve is the coincident counting rate per second per steradian of two small detectors (as a function of their separation X) divided by the product of their areas. For our zenith angles and depths, it is approximately given by $R(X)=R_0 \exp(-X/X_0)$, where $X_0 \approx 5-15$ m. The distance of a muon from the shower axis is approximately $p_T ch \sec\theta/E_\pi$, with p_T the transverse momentum of the muon's parent meson, E_π the parent's energy, h its vertical height of production, and θ its zenith angle. Typical values are $p_T = 0.5$ GeV/c, $E_\pi = 2000$ GeV, and $h \sec\theta = 20$ km, yielding a separation of 5 m.

Measurements for separation ≤ 11 m were made in the main Utah detector, which has been described elsewhere.[3] It consists of 600 cylindrical spark counters arranged in fifteen vertical planes, each 6×10 m^2, and four Cherenkov counters for triggering. The spark counters are sensitive for only 2 μsec after pulsing. Sonic ranging locates the discharge along their axes to ± 3 mm. The detector is $\approx 10^3$ g cm^{-2} thick so that muons are the only detected particles which traverse the entire apparatus. The average measured angle between reconstructed muon tracks is $<1°$ and no evidence of track divergence has been found. On the average ten sparks are associated with each muon in the detector. The original buffer memory was capable of storing information from 108 sparks and the expanded one from 1000 sparks. The average double-muon-event efficiency was 91% in the main detector and showed no change with time.

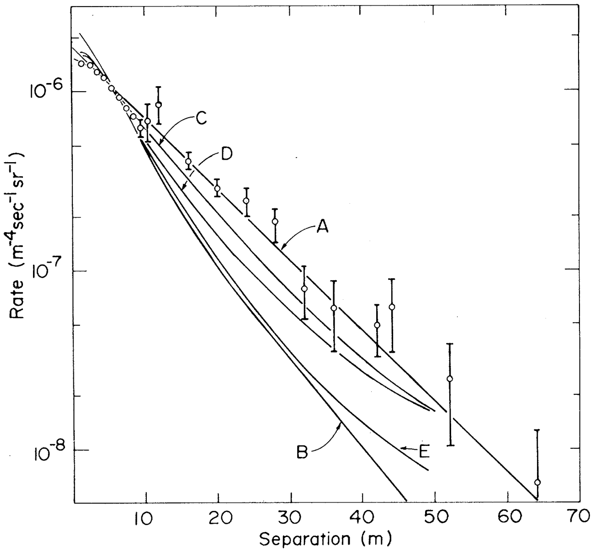

FIG. 1. Muon-pair separation distribution at a zenith angle of 47.5° and a total depth of 2.4×10^5 g cm^{-2} compared to curve A, best fit by an exponential; curve B, scaling-model prediction; curve C, scaling model with $p_T \rightarrow 1.5p_T$; curve D, p_T^{-4}-fit prediction; curve E, p_T^{-8}-fit prediction. Curves $B-E$ are normalized to data at 5 m.

Three outrigger detectors are mounted on movable mine cars in an adjacent tunnel separated from the main detector by a minimum of 800 g cm^{-2} of rock. Each outrigger has eight spark counters in each of three horizontal planes without additional absorbing material. Reconstructed tracks were required to be parallel to main-detector muon tracks to within the outrigger-detector angular resolution of 15°. To determine the spurious background rate the outrigger detectors were pulsed for the equivalent of 20% of the outrigger running time. No acceptable track was observed with three collinear sparks, one in each plane of counters. Outrigger-detector efficiency was 65% with this requirement and a main-detector trigger. The main-detector single-muon overall efficiencies were $\approx 85\%$ for more than $\frac{1}{2}$ of the running time and measured muon intensities agreed within 5% for the entire run. The possible separations were 10–78 m for pairs of a main-detector muon and an outrigger-detector muon. A more detailed description of the experiment is available.[4]

The data were consolidated in slant-depth and zenith-angle bins using parametrized centering functions. Figure 1 shows the decoherence measurements at $\theta = 47.5°$ and a slant depth of 2.4×10^5 g cm^{-2}. Curve A is an exponential fit to the data. The estimated errors are statistical only. The bin from 0 to 1 m was not used because of inefficient recognition for separations <0.3 m. Table I lists the best-fit values for X_0 obtained for individual depth and angle bins using a maximum-likelihood fit and Poisson statistics. The reduced χ^2 values χ_r^2 were computed by consoli-

TABLE I. Measured and predicted X_0.

