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4aPA8. Radiation of finite-amplitude waves from a baffled pipe

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The radiation of finite-amplitude waves from the open end of a baffled, circular pipe is considered as a direct continuation of work begun by Kuhn, Blackstock, and Wright more than three decades ago [Kuhn et al., *J. Acoust. Soc. Am.* 63, S1, S84 (1978)]. In this paper, a 1 kHz sinusoidal pulse with initial peak pressure amplitude of nearly 1.2 kPa has been propagated down a 6.1 m pipe, whose open end (5.1 cm inner diameter) has been placed off-center in a large rectangular baffle. As the steepened or shock-like waves exit the pipe, the measured waveforms are comprised of sharp impulses that are delta function-like in nature, particularly on axis. Although linear piston theory predicts similar waveform shapes, there is also evidence that nonlinear propagation of these impulses, which exceed a peak pressure amplitude of 1.5 kPa near the pipe opening, is occurring. http://asadl.org/jasa/resource/1/jasman/v63/iS1/pS84_s5

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INTRODUCTION

Propagation of finite-amplitude acoustic waves down an open-ended pipe can result in radiated waveforms that contain large positive impulses, particularly on axis. For example, Kim and Setoguchi¹ demonstrated this behavior in their study of unsteady weak shocks exiting a baffled pipe, and Nakamura and Takeuchi² performed a frequency-domain analysis of N-wave radiation from an unbaffled pipe. Although both of these examples involved transient radiation, the phenomenon extends to continuous waves. Both Hirschberg *et al.*³ and Thompson and Strong⁴ have shown that nonlinear wave steepening occurs inside trombones when played at fortissimo levels and that sharp, delta function-like peaks occur outside the bell. Similar waveforms were also documented by Gee *et al.*⁵ when the U. S. Army Research Laboratory's Mobile Acoustic Source was driven with initially sinusoidal waves at high amplitudes. In the late 1970's, Blackstock, Wright, and Kuhn^{6,7} investigated continuous-wave, finite-amplitude radiation from the end of a flanged pipe. Their setup consisted of a 3 m long pipe with a 5.1 cm inner diameter with a flanged opening. They drove the pipe using 8 kHz sinusoidal pulses at 138 dB re 20 μ Pa while pressure measurements were made at various locations outside of the pipe. An example of a single cycle is shown in Fig. 1 and is representative of the highly asymmetric pulses that were measured. The authors considered the nonlinear steepening inside the pipe but compared their results with linear piston theory outside. This paper outlines the continuation and extension of work by Blackstock *et al.*⁶ by analyzing the production and propagation of high-amplitude, continuous waveforms from the end of a baffled pipe. The experimental setup uses a longer pipe driven with a higher source amplitude and a larger baffle. Furthermore, the excitation frequency has been limited to below the first cross mode of the pipe, which will eventually facilitate comparison with piston theory. Results discussed in this paper are limited to 1 kHz sinusoidal pulses with initial peak sound pressure levels of 155 dB re 20 μ Pa.

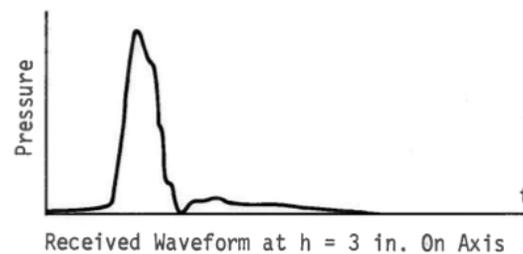


FIGURE 1. A single cycle of a radiated positive pressure impulse train, as measured by Blackstock *et al.*⁶ The data were acquired with a microphone placed on axis at a distance of 7.6 cm (3 in) from the end of a flanged pipe, driven with an 8 kHz sinusoidal pulse at 138 dB re 20 μ Pa.

EXPERIMENTAL SETUP

Like the original experiments by Blackstock *et al.*,⁶ a 5.1 cm inner diameter pipe was used, which limits the initial excitation bandwidth to below 4 kHz to prevent cross modes. A BMS 4592 compression driver with a 5.1 cm throat diameter, 1300 W peak power capability, and a nominal frequency response between 300 – 7000 Hz was used in conjunction with an 1100 W Crown XS1200 Power Amplifier. Signal output and acquisition was carried out using National Instruments PXI-446X series cards at a sampling rate of 204.8 kHz and pressure data was obtained using 3.18 mm 40DD GRAS microphones. Four microphones were placed along the pipe at distances of 5.7, 310.5, 606.4, and 607.7 cm from the driver (see Fig. 2). The microphones were mounted, without grid caps, so the diaphragms were flush with the inner wall surface of the pipe. A fifth microphone was moved to various positions at the height of the pipe centerline in order to measure the waveforms radiated from the end of the pipe. The exit of the pipe was flush mounted into a 1.23 m by 1.24 m baffle made from medium density fiberboard. The pipe axis was located 9.5 cm horizontally and 8.2 cm vertically from the center of the baffle in order to reduce waves scattered by the edges of the baffle from arriving coherently at the on-axis microphone locations. As shown in Fig. 2, the baffle was placed in an 8.71 m x 5.66 m x 5.74 m anechoic chamber with the driver side of the pipe running out the door. During all measurements the anechoic doors to the chamber were closed around the pipe to limit reflections from the hallway.

