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2aED3.   Combining theory and experiment to teach acoustic concepts

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  A rigorous theoretical development is desirable to help students at both the undergraduate and graduate levels develop a deep understanding of acoustic phenomena. However, numerous students labor through the mathematics associated with the concepts without ever developing an understanding of how that translates over into the physical world. Many acoustical phenomena lend themselves to experimental demonstrations that can greatly aid students' understanding of the physical concepts and help them connect the theoretical developments with what physically happens. These demonstrations also provide a means for introducing common issues associated with making acoustical measurements that can also be educational for students. As an example, this paper will focus on how we have developed concepts associated with vibrating strings in a class through both theoretical development and use of a relatively simple experimental apparatus. Students gain a much better understanding not only of modes associated with the string, but the relative accuracy of the underlying theory. In addition, basic signal analysis topics and measurement accuracy also surface in the process of making the measurements.

INTRODUCTION

There are a number of fundamental acoustical concepts that are typically introduced in a first year course in acoustics. A rigorous development of these concepts will invariably involve the use of mathematics to describe the phenomena. While this approach can provide a rigorous presentation of the concept being explored, it is easy for students to see the entire concept only as a mathematical problem, and not develop a good intuition of the physical response associated with that concept. Thus, it is helpful to couple a rigorous mathematical development with experimental demonstrations that allow students to "see" the phenomenon being studied and gain a deeper understanding of its characteristics. The concept of modes is one of these acoustical phenomena that are routinely covered[1,2,3]. This paper focuses on techniques that have been used to help students gain a better understanding of modes. A string is chosen as the system to study, as it also allows for easy visual study, thereby helping students gain a deeper understanding of modes.

MODES ON A STRING

A. Free Vibration of a String

Formally, modes can be developed in the context of a mathematical problem where one solves the underlying differential equation, taking into account the boundary conditions that pertain to the problem[1,2]. Physically, modes are associated with the free response of a system when given an initial excitation. In the case of a string, the modes can be identified by assuming a time-harmonic response, which results in the Helmholtz equation, given as

$$\frac{\partial^2 y}{\partial x^2} + k^2 y = 0 , \tag{1}$$

where y is the transverse displacement of the string, x is the dimension along the string, and k is the acoustic wavenumber, given by ω/c, where ω is the angular frequency and c is the phase speed in the string. This is a standard differential equation with the known general solution of

$$y = A \sin kx + B \cos kx . \tag{2}$$

To proceed, boundary conditions are assumed (such as a fixed-fixed string). The first of these two conditions will result in an expression for B in terms of A, while the second equation will then determine the values that k must assume. Formally, this corresponds to a mathematics eigenvalue problem, where the resulting eigenvalues are identified as the natural frequencies for the string, and the eigenvectors are identified as the natural modes. As an example, for the fixed-fixed string, this process leads to

$$k = k_n = \frac{n\pi}{L} ; n = 1,2,3, \dots \tag{3}$$

and

$$y_n = A_n \sin k_n x . \tag{4}$$

A similar mathematical development for a free-fixed string results in the following results for the eigenvalues (natural frequencies) and eigenvectors (modes).

$$k_n = \frac{2n-1 \, \pi}{L} ; n = 1, 2, 3, \dots \tag{5}$$

and

$$y_n = A_n \cos k_n x . \tag{6}$$

B. Forced Vibration of a String

For forced vibration of a string, application of the corresponding boundary and excitation conditions allows one to solve explicitly for the response of the string at a given frequency in terms of the general solution. While the resulting solution does not come out in terms of modes, the response of the driven string introduces the concept of resonance, and the response at resonance can be directly related to the concept of natural modes of the string.

The most common forced string problem introduced by textbooks (see for example, Ref. 1) is that of the forced-fixed string, where the string is driven by a constant time-harmonic force at $x = 0$, and is fixed at $x = L$. The resulting response can be expressed as

$$y(x,t) = \frac{F}{kT} \frac{\sin k(L-x)}{\cos(kL)} e^{j\omega t} , \qquad (7)$$

where F is the amplitude of the driving force, and T is the tension in the string. It is readily seen that a maximum displacement occurs at the resonance frequencies when $kL = (2n-1)\pi/2$, which corresponds to the natural frequencies for a free-fixed string.

While the forced-fixed string is a nice problem for introducing a driven string and the concept of resonance, it does not generally correspond to what one would measure when driving a string. A more typical configuration would be a string characterized as fixed at $x = L$, and constant displacement amplitude, Y_o, at $x = 0$. Textbooks do not generally provide this solution, but solving for these boundary conditions results in

$$y(x,t) = Y_o \frac{\sin k(L-x)}{\sin(kL)} e^{j\omega t} , \qquad (8)$$

with the resonance frequencies corresponding to the natural frequencies of the fixed-fixed string.

