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## 3aSA3. Near-field acoustic holography in conical coordinates

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Near-field acoustical holography (NAH) techniques can be optimized if the method capitalizes on the geometry of the noise source under investigation. Helmholtz-equation least squares method (HELS) uses the solutions of Helmholtz equation in spherical coordinates as basis functions for the pressure field. HELS is an efficient NAH technique if the source and the measurement surfaces are spherical in nature. For nonspherical cases, such as radiation from a plate or bar, it takes a large number of functions to represent the field. In these cases, there is also a question about where to place the origin of the wave functions. In search of a HELS-type method that could be applied to nonspherical sources, a study into the features of conical coordinates has been conducted. Because Helmholtz equation is separable in conical coordinates, the solutions can be used, in a manner similar to HELS, as basis functions to represent the pressure field. For conical coordinates, the basis functions are spherical Hankel functions and Lame functions. Thus, for a conical source, a HELS-type formulation in conical coordinates could provide a natural choice for near-field acoustical holography.[Work supported by Blue Ridge Research and Consulting and Air Force Research Laboratory.]

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## Introduction

The goal of near-field acoustical holography (NAH) is to use the acoustic pressure measured in the near-field to obtain an estimate of the pressure field on or near a vibrating surface. NAH techniques have been applied to several kinds of problems, including engine noise, car noise, and interior aircraft noise, and have been helpful in reconstructing the pressure field on or near such surfaces. Currently under investigation is the question of whether these NAH techniques be used for understanding complex, airborne noise sources such as those found in the plume of a jet engine.

Several Fourier-based methods have been used for NAH, including the planar Fourier method, SONAH, and the Helmholtz equation least-squares (HELS) method. (Some of the influential papers written on these methods are listed in the bibliography at the end of this paper.) While each of these methods has certain strengths, the primary advantage of the HELS method lies in the simplicity of the computation required to perform NAH, which can be accomplished in much less time than the other two methods. The primary, unfortunate disadvantages of the HELS method are that (1) it is intrinsically based on spherical symmetry and (2) it is an ill-posed problem unless source is at or near the origin at the center of the coordinate system.

For problem of jet noise, the HELS method is obviously not applicable because of the non-spherical geometry. However, there is a possibility that the advantages inherent to the HELS method in spherically symmetric problems may be transferred to a different coordinate system that matches the geometry of the problem. SONAH is based on separation of Helmholtz equation in cylindrical coordinates. Traditional Fourier analysis is best suited to planar problems. The question of which coordinate system best matches the cone shape of a jet plume led to the investigation of the separation of Helmholtz equation in conical coordinates and to the development of an NAH method in conical coordinates.

In this presentation consists of a short overview of the HELS method, an introduction to conical coordinates, and the development of an NAH approach in conical coordinates. The difficulties that arise when the source is not at the origin of the conical coordinate system and in transforming from Cartesian to conical geometry are addressed.

## HELS Method

Fourier-based NAH methods typically begin with the Helmholtz equation is
$\nabla^{2} p+k^{2} p=0$, where $p$ is the complex pressure of the acoustic pressure, and $k / \omega$ is the wave number. Details of separable solutions to Helmholtz equation in spherical coordinates is given in many textbooks. The resulting acoustical pressure $p$ at position $(r, \theta, \phi)$ is found by a summation involving spherical Hankel functions of the first kind, $h_{n}^{(1)}(k r)$, and spherical harmonics, $Y_{n}^{m}(\theta, \phi)$ :

$$
p(r, \theta, \phi)=\sum_{n=0}^{\infty} h_{n}^{(1)}(k r) \sum_{m=-n}^{n} A_{n}^{m} Y_{n}^{m}(\theta, \phi),
$$

where the $A_{n}^{m}$ are constant coefficients. The spherical Hankel functions of the second kind are not included because they violate the Sommerfeldt radiation condition that the
pressure must go to zero as $r$ approaches infinity. In numerical work, the above expression for $p$ is truncated at a finite number of terms:

$$
p(r, \theta, \phi)=\sum_{n=0}^{N} h_{n}^{(1)}(k r) \sum_{m=-n}^{n} A_{n}^{m} Y_{n}^{m}(\theta, \phi)
$$

where $N$ is the number of spherical Hankel functions used in the expansion to find $p$. The total number of terms in the expansion and thus the total number of coefficients $\left\{A_{n}^{m}\right\}$ is $J=(2 N+1)^{2}$.

