correspondence on this point.

[15]Andrews *et al.* (Ref. 13). They cite a phase ambiguity which I have resolved by choosing the solution consistent with SU(3) symmetry. For this solution $\Gamma(\rho\to\eta\gamma) = 50\pm 13$ keV. (The other solution would give $\xi^2 = 0.8 \pm 0.2$.)

[16]I am grateful to Tom Weiler for emphasizing this to me.

Measurement of $\pi^- p \to \pi^- \pi^+ n$ near Threshold and Chiral-Symmetry Breaking

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(Received 2 October 1979)

$d^2\sigma/d\Omega\, dT$ for π^+ mesons produced in $\pi^- p\to\pi^-\pi^+ n$ was measured at seven incident energies between 203 and 357 MeV and the integrated reaction cross section was calculated. The matrix element, when extrapolated to threshold and compared with soft-pion calculations, determined the chiral-symmetry-breaking parameter $\xi = 0.05\pm 0.26$, which is consistent with the Weinberg Lagrangian. The large hard-pion contributions at 203 MeV demonstrated the absolute necessity for comparing at threshold.

Soft-pion calculations for several processes are unambiguously determined with but one free parameter, the pion decay constant f_π, and have been found reasonably consistent with measurements. Pion decay, the Goldberger-Treiman relation, πN S-wave scattering lengths, and the $\pi\pi$ P-wave isovector scattering length all yield values for f_π between 82 and 94 MeV.[1] This degree of success is remarkable since the hypotheses of soft-pion theory are strictly valid only for pions with vanishing four-momenta, not for physical pions with mass. Variations in f_π may reflect both the differing importance of the m_π/M_N corrections to the various soft-pion predictions and the limited accuracy in the measurements.

Soft-pion calculations for other processes to which $\pi\pi$ scattering can contribute depend on an additional parameter ξ which is determined by the nature of the chiral-symmetry breaking. The validity of soft-pion calculations for these processes has not been critically tested. Single pion production in pion-nucleon scattering $\pi N\to\pi\pi N$, which depends on the $\pi\pi$ S-wave isoscalar and isotensor scattering lengths, and hence on ξ, is such a process and has been calculated by several authors.[2-5] Although the charge state $\pi^- p \to \pi^-\pi^+ n$ is amenable to measurement, significant comparison with experiment to determine ξ has been elusive. The limited accuracy with which the small cross sections near threshold could formerly be measured have made extrapolation of the matrix element to threshold, where comparison must be made, less than convincing. Nevertheless, calculations with ξ between ± 1 have agreed roughly with experiments. In an alternative analysis,[6] comparison of the nonresonant $SP11(\epsilon N)$ wave of an isobar model, fitted to bubble-chamber events between 324 and 396 MeV with the $SP11(\epsilon N)$ portion of the soft-pion prediction gave $\xi = -0.3\pm 1.6$.

This Letter reports a study of the reaction $\pi^- p \to \pi^-\pi^+ n$ with the improved accuracy made possible by the intense π beams at the Clinton P. Anderson Meson Physics Facility (LAMPF) in an experiment specifically designed for extrapolation to threshold and comparison to soft-pion calculations. The doubly differential cross section $d^2\sigma/d\Omega\, dT$ for π^+ mesons produced was measured at locations distributed uniformly over the accessible portion of T-$\cos\theta$ space in the center-of-momentum system for seven incident energies between 203 and 357 MeV, immediately above the 172.4 MeV threshold. Inevitably, the fraction of T-$\cos\theta$ space studied varied from 85% at 357 MeV to 35% at 203 MeV. With a π^- beam incident on a cylindrical liquid hydrogen target, particles of appropriate charge, angle, and momentum passed through a 180°, double-focusing magnetic spectrometer *in vacuo*, were focused onto a ladder of solid-state dE/dx detectors, and continued outside the vacuum to the trigger detectors and a fluorocarbon, threshold Cherenkov detector. π^+ from double charge exchange, e^+ from single charge exchange, and $e^\pm$ from nuclear reactions all contributed a background. These were eliminated by dE/dx, the Cherenkov detector, absorbing low-energy $e^\pm$, and subtracting the rate of events not from hydrogen which was measured with the target empty.

The reaction cross sections were normalized to π^-p elastic scattering. At each incident energy, relative π^-p elastic cross sections were measured at several angles and the collection fitted to the "known cross sections" with a scale factor which calibrated the system as a whole. Below 300 MeV, "known cross sections" were generated by a subroutine SCATPI[7] which had been derived from the phase-shift analysis of Carter, Bugg, and Carter.[8] Above, they were extrapolated graphically directly from measurements in this energy range.[9] The same observations of elastic scattering also measured the momentum of the beam and its rms width which were required in subsequent analysis.

