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1pBB9. Cooperative radiation and losses in bubble clusters

Derek Thomas*, Yurii A. Ilinskii and Mark Hamilton

***Corresponding author's address: Applied Research Laboratories, The University of Texas at Austin, Austin, Texas 78758, derekcthomas@gmail.com**

In existing models for single bubble dynamics it is necessary to account for radiation damping, a consequence of fluid compressibility. A similar correction is necessary when modeling systems of coupled bubbles. The coupling alters the collective dynamics and therefore the acoustic power radiated. In the linear approximation and for compact clusters in which the bubbles pulsate in phase, the radiation damping per bubble increases in proportion to the number N of bubbles in the cluster, and the acoustic power is N times greater than is radiated by the cluster in the absence of bubble interaction. The latter effect is relevant to passive detection of cavitation noise as an indicator of the onset and degree of cavitation in therapeutic applications such as shock wave lithotripsy and high‐intensity focused ultrasound. The effect that collective radiation damping has on the dynamics of several simple systems is considered for both small and large pulsations. [Work supported by the ARL:UT McKinney Fellowship in Acoustics and NIH DK070618.]

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Motivation

The primary motivation for this research is shock wave lithotripsy (SWL), a procedure for treating kidney disease. In SWL, a high-amplitude shock wave is generated outside the body and focused on the kidney stone. Repeated application of shock waves results in comminution of the stone. It has been shown that the lithotripsy shock waves produce cavitation bubbles, and that the bubble activity impacts the treatment.[1] However comparison between model and experiment has been primarily qualitative. The goal of this research is to develop a more accurate model for interacting bubbles in a compressible medium that can be used to better understand the relevant processes in lithotripsy.

A model for the dynamics of bubbles observed in lithotripsy experiments requires the ability to model the complex, highly nonlinear motion of the bubbles in response to the high-amplitude pressures observed in lithotripsy. Rather than solve the governing partial differential equations for the fluid and the bubbles, the bubbles are modeled as coupled oscillators.

Theory

Prototypical equations of motion for the bubble radius are

$$R_i\ddot{R}_i + \frac{3}{2}\dot{R}_i^2 = \frac{P_i - P_0}{\rho} + \frac{\dddot{V}_i}{4\pi c} - 4\eta\frac{\dot{R}_i}{R_i} + Q_i^R - \sum_{k\neq i}\frac{\ddot{V}_k}{4\pi d_{ik}} \tag{1}$$

based on the assumption that propagation delays can be neglected, but that radiation damping must be included, and

$$R_i\ddot{R}_i + \frac{3}{2}\dot{R}_i^2 = \frac{P_i - P_0}{\rho} + \frac{\dddot{V}_i}{4\pi c} - 4\eta\frac{\dot{R}_i}{R_i} + Q_i^R - \left[\sum_{k\neq i}\frac{\ddot{V}_k}{4\pi d_{ik}}\right]_{t-d_{ik}/c} \tag{2}$$

which includes the propagation delays in bubble interactions. In both equations, R_i is the bubble radius, d_{ik} is the separation distance between bubbles i and k, c is the small-signalllj sound speed in the liquid without the bubbles, μ is the liquid shear viscosity coefficient, P_i is the pressure inside the ith bubble and Q_i^R is the effect of an external source as given in Eq. (36) of Ilinskii et al.[2] Overdots are used to indicate differentiation with respect to time. As proposed by Ilinskii and Zabolotskaya[3] the radiation damping is included with the $\dddot{V}$ term. The volume derivatives are related to the bubble radius by

$$\ddot{V} = 4\pi R^2\ddot{R} + 8\pi R\dot{R}^2,$$

$$\dddot{V} = 4\pi R^2\dddot{R} + 24\pi R\dot{R}\ddot{R} + 8\pi\dot{R}^3.$$

Bubble translation is neglected here. Equation (2) represents a set of coupled delay differential equations (DDEs). In order to use the model equations in Eq. (2), it is necessary to solve a large system of coupled DDEs.