To create the radiation of these sharp impulses, a steepened or shock-like wave must be generated in the pipe. However, an initial sawtooth response cannot be used because the higher harmonics would likely generate cross modes within the pipe. Thus, to produce a shock-like response at the end of the tube, significant nonlinear steepening must occur from an initial signal with dominant frequencies below the ~ 4 kHz cutoff frequency of the first cross mode. This ensures a planar wavefront across the face of the fluid piston at the pipe opening. The pipe length is a key factor in this process because it determines the number of shock formation distances the wave travels before exiting the pipe. The calculated shock formation distance for the parameters used by Blackstock *et al.*⁶ (an initial 8 kHz sinusoid with 138 dB re 20 μ Pa in a 3 m pipe) reveals that the waveforms had not yet reached one shock formation distance upon exiting the pipe. This relatively weak nonlinear steepening differs considerably from the unsteady shock experiment of Kim and Setoguchi.¹ For our experiment, space constraints limited our pipe to 6.1 m, which meant for our 1 kHz, 155 dB re 20 μ Pa waveform, the tube was approximately two shock formation distances in length. This condition result in a waveform approaching a sawtooth-like condition at the end of the pipe,⁸ in the absence of the interior reflection occurring from the open end of the pipe. In order to measure the initial waveform near the driver without the influence of reflections, sinusoidal pulses of approximately 35 ms in length were used.

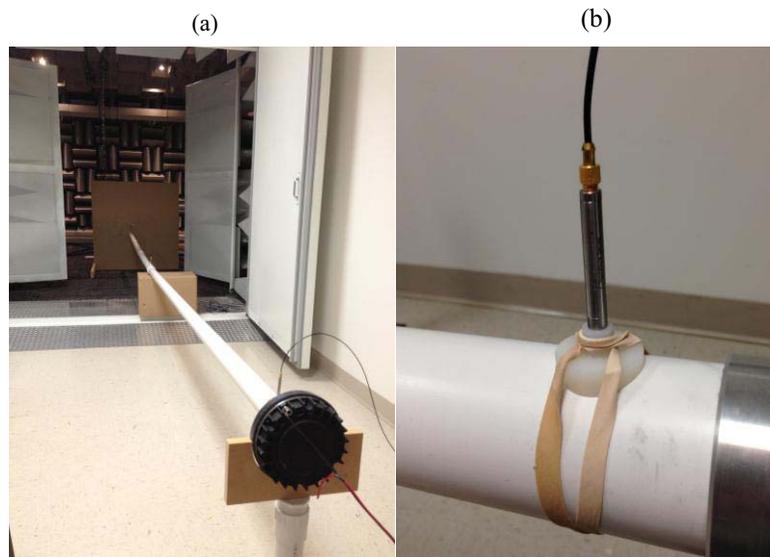


FIGURE 2. Photographs of experimental setup. (a) Complete tube set up with microphones in place, the baffle is located inside an anechoic chamber. (b) An example of a 3.18 mm 40DD GRAS microphone mounted outside the pipe (with gridcap removed) such that the diaphragm is flush with the inner surface of the pipe.

RESULTS AND DISCUSSION

Measured results provide evidence of nonlinear effects not only inside of the pipe but, potentially, also in the radiation from the baffled opening. The data displayed in Fig. 3 clearly demonstrate steepening inside of the pipe. Figures 3(a) – 3(c) show steepening of the sinusoid into a shock-like waveform as the sinusoid propagates from 5.7 to 310.5 and then to 607.7 cm, respectively. The data shown in Fig. 3(d) are from a microphone placed on axis, 3.8 cm from the edge of the baffle. Figure 3(a) shows that the pressure at the driver is sinusoidal with amplitude of about 1.2 kPa. Figure 3(b) shows a slight reduction in amplitude from the original signal and a definite steepening towards shock formation approximately halfway down the pipe. At the microphone closest to the pipe opening, the waveform shape, as displayed in Fig. 3(c), is distorted with an appreciable increase in amplitude due to the reflection from the boundary at the end of the pipe. Even as the wave becomes more asymmetric due to these reflections,

shock wave characteristics are still noticeable. Figure 3(d) depicts sharp narrow pressure peaks produced on-axis 3.8 cm outside of the tube, similar to those proposed in previous studies, but higher in amplitude than the continuous wave examples seen in Refs. 3, 4, and 6.