In the HELS method, the acoustic pressures recorded at the measurement locations are used to find the coefficients $\left\{A_{n}^{m}\right\}$ via a least-squares method that minimizes the difference between the measured pressure and the computed pressure for $J$ terms in the above expansion. The matrix computations behind this procedure are explained in Ref. 2. The computed $A_{n}^{m}$ are then used to compute the pressure at the reconstruction locations.

Like all methods, the HELS method has both strengths and weaknesses. The advantages of the HELS method are (1) the ability to reconstruct over a larger area than the measurements, and (2) the speed of the numerical computations. Unfortunately, the HELS method works best in a spherically symmetric problem where the source is essentially centered on the origin. For an arbitrarily shaped source, a large number of terms with a strict regularization technique must be used in the expansion, and an estimate must be made as to the effective origin of the problem. The problem with the origin arises because the solution of the Helmholtz equation presented here has only one boundary condition, the Sommerfeldt radiation condition. Typically for a second-order differential equation, two boundary conditions are needed to completely specify the solution.

The goal of this research is to investigate if the advantages of a HELS-based NAH method can be extended to a more flexible coordinate system that would match the noise sources in a jet plume. Conical coordinates are chosen for this study because they appear to match the geometry of a jet plume and because the Helmholtz equation is separable in conical coordinates.


Figure 1 Graphical representation of conical coordinates. Courtesy of Wikipedia Commons.

## Conical coordinates

In conical coordinates system, the orthogonal coordinate surfaces are a sphere and two cones. The three surfaces are shown in Figure 1. The sphere has radius $r$ and is centered on the origin as in spherical coordinates. The first cone (blue) is aligned with the positive $z$-axis and corresponds to a surface of constant $\mu$. The second cone (yellow) is aligned with the positive $x$-axis and corresponds to a surface of constant $v$. A point $P$, shown as a black dot in Fig. 1, is identified in conical coordinates by the three vector $(r, \mu, v)$.


Figure 2 Portion of cones that correspond to different ratios $b / c$. The inner, nearly circular cone has $b / c=0.9995$. The other cones have $b / c=0.5$ (middle) and 0.25 (outer).

The transformation between conical and Cartesian coordinates is complex. First two parameters, $b$ and $c$, need to be chosen. The ratio $b / c$ is related to the eccentricity of the cross-section of the cones as shown in Fig. 2. When $b / c$ is almost unity, the corresponding cone of constant $v$ has an almost circular shape. As $b / c$ decreases, the sides of the cone become flattened out. In addition, there are limitations on the relative
allowed values of $r, \mu, \nu, b$, and $c$ in conical coordinates, namely, $0<v^{2}<b^{2}<\mu^{2}<c^{2}<r^{2}$. Once $b$ and $c$ are chosen, the orthogonal coordinate surfaces are given by the following set of equations:

$$
\begin{aligned}
& x^{2}+y^{2}+z^{2}=r^{2} \\
& \frac{x^{2}}{\mu^{2}}+\frac{y^{2}}{\mu^{2}-b^{2}}+\frac{z^{2}}{\mu^{2}-c^{2}}=0 . \\
& \frac{x^{2}}{v^{2}}+\frac{y^{2}}{v^{2}-b^{2}}+\frac{z^{2}}{v^{2}-c^{2}}=0
\end{aligned}
$$

The above equations can be rearranged to show the conversion between Cartesian coordinates $(x, y, z)$ and conical coordinates $(r, \mu, v)$ :

$$
\begin{aligned}
& x^{2}=\left(\frac{r \mu \nu}{b c}\right)^{2} \\
& y^{2}=\frac{r^{2}}{b^{2}} \frac{\left(\mu^{2}-b^{2}\right)\left(b^{2}-v^{2}\right)}{\left(c^{2}-b^{2}\right)} \\
& z^{2}=\frac{r^{2}}{c^{2}} \frac{\left(c^{2}-v^{2}\right)\left(c^{2}-\mu^{2}\right)}{\left(c^{2}-b^{2}\right)}
\end{aligned}
$$

To complete the transformation, the values of $(r, \mu, v)$ and the signs of $\sqrt{c^{2}-\mu^{2}}$ and $\sqrt{b^{2}-\mu^{2}}$ must be known. Specifically, $\sqrt{c^{2}-\mu^{2}}$ is taken with positive sign for a point on the cone where $z$ is positive, and negative when $z$ is negative; $\sqrt{b^{2}-\mu^{2}}$ is taken to be positive for a point where $y$ is positive; $v$ is taken to be positive when x is positive. (From Ref. 12, Byerly - p.251) For the work presented herein, the positive radicals are chosen indicating that all measurement and reconstruction points are located in the first quadrant.