C. Experimental Demonstration

These concepts of modes and resonances can be nicely demonstrated through the use of a "string" utilizing surgical tubing. This type of a string has a number of advantages. It is inexpensive and easy to use. It is also flexible enough to have vibration amplitudes that are readily visible, so that students can easily observe the response of the string. Lastly, in spite of the large amplitudes that result, the string response comes very close to what is predicted by simple linear string theory.

To implement this demonstration, the following items are needed: surgical tubing, a vibration shaker, an amplifier (if needed to drive the shaker), and a signal generator. I have found that various types of surgical tubing work quite well, but I have generally used surgical tubing that is about 1/8 in (3.2 mm) in diameter and 2-3 m in length. A longer string will shift the fundamental resonance to a lower frequency, but it will also make it easier to observe higher-order resonances. I have found that this type of string is also very robust in terms of the fixed end. Very good results are obtained simply by tying the string around some fixed object for the boundary, such as on the arm of an overhead projector, on a door knob, etc. Thus, one of the major advantages of this demonstration is its simplicity, and yet good agreement with theory. In principle, any signal generator can be used, although a digital signal generator is advantageous since it is much easier to precisely control the driving frequency. The general configuration can be seen in Fig. 1.

With this string made from surgical tubing, both concepts of normal modes and resonances can be readily demonstrated. If one displaces the string near the middle of the string and releases it, the resulting motion will correspond nearly perfectly to the fundamental mode (for a fixed-fixed string). To demonstrate higher order modes, lightly place your thumb and finger at the location of one of the nodal positions for that desired mode, then displace the string at an antinode for that mode and release. Thus, for the second mode, the fingers should be placed at $L/2$, for the third mode, they should be placed at $L/3$, and so forth. When the string is released, the fingers can also be removed, and the string will vibrate in the desired mode.

For the driven string, it is educational for the students to ask them to predict the relationship between the resonances of the string before actually doing the demonstration. Invariably, they often predict the resonance frequencies associated with the forced-fixed string (Eq. (7)), since that will be the problem they will tend to be familiar with, and they will tend to automatically associate the vibration shaker with a constant force excitation. To show the effects of resonance and anti-resonance, one can choose any arbitrary frequency to drive the string at and observe that if one is not near resonance a poor response results, corresponding to the condition when the

FIGURE 1. General experimental configuration used. The vibration shaker can be seen on the left, with the power amplifier located behind. The string is stretched from the shaker over to a doorknob, with it simply being tied at both ends. The signal general is located just to the left of this picture.

denominator in Eq. (8) is relatively large. Then one can identify the frequencies where resonance occurs, beginning with the fundamental, f_o. Based on typical responses, the students will often predict the next resonance to be at $3f_o$, and may be a little surprised to find that it happens at $2f_o$. That provides a great context in which to discuss what the driving conditions really are and to solve for the fixed displacement driving condition, as given in Eq. (8). It can then be observed that the next resonances are at $3f_o$, $4f_o$, etc. My experience is that the harmonic relationship of the resonances is very close to those corresponding to the ideal constant displacement-fixed string given in Eq. (8). The flexibility of the string also allows one to drive the string at higher-order resonances (on the order of n = 10-15) and still have enough amplitude to be able to clearly discern the mode shape associated with that resonance.

The resonance frequencies can be adjusted by changing the length and the tension in the string. For lengths of approximately 2 m, the fundamental frequency will generally be in the range of 3-7 Hz, which can be fine-tuned by adjusting the tension of the string (stretching or loosening the string a little). It is often convenient to adjust the tension a little so that the fundamental corresponds to an integer frequency. Figure 2 shows the response of this string at 12 Hz, with the tension adjusted so that the fundamental was at 4 Hz. The spatial response corresponding to the third mode of a fixed-fixed string can be clearly seen. It is readily apparent that the small constant amplitude displacement, Y_o, which characterizes the driven end behaves very nearly like a fixed boundary condition, rather than the constant force condition that is generally provided as an example in textbooks. For this configuration, it was also observed that the resonance frequencies corresponded to multiples of 4 Hz up to as high as tested, which for this case was n = 10.

CONCLUSIONS

While a rigorous mathematical development is helpful for students to understand the theory governing acoustic behavior, in many cases it is also possible to supplement these developments with demonstrations that can solidify the concepts that are taught. This paper has outlined how a surgical tubing string can be used that results in a large amplitude response that nonetheless comes very close to matching the simple theoretical response. Similar demonstrations can be developed for standing waves in rods, beams, and fluid-filled tubes. Not only do students tend to enjoy seeing and participating with these demonstrations, but they gain a much greater intuition into how systems respond to various excitation stimuli, changing boundary conditions, and so forth.

FIGURE 2. Response of the string when driven at the frequency of the third resonance (12 Hz). The two nodes associated with this response are clearly seen, and the antinodes are of sufficient nature that the spatial response of the string is also readily observable.

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