The waveform shape outside the pipe can be qualitatively explained by considering the linearized momentum (Euler's) equation in one dimension (x), which can be written as $\rho_0 \dot{u} = -p_x$. Under this linear approximation, the particle acceleration, \dot{u} , and pressure gradient, p_x , are proportional to one another via the ambient density, ρ_0 . The rapid spatial change in the pressure due to the acoustic shocks produces large accelerations at the end of the pipe. From linear piston theory, the far-field, on-axis pressure is proportional to the particle acceleration at the fluid piston face. Although the Rayleigh distance for the piston varies between 0.59 cm and 59 cm over a frequency range of 1 – 100 kHz, such that the measurement in Fig. 3(d) is well within the near field, the derivative of the pressure in Fig. 3(c) can still be matched in a qualitative sense to the radiated shape in Fig. 3(d). This positive impulse is similar to those seen by both Kim and Setoguchi¹ and Hirschberg *et al.*³ but appears more shock-like than those measured previously by Blackstock *et al.*,⁶ Gee *et al.*,⁵ and Nakamura and Takeuchi.³ Note further that, unlike the initial sinusoid in Fig. 3(a), the amplitude of the waveform at the end of the pipe in Fig. 3(c) is asymmetric, with greater positive values that occur over a shorter time duration. Although this phenomenon requires further investigation, this is possibly attributed to nonlinear reflections occurring at the end of the pipe.

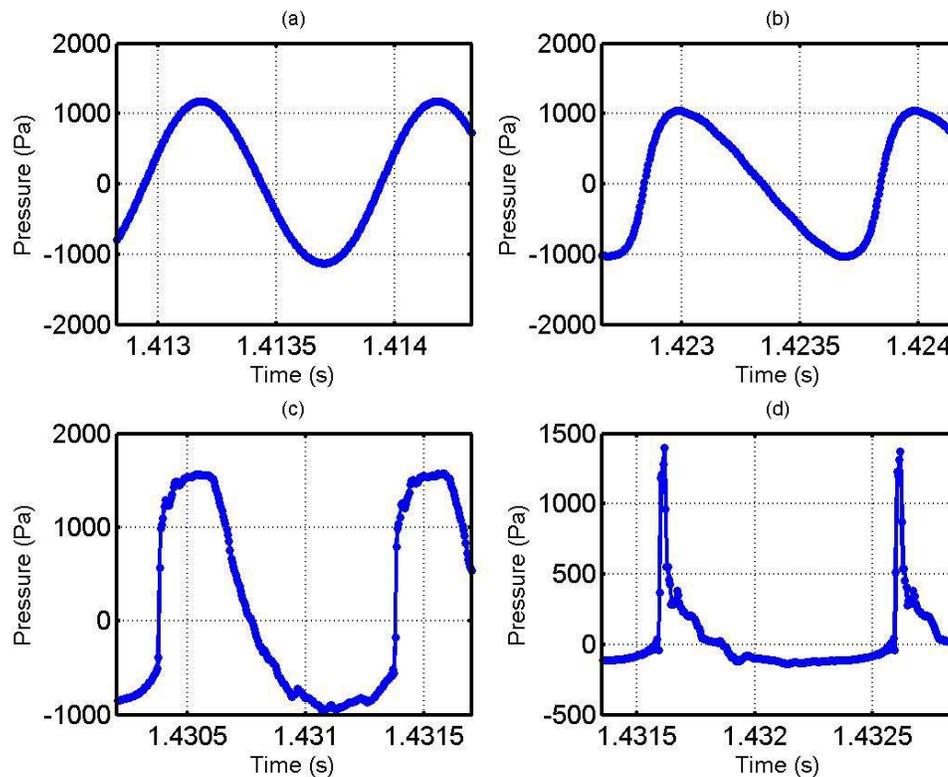


FIGURE 3. Results for 1 kHz sinusoid with initial peak level of 155 dB re 20 μ Pa. Microphone positions, relative to the driver, are located in the pipe at (a) 5.7 cm, (b) 310.5 cm, and (c) 607.7 cm. The last microphone (d) is located beyond the baffle on axis with the pipe at a distance of 3.8 cm, measured from the end of the pipe.