Direct solution of the delay differential equations of motion (Eq. (2)) for large numbers of bubbles is computationally prohibitive. To overcome this limitation, we propose to model a large bubble cluster as a system of smaller, coupled subclusters. The subclustering concept is illustrated in Fig. (1). Rather than considering the direct, delayed interaction of all bubbles in the cluster, a set of ordinary differential equations (ODEs) that approximate Eq. (2) for closely spaced bubbles (small delays) is used for bubbles within a subcluster (green cell in the figure). The action of the bubbles in the red cell on the bubbles in the green cell is found by computing the average pressure produced by the motion of the bubbles in the red cell and treating this pressure as an external source in the motion of the bubbles in the green cell. This approach dramatically reduces the number of delayed quantities that must be considered, especially for dense clusters.

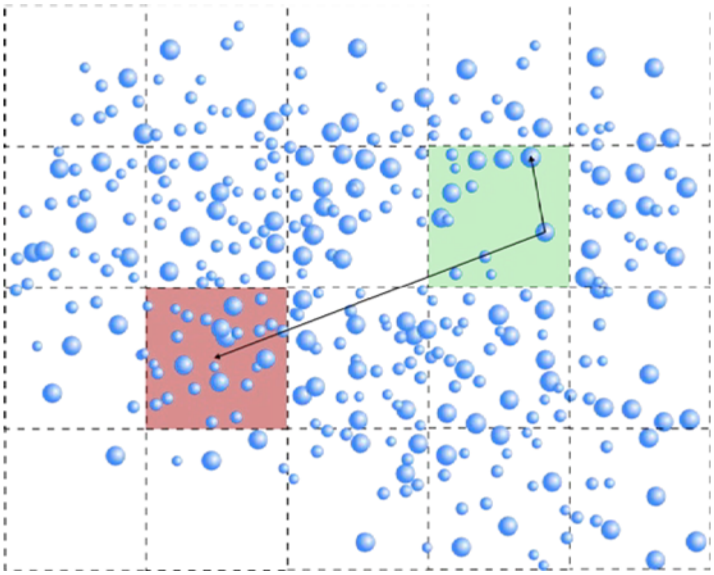

Figure 1: Bubble cluster with grid for subcluster divisions. The dynamics of bubbles within each subcluster are modeled with an approximation (Eq. (4)) to the delay differential equations (Eq. (2)), while delayed, averaged values are used to account for delayed interactions with bubbles in other subclusters.

We now seek an approximation to Eq. (2) for use within the subclusters. For systems of closely spaced bubbles, the delayed interaction terms on the right hand side can be expanded in a Taylor series about $d_{ik}/c = 0$:

$$\left[\sum_{k\neq i}\frac{\ddot{V}_k}{4\pi d_{ik}}\right]_{t-d_{ik}/c} \approx \sum_{k\neq i}\left[\frac{\ddot{V}_k}{4\pi d_{ik}}-\frac{\dddot{V}_k}{4\pi c}\right]. \tag{3}$$

This expansion for delayed terms was proposed by Ilinskii and Zabolotskaya. The approximate form of Eq. (2) for closely spaced bubbles is.

$$R_i\ddot{R}_i+\frac{3}{2}\dot{R}_i^2=\frac{P_i-P_0}{\rho}+\frac{\dddot{V}_i}{4\pi c}-4\eta\frac{\dot{R}_i}{R_i}+Q_i^R-\sum_{k\neq i}\frac{\ddot{V}_k}{4\pi d_{ik}}+\sum_{k\neq i}\frac{\dddot{V}_k}{4\pi c}. \tag{4}$$

The leading-order derivatives in Eqs. (1), (2), and (4) are multiplied by small coefficients, and thus all three equations represent singular problems which are numerically unstable for time-domain integration. This instability is due to the approximation that produced the third-order derivatives. In order to obtain a stable form, an iterative substitution procedure is used to eliminate the third-order derivatives. In this procedure, the governing equation is solved for $\ddot{R}_i$ and differentiated to obtain an expression for $\dddot{R}_i$. The result is substituted into the governing equation and terms of a desired order are retained. If necessary, the procedure is repeated. For the problem at hand, compressibility terms are retained to $O(1/c)$ and interaction terms are retained to $O(R/D)$. It is assumed that terms of $O(R/D)$ that are also multiplied by $1/c$ are negligible. For a single bubble, this procedure produces the Keller-Miksis equation. The resulting expression for $\dddot{V}_i/4\pi c$ is

$$\frac{\dddot{V}_i}{4\pi c} = \frac{1}{c}\frac{d}{dt}\left\{R_i\left[\frac{P_i - P_0}{\rho} + \frac{1}{2}\dot{R}_i^2\right]\right\} + O\left(\frac{1}{c^2}\right) + O\left(\frac{1}{c}\right) \times O\left(\frac{R}{D}\right). \quad (5)$$