The doubly differential cross section varies rapidly with kinematic quantities through its dependence on the density in phase space, but the matrix element itself is expected to vary slowly. Hence, its squared modulus $[|M|^2]_{av}$, weighted and averaged over unobserved variables of phase space, incident momentum, and angular and momentum acceptance of the spectrometer, was calculated directly from the measured rate of events for each setting of angle and momentum. Kinematic variables were calculated with the same average. From these, the doubly differential and integrated reaction cross sections could be derived.

An integrated reaction cross section σ and the squared modulus of the matrix element corrected for Coulomb attraction in the final state and averaged over all phase space $\langle|M_c|^2\rangle$ were computed

TABLE I. Integrated reaction cross section and $\langle|M_c|^2\rangle$ with error.

$T_{\rm inc}$ (MeV)	σ (μb)	$\langle\lvert M_c\rvert^2\rangle$ (m_π^{-6})	Error (%)
203	13.8	16.7	10.9
230	60.3	20.2	5.3
255	166	25.8	3.7
279	374	33.7	3.9
292	546	38.6	5.6
331	1160	44.9	4.5
357	1880	52.7	4.1

for each energy of incident beam and are given in Table I. All $[|M|^2]_{av}$ for a given incident beam were fitted to a function $\overline{C}F$, where $\overline{C}=[\exp(\pi\alpha/\beta)]_{av}$ and $F=a+bp\cos\theta+c(p\cos\theta)^2+dp^2+ep\cos\theta/(1+fp^4)$. $\overline{C}$ is the Coulomb correction for the final state averaged over the unobserved variables. The first four terms of F were the beginning of an expansion of $[|M|^2]_{av}$ in the measured, kinematic variables.[10] The last term, with f large, was contrived to fit a sizable variation with $\cos\theta$ at small, but not zero, p which appeared at 331 and 357 MeV, energies within the half width for the $\pi^-\Delta^+$ intermediate state. The minimum number of constants a through f which gave a satisfactory χ^2/ν were adjusted. The remaining constants were set to zero. The incident energy given in Table I was the average of the incident energies for the individual $[|M|^2]_{av}$ weighted by the inverse squared fractional error in $[|M|^2]_{av}$. σ and $\langle|M_c|^2\rangle$ were calculated from the definitions $\sigma=\int\overline{C}F(d^2\sigma_0/d\Omega\,dT)\,d\Omega\,dT$, and $\langle|M_c|^2\rangle=\int F(d^2\sigma_0/d\Omega\,dT)\,d\Omega\,dT/\sigma_0$. Integration extended over all phase space. In this expression, $d^2\sigma/d\Omega\,dT$ is the density in phase space integrated over unobserved variables and multiplied by the flux factor and momentum squared of an incident particle all in the center-of-momentum system.[11]

Figure 1 displays the integrated reaction cross sections from this experiment with their errors, both statistical and systematic, a curve generated by the function $(g+hT_{\rm tot})\sigma_0$, with $T_{\rm tot}$ equaling total energy minus threshold energy, and also a selection of the more accurate integrated reaction cross sections from previous experiments with their stated errors.[12-19] Measurements with errors 40% or greater are not shown. The new cross sections agree particularly well with previous measurements of comparable accuracy at higher energies. However, they are systematically higher than most remaining previous mea-

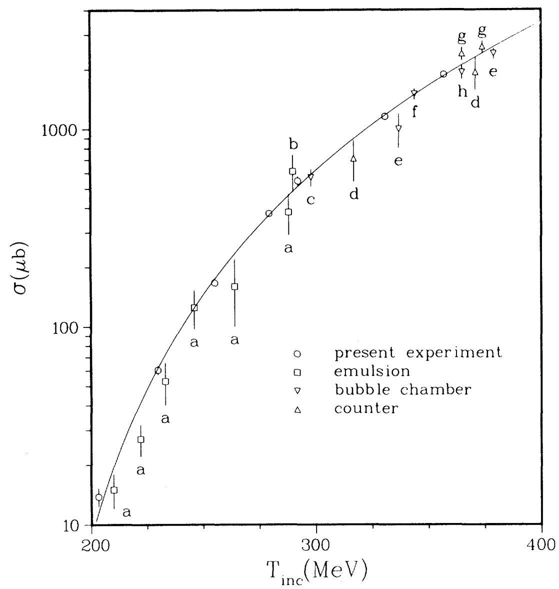

FIG. 1. Integrated reaction cross section for $\pi^- p \rightarrow \pi^- \pi^+ n$ vs incident kinetic energy from the present and selected previous measurements: *a*, Ref. 12; *b*, Ref. 13; *c*, Ref. 14; *d*, Ref. 15; *e*, Ref. 16; *f*, Ref. 17; *g*, Ref. 18; and *h*, Ref. 19.

surements of lesser accuracy which span the entire range of energy.