θ (deg)	Depth (10^{-5} g cm^{-2})	X_0 meas. (m)	χ_r^2	X_0 pred. (m)
47.5	2.4	11.2 ± 0.4	1.5	6.5 ± 0.2
47.5	3.2	6.9 ± 0.3	1.5	4.9 ± 0.2
62.5	3.2	14.9 ± 1.6	3.3	8.5 ± 0.5
62.5	4.0	10.0 ± 1.1	1.7	6.3 ± 0.6
62.5	4.8	7.9 ± 1.4	0.5	4.4 ± 0.8

dating bins with <5 counts. Errors are multiplied by the square root of χ_r^2 for $\chi_r^2 < 1$. There is some indication that $R_0 \exp(-X/X_0)$ does not describe the measurements adequately. At $\theta = 45°$ and a depth of 2.5×10^5 g cm^{-2} our X_0 is 15% lower than that of Coats *et al.*[1]

For purposes of comparison, Monte Carlo calculations have been done using a detailed scaling model described elsewhere.[5] The calculations sample a primary cosmic-ray spectrum proportional to $E^{-2.75}$ and a composition from low-energy observations to develop atmospheric hadronic cascades. Decaying mesons yield high-energy muons. Multiple scattering, geomagnetic deflection, and muon energy losses, with fluctuations, are simulated. The decoherence calculation is primarily sensitive to the assumed p_T distributions. The energy-independent form of this distribution for pions with $x = p_L/p_{\max} > 0.01$ was

$$d\sigma/dp_T \propto p_T^{1.47} \exp[-p_T/\alpha(x)], \tag{1}$$

with $\alpha(x) = 0.141 + 0.172x - 0.172x^2$. This was obtained from a fit to 19.2-GeV p-Be and p-Al inclusive data[6] with $x \gtrsim 0.3$ together with the x dependence of $\langle p_T \rangle$ from the "sea-gull effect" observed in 24-GeV/c p-p interactions.[7] For $x > 0.01$ the assumed $\langle p_T \rangle$ for mesons (both pions and kaons) is 0.41 GeV/c.

The decoherence calculations are somewhat sensitive to the total hadron-air inelastic cross sections and to the details of the assumed primary-cosmic-ray spectrum and composition (compositions ranging from predominantly protons to predominantly Fe nuclei were tried). Resonable variations of these quantities, including variation of the primary spectrum from $E^{-2.50}$ to $E^{-2.85}$, change the calculated average separation, X_0, by approximately 15%. Uncertainties due to assumed inclusive particle spectra, etc., are less important if scaling is approximately valid at the relevant energies.

Curve B of Fig. 1 is predicted by the above model. The measured and calculated values of X_0 are compared for all five depth-angle combinations in Table I. Roughly, X_0 is proportional to the parent meson's $\langle p_T \rangle$. The ratio of measured to calculated X_0 with minimum χ^2 for the values in Table I is 1.6. The reduced χ^2 is 1.9, indicating that use of a single ratio for all depths and angles is too simplistic or that there are neglected systematic errors. Multiplying all Monte Carlo p_T by 1.5 improves the agreement with the observations. Curve C of Fig. 1 shows the result at 47.5° and 2.4×10^5 g cm^{-2}. For these predictions, the best ratio of measured to predicted X_0 is 1.08, again implying a factor of $(1.08)(1.5) = 1.6$ increase in $\langle p_T \rangle$. Thus, $\langle p_T \rangle$ for all mesons with $x > 0.01$ in hadron-air collisions with incident particle energies of about 8–20 TeV is found to be 0.66 ± 0.10 GeV/c. Comparison of $\langle p_T \rangle$ for particles with $x > 0.01$ with that from extremely high-energy data which include particles from the region $x \approx 0$ is misleading since $\langle p_T \rangle$ is strongly affected by the numerous particles with $x < 0.01$ with lower p_T because of the sea-gull effect and the E^{-1} dependence of $d^3\sigma/dp^3$. The region with $x < 0.01$ is not so important for our data or for lower-energy measurements made in the entire x region.