With the approximation given in Eq. (5), Eq. (4) becomes

$$R_i\ddot{R}_i + \frac{3}{2}\dot{R}_i^2 = \frac{P_i - P_0}{\rho} - 4\eta\frac{\dot{R}_i}{R_i} + Q_i^R - \sum_{k\neq i}\frac{\ddot{V}_k}{4\pi d_{ik}} + \sum_k \frac{1}{c}\frac{d}{dt}\left\{R_k\left[\frac{P_k - P_0}{\rho} + \frac{1}{2}\dot{R}_k^2\right]\right\}. \quad (6)$$

Note that in Eq. (6), the first sum is over all bubbles in the system except the current bubble ($k = i$) while the second sum is over all bubbles in the system. The second sum represents "mutual compressibility" effects in the system; that is, corrections to the equations of motion due to interaction effects in a compressible liquid.

Results

In order to compare the dynamics predicted by Eq. (1) to those predicted by Eq. (6), we consider a simple system. The chosen system consists of two bubbles of equilibrium radii R_{01} and R_{02}. The natural frequency of bubble 1 is ω_0. The bubble centers are separated by $10R_{01}$, subjected to a tone burst of amplitude p_e with a duration of 10 times the period of bubble 1 ($T_0 = 2\pi/\omega_0$) at a frequency of ω. The system geometry is shown in Fig. 2. The bubbles are oriented so that the pulse arrives at both bubbles at the same time. The response of the system is simulated for the following parameter values:

- R_{02}/R_{01} ranges from ½ to 2
- Drive amplitude p_e ranges from 10^{-3} to 2 atm
- Drive frequency ω ranges from $\omega_0/2$ to $2\omega_0$

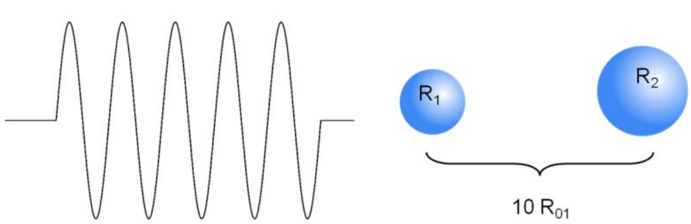

Figure 2: Bubble system geometry used in simulations presented in Figs. 3-5. The bubbles are oriented so that both bubbles experience the same pressure at the same time.

The responses of the systems are compared by calculating the total energy in each system, which is the sum of the potential energy stored in the compressed gas inside the bubble and the kinetic energy due to the motion of the fluid surrounding bubble. The normalized, relative energy is calculated by subtracting the equilibrium energy of the bubble system (E_0) from the total energy and then dividing the result by E_0. The equilibrium energy is found by evaluating the energy expressions given by Ilinskii et al.[2] with all bubbles in the system at rest. The normalized, relative energy predicted by Eq. (1) (labeled single) and Eq. (6) (labeled mutual) is shown in Figs. 3-5 for a range of parameters. In all three figures, the time when the source is turned on is marked by dashed lines and a tan background.

Figure 3 shows the normalized relative energy in the bubble system as a function of nondimensional time ($\tau = t\omega_0/2\pi$) in response to ten cycle pulses at the natural frequency of the single bubbles ($\omega/\omega_0 = 1$) for a range of pressure amplitudes (p_e/P_0=9.87e-4, 6.59e-1, 1.32, 1.97). The two bubbles have the same equilibrium size ($R_{02}/R_{01} = 1$). It can be seen that for low amplitude excitation (top left of Fig. (3)), the mutual compressibility effects increase the damping in the system as predicted by Feuillade.[4] For moderate amplitude excitation (top right and bottom left of Fig. 3), the predicted dynamics are somewhat similar for both models although the change in damping can be seen after the source is turned off at $\tau = 11$. It can be seen that the dynamics predicted by Eq. (6) increase the strength of the coupling between the source and the bubble system. As the amplitude increases, the predicted dynamics diverge dramatically (bottom right of Fig. 3).