## Helmholtz Equation in Conical Coordinates

Helmholtz equation is separable in conical coordinates. In conical coordinates, using the conical expression for $\nabla^{2}$, Helmholtz equation is

$$
\frac{\partial^{2} p}{\partial \alpha^{2}}+\frac{\partial^{2} p}{\partial \beta^{2}}+\left(\mu^{2}-v^{2}\right) \frac{\partial}{\partial \lambda}\left(\lambda^{2} \frac{\partial p}{\partial \lambda}\right)=0
$$

where

$$
\begin{aligned}
& \alpha=\int_{0}^{\mu} \frac{\mathrm{d} \mu}{\sqrt{\left(\mu^{2}-b^{2}\right)\left(c^{2}-\mu^{2}\right)}} . \\
& \beta=\int_{0}^{v} \frac{\mathrm{~d} v}{\sqrt{\left(b^{2}-v^{2}\right)\left(c^{2}-v^{2}\right)}}
\end{aligned}
$$

Substitution of a separable solution, $p(r, \mu, v)=R(r) f(\mu) g(v)$, into Helmholtz equation, results in three ordinary differential equations, the spherical Hankel equation for $r$, and
two Lamé equations, one for $\mu$ and one for $v$. The resulting expression for the pressure field in conical coordinates is

$$
p(r, \mu, v)=\sum_{n=0}^{\infty} h_{n}^{(1)}(k r) \sum_{m=0}^{2 n} A_{n}^{m} E_{n}^{m}(\mu) E_{n}^{m}(v) .
$$

The radial portion of the solution is $h_{n}^{(1)}(k r)$, a spherical Hankel function of the first kind of order $n$, for outgoing waves, as in spherical coordinates. The other two parts of the solution, $E_{n}^{m}(\mu)$ and $E_{n}^{m}(v)$, are Lame functions of the first kind of order $n$ and degree $m$.

There are four different classes of Lamé functions of the first kind, which are often referred to as $K, L, M, N$. (See references in bibliography on conical coordinates.) All of these must all be used to get the $2 n+1$ independent terms for the $n$th part of the expansion. This is analogous to the solution of Helmholtz's equation in spherical coordinates for the pressure outside an acoustical source:

$$
p(r, \theta, \varphi)=\sum_{n=0}^{N} h_{n}^{(1)}(k r) \sum_{m=-n}^{n} A_{n}^{m} Y_{n}^{m}(\theta, \phi),
$$

where $2 n+1$ spherical harmonics are included for each $n$ value in the summation. For convenience, the four different classes of the Lamé functions of the first kind are represented with as $E_{n}^{m}$. The Lamé functions of the first kind are called the ellipsoidal harmonics. For low order $n$, the ellipsoidal harmonics are tabulated in Ref. 12 (Byerly, p.257) and shown in Table 1.

Table 1 Ellipsoidal harmonics of low order as tabulated in Ref. 12.

|  | $n=0$ | $n=1$ | $n=2$ |
| :--- | :--- | :--- | :--- |
| $m=0$ | $E_{0}^{0}(\lambda)=1$ | $E_{1}^{0}(\lambda)=x$ | $E_{2}^{0}(\lambda)=\lambda^{2}-\frac{1}{3}\left[b^{2}+c^{2}-\sqrt{\left(b^{2}+c^{2}\right)^{2}-3 b^{2} c^{2}}\right.$ |
| $m=1$ |  | $E_{1}^{1}(\lambda)=\sqrt{\lambda^{2}-b^{2}}$ | $E_{2}^{1}(\lambda)=\lambda^{2}-\frac{1}{3}\left[b^{2}+c^{2}+\sqrt{\left(b^{2}+c^{2}\right)^{2}-3 b^{2} c^{2}}\right.$ |
| $m=2$ |  | $E_{1}^{2}(\lambda)=\sqrt{\lambda^{2}-c^{2}}$ | $E_{2}^{2}(\lambda)=\lambda \sqrt{\lambda^{2}-b^{2}}$ |
|  |  |  | $E_{2}^{3}(\lambda)=\lambda \sqrt{\lambda^{2}-c^{2}}$ |
|  |  |  | $E_{2}^{4}(\lambda)=\sqrt{\left(\lambda^{2}-b^{2}\right)\left(\lambda^{2}-c^{2}\right)}$ |