Our scaling-model calculations show that the average energies per nucleon of primary nuclei yielding pairs of muons are independent of the primary's atomic weight, A, and are 30–70 TeV/nucleon for depths 2.4×10^5–4.8×10^5 g cm^{-2}. Muons in observed pairs are typically produced in several generations of atmospheric interactions. Median laboratory energies of these collisions are about 8 TeV at 2.4×10^5 g cm^{-2} and 20 TeV at 4.8×10^5 g cm^{-2} and mesons and nucleons are of comparable importance as incident particles. Most pairs result from separate collisions, especially when the primary is not a proton.

If the p_T distributions consist of two components, with the low-p_T component given by Eq. (1), then comparison of the data and predictions indicates that $\geqslant 30\%$ of the mesons are in the high-p_T component. This component could conceivably result from direct decay to muons of such "new" particles as the $\psi(3.1)$. But muons from particles with lifetimes $\tau \ll 10^{-8}$ sec (or from direct production processes) would not display the $\sec\theta$ zenith-angle dependence of muons from pion and kaon decay. The Utah muon-intensity work[8] shows that $\leqslant 10\%$ of the muons at 3.2×10^5 g cm^{-2} may be produced without the $\sec\theta$ dependence (isotropic production), inconsistent with the $\geqslant 30\%$ of the muons in the high-p_T component. An isotropic high-p_T component would also give relatively fewer high-p_T muons at 62.5° than for 47.5° at a depth of 3.2×10^5 g cm^{-2}, contrary to the data in Table I. High-p_T muons apparently result mainly from the decay of particles with $\tau \gtrsim 10^{-8}$ sec, such as pions and kaons.

Accelerator experiments at CERN intersecting storage rings and Fermilab, other cosmic-ray experiments, and theoretical arguments suggest[9] that for high p_T

$$E\, d^3\sigma/dp^3 \propto p_T^{-N} f(x_T), \tag{2}$$

where $x_T = p_T/p_{\max}$ and f is energy independent. Halzen and Luthe[10] have found expressions giving good descriptions of accelerator high-p_T interactions with either $N=4$ or $N=8$ ultimately prevailing. We have calculated the coherence curves at 47.5° using the p_T dependences given by the p_T^{-4} and p_T^{-8} Halzen-Luthe formulas. The high-p_T component is assumed to be enhanced by $A^{1/3} = 2.45$ for p-air relative to p-p collisions.[11] Otherwise the calculations follow our ordinary model.

Curves D and E in Fig. 1 are predictions for these models. The shape of the p_T^{-8} curve (curve E) is in poor agreement with the 2.4×10^5-g-cm^{-2} data but the p_T^{-4} curve is in better but not good agreement, thus weakly favoring the p_T^{-4} formula. In the energy range of interest, this formula gives $\langle p_T\rangle = 0.50-0.67$ GeV/c and for $N=8$ $\langle p_T\rangle = 0.39-0.40$ GeV/c. Our calculations show that $\langle p_T\rangle$ is model dependent and increases from 0 to ~2 GeV/c as a function of separation in the range 0-50 m, implying that $x_T = p_T/p_{\max} \lesssim 0.03$ and the c.m. system angles are $\lesssim 10°$. Several assumptions we used should be emphasized. We assumed that the p_T distribution shapes are x independent and identical for incident mesons and nucleons. (If incident mesons produce relatively more high-p_T mesons than incident nucleons, the p_T^{-8} law might agree better with the data.) Correlations of p_T for particles produced in the same interaction were neglected, perhaps lowering the predicted pair rate at large separations.

Good agreement with multiple-muon rates for events with ≤ 30 muons was obtained previously using our scaling model with $\langle p_T\rangle = 0.41$ GeV/c.[5] Fair agreement is still possible[12] with the multiple-muon rates using the broadened p_T distributions, but this requires a higher primary-cosmic-ray intensity or a greater abundance of heavy primary nuclei at energies from 10^{14} to 10^{16} eV than was obtained with $\langle p_T\rangle = 0.41$ GeV/c. For example, a spectrum fitted[12] to the muon data from a model in which all p_T values were increased by a factor of 1.5 yielded a primary rate at 3×10^{15} eV which was about 1.4 times higher than that obtained with $\langle p_T\rangle = 0.41$ GeV/c. We have discussed the problem of the consistency of the primary spectrum obtained from this work with that from extensive air-shower data elsewhere.[13]

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