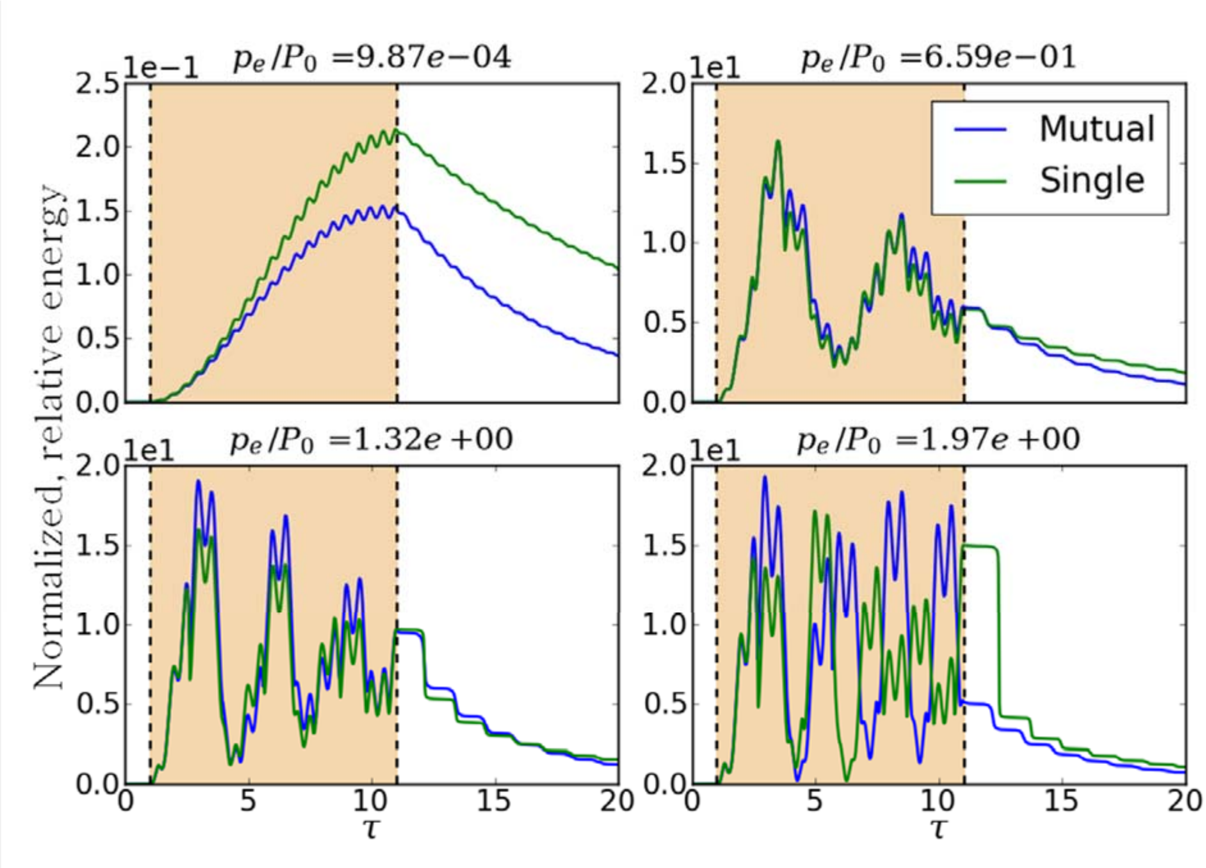

Figure 3: Normalized relative energy in the bubble system as a function of nondimensional time ($\tau = t\omega_0/2\pi$) in response to ten-cycle pulses at the natural frequency of the single bubbles ($\omega/\omega_0 = 1$) for a range of pressure amplitudes (p_e/P_0=9.87×10^{-4}, 6.59×10^{-1}, 1.32, 1.97). The two bubbles have the same equilibrium size ($R_{02}/R_{01} = 1$).

Figure 4 shows the normalized relative energy in the bubble system driven by the same external pressure amplitude as in the bottom right of Fig. 3 ($p_e/P_0 = 1.97$). The bubbles are again the same size, but the driving frequency ranges from $\omega/\omega_0 = 0.5$ to $\omega/\omega_0 = 1.2$. The bottom left part of Fig. 4 has the same parameters as the bottom right part of Fig. 3. The only difference is the scale of the vertical axis. Again it can be seen that the inclusion of corrections for mutual compressibility (blue lines) impacts the predicted dynamics as compared to the equations of motion with only single bubble compressibility effects (single bubble radiation damping) (green lines).

