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Citation: AIP Conference Proceedings **838**, 572 (2006); doi: 10.1063/1.2210421
View online: http://dx.doi.org/10.1063/1.2210421
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/838?ver=pdfcov
Published by the AIP Publishing

Investigation of a Single-Point Nonlinearity Indicator in One-Dimensional Propagation

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Abstract. The influence of nonlinear effects in jet noise propagation is typically characterized by examining changes in the power spectral density (PSD) of the noise as a function of propagation distance. The rate of change of the PSD is an indicator of the importance of nonlinearity. Morfey and Howell [AIAA J. 19, 986-992 (1981)] introduced an analysis technique that has the potential to extract this information from a measurement at a single location. They develop an ensemble-averaged Burgers equation that relates the rate of change of the PSD with distance to the quantity Q_{p^2p}, which is the imaginary part of the cross-spectral density of the pressure and the square of the pressure. With the proper normalization, spreading and attenuation effects can be removed, and the normalized quantity represents only spectral changes which are due to nonlinearity. Despite its potential applicability to jet noise analysis, the physical significance and utility of Q_{p^2p} have not been thoroughly studied. This work examines a normalization of Q_{p^2p} and its dependence on distance for the propagation of plane waves in a shock tube. The use of such a controlled environment allows for better understanding of the significance of Q_{p^2p}.

Keywords: nonlinearity indicators, jet noise
PACS: 43.25.Ba, 43.25.Cb

INTRODUCTION

The study of the propagation of jet noise often uses the power spectral density (PSD) as the quantity of interest in assessing impact on the surrounding community. Many factors can influence the evolution of the PSD as the noise propagates, not least of which is nonlinearity. In any jet noise prediction scheme, it is important to determine whether nonlinearity affects the propagation and, if so, to account for it.

It has been shown that a linear prediction model does not accurately represent the propagation of noise from full-scale [1] and model-scale [2] jets. For these cases, the presence of nonlinearity was established by examining changes in the PSD relative to a linear prediction, especially for higher frequencies and larger propagation distances. This requires measurements at multiple locations. There can be difficulties and limitations with such a method. For full-scale measurements, factors such as ground reflections, wind and temperature gradients, and the complex nature and directivity of the source can affect the evolution of the PSD. It can be difficult to separate the influence of these effects from that of nonlinearity. For model-scale measurements, the ability to capture spectral changes due to nonlinearity is often limited by the

CP838, *Innovations in Nonlinear Acoustics: 17th International Symposium on Nonlinear Acoustics*, edited by A. A. Atchley, V. W. Sparrow, and R. M. Keolian

frequency bandwidth of the measurement system and the maximum propagation distances dictated by the measurement space. In light of these limitations, it would be beneficial to be able to determine the presence or importance of nonlinearity with a measurement at a single location.

Morfey and Howell [3] derive an expression containing a quantity that has the potential to serve as a single-point nonlinearity indicator. A normalization of this quantity, often referred to as "Q/S" or "the Morfey-Howell nonlinearity indicator", has recently been used by several researchers [1, 2, 4] in the analysis of high-amplitude nose. However, its physical meaning is not well understood and has not yet been thoroughly investigated.

THEORY

The present analysis follows that of Morfey and Howell [3]. Its basis is the Burgers equation,

$$\frac{\partial p}{\partial x} - \frac{\beta}{\rho_o c_o^3} p \frac{\partial p}{\partial \tau} = \frac{\delta}{2} \frac{\partial^2 p}{\partial \tau^2}, \tag{1}$$

where p is acoustic pressure, x the distance from the source, β the coefficient of nonlinearity, ρ_o the ambient density, c_o the equilibrium sound speed, τ the retarded time, and δ the diffusivity of sound [5]. After some manipulation, including transformation to the frequency domain and ensemble-averaging, they obtain an equation similar to

$$\frac{d}{dx}\left(e^{2\alpha x} S_p\right) = -\omega \frac{\beta}{\rho_o c_o^3} e^{2\alpha x} Q_{p^2 p}, \tag{2}$$

where S_p is the power spectral density and Q_{p^2p} the imaginary part of the cross spectral density of p^2 and p. The term in parentheses on the left-hand side of Eq. (2) is the absorption-corrected PSD. According to linear theory, the spatial derivative of this quantity is zero. Thus, Q_{p^2p} is a measure of the rate of nonlinear distortion. For a given frequency and propagation distance, a positive value for the right-hand side of Eq. (2) indicates a gain of energy, and a negative value indicates a loss of energy.