The propagation of the radiated wave outside the pipe is now considered. Figure 4 shows data taken at increasing on-axis distances outside of the pipe. As the waveform propagates from 1.3 cm out to 91.5 cm, the peak-

like structures in the waveforms become more shock-like and their positive duration shortens noticeably. There are two possible explanations for this phenomenon. First, linear piston theory and the range of Rayleigh distances for the harmonics up to 100 kHz indicate that only the measurement in Fig. 4(d) is in the geometric far field. As axial distance increases, a reduction of path length differences from points on the piston face to the observation position could result in a narrowing of the peak as is seen in Figs. 4(c) and 4(d). However, the peak pressure amplitude just outside the pipe exceeds 1500 Pa. Consequently, it is also possible that shock-like behavior is the result of nonlinear propagation of the positive impulse as it propagates on axis. Further analysis is in progress to examine the pulse propagation as a function of angle, frequency, distance, and input amplitude in order to determine the relevance of nonlinear and linear propagation effects.

We are currently working to develop a suitable numerical model for comparison with the experiment. Blackstock *et al.*⁶ predicted impulse-like waveforms produced by a baffled piston by applying the Greens' function to time-dependent pressures⁹ achieved from particle velocities at the pipe boundary. However, the cross modes in the pipe and open-end reflections were not included in the model. Nakamura and Takeuchi⁵ achieved reasonable agreement with their N-wave measurement using a frequency-domain piston theory model that incorporated the radiation impedance of a virtual piston at the pipe opening. In our model, the steepening process inside the pipe will be modeled using the generalized Burgers equation with appropriate loss mechanisms and coupled with the modeled piston radiation impedance to provide a prediction of the velocity at the end of the pipe. For comparison, the measured velocity will be estimated utilizing the two closely spaced microphones at the end of the pipe.¹⁰ These velocity estimates will serve as inputs to a (linear) piston model that will be used to predict the radiated field for comparison with measurements outside the pipe and to examine the pipe data for evidence of nonlinear reflections.

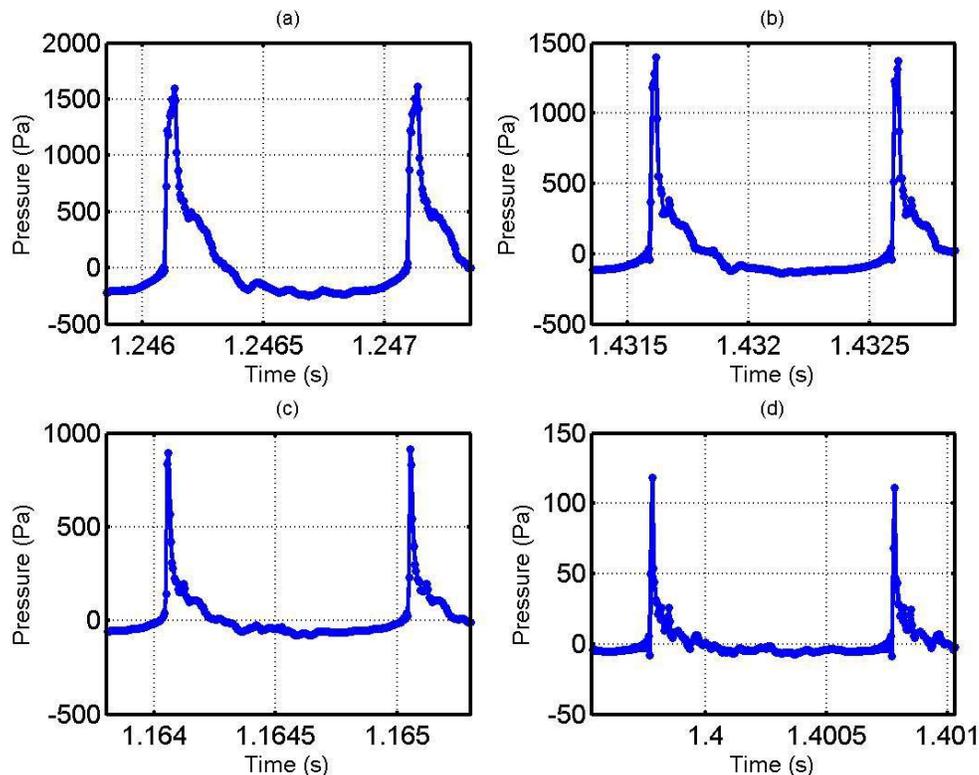


FIGURE 4. Results for 1 kHz sinusoid with initial peak level of 155 dB re 20 μ Pa. On-axis microphone positions, relative to the pipe exit, are located as follows: (a) 1.27 cm, (b) 3.18 cm, (c) 7.62 cm, and (d) 91.44 cm.

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