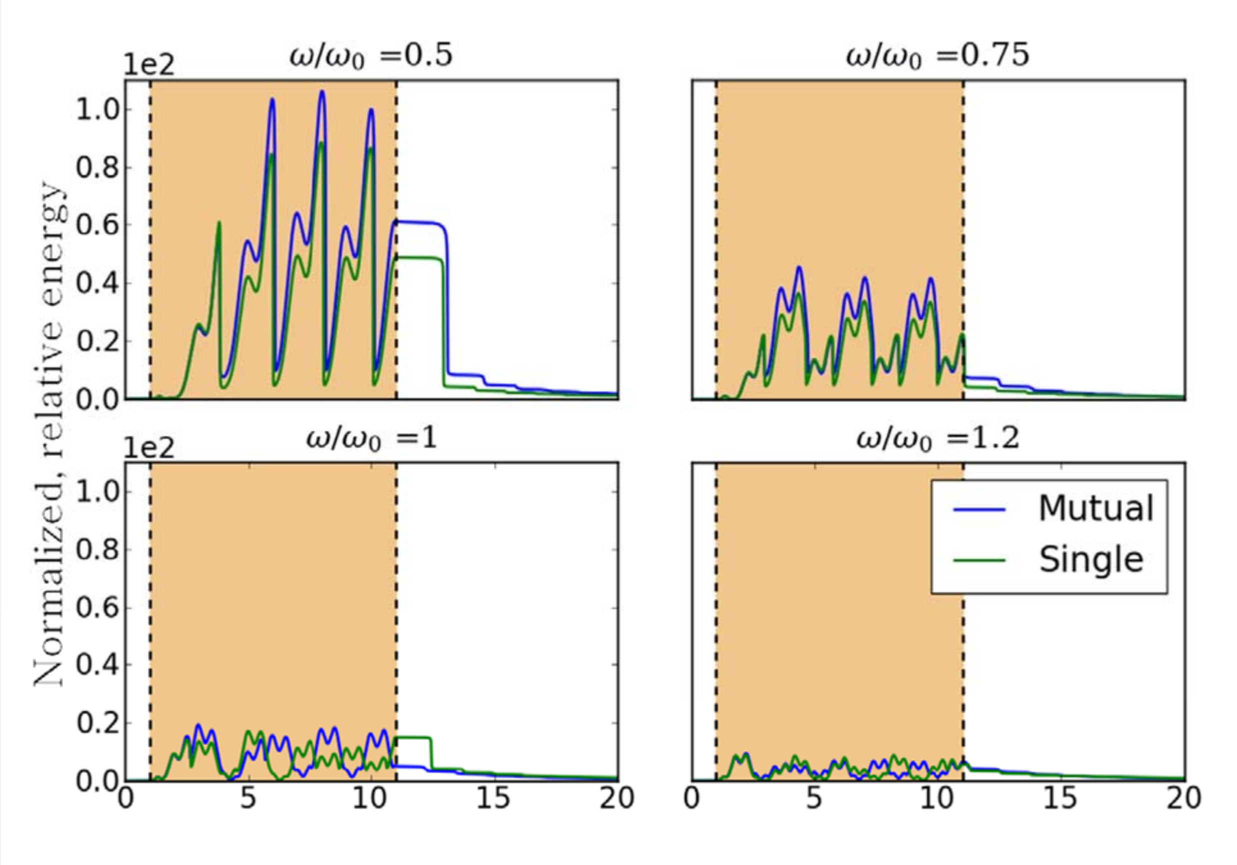

Figure 4: Normalized relative energy in the bubble system as a function of nondimensional time ($\tau = t\omega_0/2\pi$) in response to pulses at a range of frequencies ($\omega/\omega_0 = 0.5, 0.75,\ 1, 1.2$) for a pressure amplitude p_e/P_0= 1.97. The pulses are ten times as long as the natural period of a single bubble. The two bubbles have the same equilibrium size ($R_{02}/R_{01} = 1$).