A physical interpretation of this equation can be drawn from the normalized harmonic amplitudes of an initially sinusoidal wave undergoing nonlinear propagation. If these amplitudes are plotted as functions of distance, as in Fig. 4 of Blackstock [6], the slope of a curve at any given point is qualitatively represented by the spatial derivative on the left-hand side of Eq. (2). With this interpretation, theory suggests that the fundamental and second harmonic will be of most importance near the source, and the higher harmonics will gain importance as propagation distance increases. The data presented in this paper are for a narrowband noise source. The theory for such a source, developed by Gurbatov & Rudenko [7], predicts that the spectral evolution of a noise source is qualitatively similar to that of a sinusoidal source but happens more quickly.

EXPERIMENTS

The data presented in this paper were obtained in a plane wave tube constructed of PVC pipe with an inner diameter of 5.21 cm and a total usable length of 9.68 m. It is fitted with two JBL 2402H drivers at one end, four B&K 6.35 mm type 4938 microphones spaced equally along its length, and a fiberglass anechoic termination at the other end. The first cross mode of the tube occurs at approximately 3.8 kHz. For a more complete description of the apparatus and a validation of the sound propagation in the tube, see Refs. [8] and [9]. All data shown here have a source frequency band centered at 2.9 kHz with half-power points at approximately 2.8 and 3.0 kHz.

RESULTS

The dependence of Q_{p^2p} on source amplitude and propagation distance was investigated by plotting the right-hand side of Eq. (2) for various experimental conditions. For Figs. 1(a) and 1(b), measurements from the first microphone (0.10 m propagation distance) were used to calculate Q_{p^2p} for four different source amplitudes.

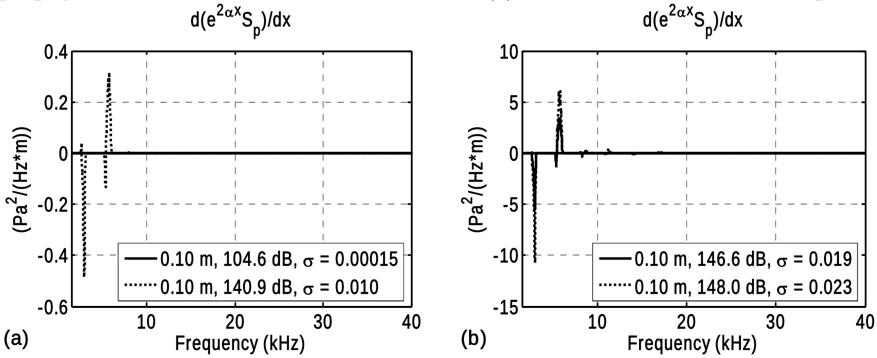

FIGURE 1. Right-hand side of Eq. (2) at a fixed distance of 0.1 m for four different source amplitudes.

In all cases in Fig. 1, energy is being lost in the fundamental frequency region and gained in the second harmonic frequency region. This is consistent with the theory discussed above. It should be noted that although the values for the lowest amplitude case (104.6 dB re 20 μPa in Fig. 1(a)) appear to be zero, they exhibit the same trend as in the other plots but at a much smaller amplitude.

Figures 2(a) and 2(b) depict the right-hand side of Eq. (2) at all four microphone locations for the 148 dB source condition. The value for the fundamental frequency region is always negative, and the (nonzero) values for the higher harmonic regions are always positive, indicating that energy is continually transferred upward in the spectrum. The higher harmonic regions do not show significant nonlinear growth until larger propagation distances. These observations are similar to those made of an initially sinusoidal signal [8].

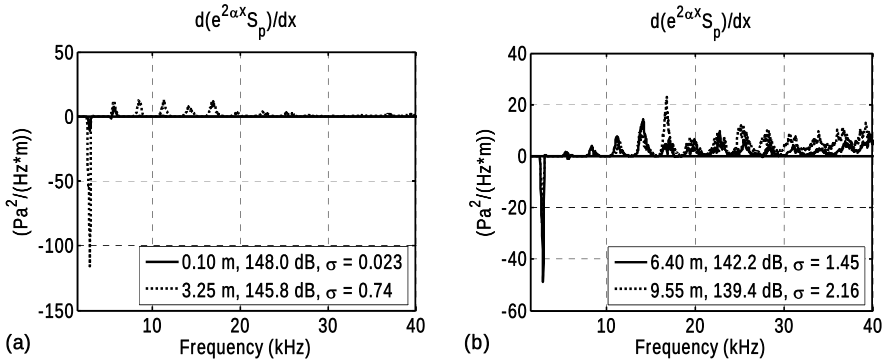

FIGURE 2. Right-hand side of Eq. (2) at four different distances for the same 148 dB source condition.

CONCLUSIONS

The quantity Q_{p^2p} has been identified as being related to the nonlinear distortion rate of narrowband noise. The quantity has been shown to behave qualitatively as would be expected given the theory of the evolution of an initially narrowband noise signal. Its dependence on amplitude and on propagation distance have been demonstrated.

ACKNOWLEDGMENTS

The authors would like to thank the National Science Foundation, the Office of Naval Research, and the Strategic Environmental Research and Development Program for supporting this work.

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