Figure 2 displays the $\langle |M_c|^2 \rangle$ from this experiment with their errors and a curve generated by the function of $r + sT_{tot}$ which was fitted to these with $\chi^2/\nu = 1.17$. This curve, when extrapolated to threshold, gave a plausible estimate of the matrix element

$$\langle |M_c|^2 \rangle^{1/2} = (2.42 \pm 0.28) m_\pi^{-3}$$

which could be directly compared to the soft-pion prediction[3] of $\pm \langle |M_c|^2 \rangle^{1/2} = (-2.475 + 1.093\xi) m_\pi^{-3}$ to determine ξ. This expression assumed $g_A/g_V = -1.253$ and $G^2/4\pi = 14.6$ which corresponded to $f_\pi = 86.9$ MeV via the Goldberger-Treiman relation. If the negative sign were selected, the symmetry-breaking parameter became[20]

$$\xi = 0.05 \pm 0.26.$$

If the positive sign were selected, $\xi = 4.48 \pm 0.26$. The ambiguity in sign was partly resolved in favor of the first choice by comparing the soft-pion predictions implied by the ξ with the rudimentary measurements for the $\pi^+\pi^0 p$, $\pi^+\pi^+ n$, and $\pi^0\pi^0 n$ final states available.[21] The ξ determined by the procedure necessarily depended on the m_π/M_N corrections introduced and ignored in the calculation, since the extrapolation did not reach the

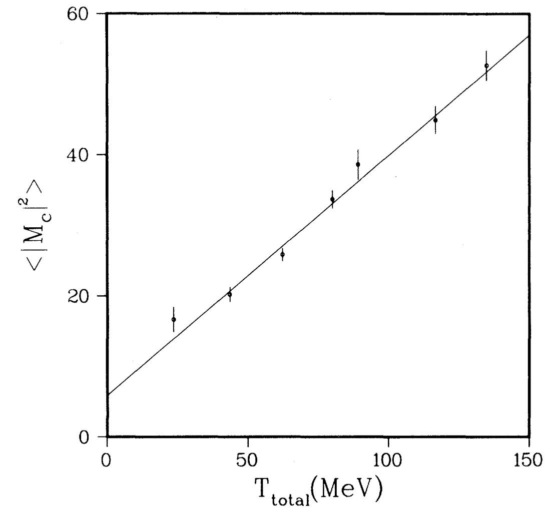

FIG. 2. Squared modulus of the matrix element $\langle |M_c|^2 \rangle$ vs total kinetic energy. The curve indicates extrapolation to threshold.

nonphysical limit where the pion four-momenta vanish. It is clearly compatible with $\xi = 0$ required for the Weinberg Lagrangian[22] which is substantiated by the present quark models; it is inconsistent with $\xi = 1$ or -2 required for the two Schwinger Lagrangians.[23]

The absolute necessity of systematically extrapolating measurements of $\pi N \rightarrow \pi\pi N$ to threshold before comparing with soft-pion calculations which are intended to be valid at threshold is strikingly demonstrated by Fig. 2. The variation of the averaged, squared moduli with incident energy measured the hard-pion contribution. Even at the closest approach to threshold 203 MeV, $\langle |M_c|^2 \rangle$ was about half the hard-pion contribution. Since this cross section is already small at 203 MeV and phase space varies as about T_{tot}^2, no measurement would be practical at a single incident energy near enough threshold to represent validly the soft-pion limit.

Soft-pion calculations[3] also predicted the isoscalar and isotensor S-wave $\pi\pi$ scattering lengths with the same f_π and ξ to be $a_0 = 0.180(1 - 5\xi/14) m_\pi^{-1}$ and $a_2 = -0.026(2 + \xi) m_\pi^{-1}$. These assumed the coupling constants given above. With ξ found in this analysis,

$$a_0 = (0.177 \pm 0.017) m_\pi^{-1}$$

and

$$a_2 = (-0.053 \pm 0.007) m_\pi^{-1}.$$

The isoscalar $\pi\pi$ scattering length found here agreed reasonably with the $a_0 = (0.28 \pm 0.05) m_\pi^{-1}$

deduced from K_{e4} decay.[24] This satisfactory comparison of a_0 further resolved the ambiguity in sign noted above; the positive sign would have led to $a_0 = (-0.108 \pm 0.017) m_\pi^{-1}$ and $a_2 = -0.168 \pm 0.007$, a significant disagreement with K_{e4} decay.

The above conclusions are substantiated by an alternative analysis in the following Letter[25] in which both the σ reported here and bubble-chamber events between 324 and 396 MeV were fitted to an isobar model with a chiral-symmetry background. The present experiment and analysis have established ξ accurately enough to distinguish among several of the effective Lagrangians suggested for soft-pion calculations. With ξ now reasonably constrained, an experiment on another final state for $\pi N \to \pi\pi N$ suitable for extrapolation to threshold would critically test application of soft-pion calculations to this reaction. The ξ found for a new final state must agree approximately with that from the present experiment or the application of soft-pion calculations would be invalid.

Several of us gratefully acknowledge the hospitality of LAMPF and the assistance of its staff which made this experiment possible. This research was supported in part by the U. S. Department of Energy under Contract No. EY-76-5-02-2197 and W-7405-ENG-36.

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