## Conical HELS

To formulate a Helmholtz equation, least-squares method for NAH in conical coordinates, one follows steps similar to those outlined in the review of HELS in spherical coordinates. First, the expression for the pressure field in conical coordinates is changed from an infinite summation to a finite summation:

$$
p(r, \mu, v)=\sum_{n=0}^{N} h_{n}^{(1)}(k r) \sum_{m=0}^{2 n} A_{n}^{m} E_{n}^{m}(\mu) E_{n}^{m}(v) .
$$

Second, the values of the coefficients $\left\{A_{n}^{m}\right\}$ are found by minimizing the difference between acoustic pressure at a set of measurement positions and the corresponding spherical Hankel and Lame function values for the corresponding locations. Third, the
coefficients $\left\{A_{n}^{m}\right\}$ are used to estimate the acoustic pressure at the reconstruction locations.

## Simulated Data

As an initial testing of conical NAH, two surfaces of constant $\mu$ are chosen as the measurement locations and the reconstruction locations. In both cases, $b=0.05$ and $\mathrm{c}=$ 0.1 , and the distances of the measurement and reconstruction positions from the origins, $r$, range from 0.2 to 1.0, as shown in Fig. 3. The 100 measurement locations cover one quarter of a cone with $\mu=0.0447$. The $\square 100$ reconstruction $\square$ positions lie on a similar surface with $\mu=0.0490$. A monopole radiating at 1700 Hz is used.


Figure 3 Front and back views of two conical shapes with $\mathbf{b}=0.05$ and $\mathbf{c}=0.1$. The outer cone represents the measurement surface and the inner one represents the reconstruction surface.

In this initial testing, the location of the source is the factor that most strongly influences the performance of conical HELS in reconstructing an accurate pressure field. When the source is relatively far from the origin, at $x=0.2$, the blue dot in Fig. 3, the resulting reconstructed field does not match the predicted field for a monopole. Figure 4 shows the reconstruction results in red when $N=0,1,2$, and 3 , respectively are use in conical HELS. The blue line is the pressure field calculated from a monopole at $x=0.2$ at the reconstruction locations shown in Fig. 3. The horizontal axis in Fig. 4 counts across the 100 reconstruction locations form the lower edge to the top edge of the cone.


Figure 4 Calculated.pressure field (blue) for a monopole at $x=0.2$ compared to the reconstruction results with $N=0$ (red), 1 (green), 2 (black), and 3 (magenta) in conical HELS.

If the source is moved to the origin, then the conical HELS method provides a perfect match with $N=0$, as should be expected for a monopole. This shows that the proximity of the source to the origin of the conical coordinate system is very important. If the source is not at the center of the coordinate system, then extra terms are needed in the expansion to get a relatively good estimate. But if $N=3$ or greater, regularization techniques are necessary because the matrices are badly scaled in the least-squares operation. Thus, the need for regularization techniques arises because the problem is ill-posed when the source is not at or near the origin.


Figure 5 Drawing of the experimental set-up. The pressure from the loudspeakers is recorded on an 8x8 microphone array at various positions along an angle of $\mathbf{3 0}$ degrees (purple) and 10 degrees (green) relative to the face of the loudspeakers.

## Experiment

In an attempt to perform a laboratory experiment that in some way resembles the distributed source of a jet plume, the experiment illustrated in Fig. 5 was performed. The signals from the four loudspeakers overlap somewhat in the operating frequency band and are uncorrelated. An $8 x 8$ microphone array, represented by the purple rectangles in Fig. 5, was moved to eight different locations along a line extending at an angle of 30 degrees relatively to a line drawn from the front of the loudspeakers to an origin 1 m from loudspeaker 1 . Data from the 30 -degree case is
the input for the conical NAH. The conical NAH reconstructed the pressure field along the 10 -degree line, represented by the green line. The measurements were taken along the 10-degree line for comparison with the reconstruction results.

To test the conical NAH method, a simple sample of 1700 Hz data is analyzed. Only loudspeaker 1 is emitting sound at this frequency. Data from the fifth scan is used because the distance between loudspeaker 1 and the microphone array is the smallest for this array position. Initially the conical coordinates for the microphones are calculated based on the origin shown in Fig 7(a), where loudspeaker 1 is 1 m from the origin.

The conical NAH results for different values of $N$ are shown in Fig. 6. The reconstruction with only a single monopole term, $N=0$, shown as the red line, differs greatly from the data, shown as the blue line. For $N=1,2$, and 3 in the expansion, the green, black and magenta lines show the reconstruction results, respectively. The $N=1$ results approach the data but do not capture the exact details. The results for $N=2$ and 3 are bad largely because the matrix used in the least-square calculation for the coefficients is poorly conditioned in both cases. Regularization has not yet been implemented.


Figure 6 The 10-degree data on 29 microphones at 1700 Hz (blue) is compared to the reconstruction results, based on the origin shown in Fig. 7 (a), for conical NAH with $\mathrm{N}=0$ (red), 1 (green), 2 (black), 3 (magenta).

The reconstruction results are improved dramatically if the origin of the conical coordinate system is moved closer to the location of the speaker. Because the measurement and reconstruction points need to be confined to the first quadrant, care must be taken to rotate the axes in addition to shifting them. Specifically, the origin is first translated from the original origin to the location of the loudspeaker [Fig. 7(b)]. Then the coordinate system is rotated 180 degrees about the $z$-axis [Fig. 7 (c)]. Because the $+z$-axis goes into the page and the origin is at the middle of the 8 x 8 array, the microphones on the lower half of the array lie in the first quadrant. The locations of these microphones in Cartesian coordinates are transformed to conical coordinates and used with the data in conical NAH.


Figure 7 (a) The data used for the conical NAH is measured at an angle of 30 degrees with respect to the origin that is 1 m from loudspeaker 1 . (b) The coordinate system is translated in the $x$-direction so the loudspeaker is at the origin. (c) The coordinate system is rotated 180 degrees about the $z$-axis, such that microphones on the lower of half of $8 \times 8$ array (represented by the purple box) are in the first quadrant.


Figure 8 Reconstruction results of conical NAH compared to the data recorded on 29 microphones at a frequency of 1700 Hz (blue) when the approximate origin is used (green) and when the optimal origin is used (red). $\mathrm{N}=0$ was used for these monopole reconstructions.

Figure 8 shows the results of conical NAH reconstruction when the origin is shifted as in Fig. 7(c). The blue line is the data recorded on the 10-degree line on 29 microphones, at 1700 Hz . The green line is the reconstruction results at the 10degree locations obtained from conical NAH on the 30-degree data from 29 microphones and with the origin shifted to the estimated location of the loudspeaker, measured in the experiment to be the center of the loudspeaker. The agreement between the reconstruction results based on the shifted origin is significantly better than any of the reconstruction results obtained in Fig. 6 with the original origin.

The reconstructed pressures can be improved if a quick search is done to find an optimal origin. The optimal origin is the location of a monopole that minimizes the difference between the recorded signal and the field from the monopole. The optimal source position is located 8.4 cm in the $x$ direction and 0.5 cm in the $z$ direction from the estimated origin. By shifting the origin of the conical coordinates
to the optimal origin and then performing the conical NAH, the reconstruction results (shown in red in Fig. 8) provide a closer match to the measured values.

## Conclusion

In an attempt to overcome the restrictions of the HELS method to spherical problems, a conical coordinates based NAH method has been explored. The separable solution to the Helmholtz equation makes it possible to implement the conical NAH least-square method in a manner similar to spherical HELS. However, the coordinate transformations required to work in conical coordinates are difficult and impose the restriction of working in the first quadrant unless one wanted to be extremely careful in choosing the positive or negative signs on the radicals in the coordinate transformations. This investigation also provided an opportunity to better appreciate why and how the NAH problem is ill-posed if the source of interest in not at or near the origin of the coordinate system. Because of these restrictions, conical and spherical, Helmholtz-equation, least squares NAH methods are not applicable for jet noise which is a distributed source.

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