Figure 5 shows results of simulations for a variety of equilibrium bubble sizes. The bubbles are driven by the same external pressure amplitude as in the bottom right of Fig. 3 ($p_e/P_0 = 1.97$). The relative equilibrium size of the bubbles has a range of values (R_{02}/R_{01} between 0.5 and 2). The driving frequency is the natural frequency of a single bubble ($\omega/\omega_0 = 1$). Again it is observed that the inclusion of corrections for mutual compressibility (blue lines) impacts the predicted dynamics as compared to the equations of motion with only single bubble compressibility effects (green lines) even more dramatically than the previous figure.

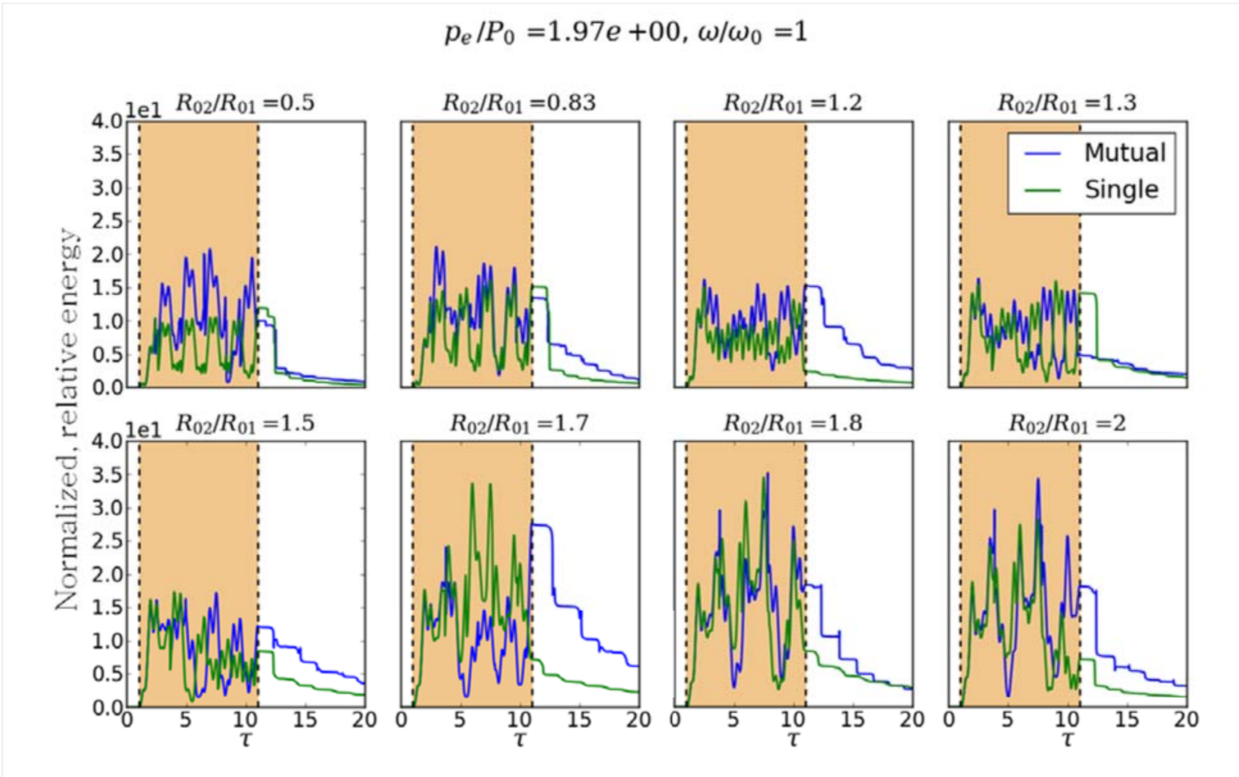

Figure 5: Normalized relative energy in the bubble system as a function of nondimensional time ($\tau = t\omega_0/2\pi$) in response to ten-cycle pulses at the natural frequency of the single bubbles ($\omega/\omega_0 = 1$) for pressure amplitude p_e/P_0= 1.97. The two bubbles have a range of relative equilibrium sizes (R_{02}/R_{01}).

Conclusions and future work

An approximate model for the coupled dynamics of a system of closely space bubbles in a compressible medium (Eq. (6)) undergoing nonlinear oscillations was developed and integrated in the time domain. Previous work on coupled bubbles in a compressible liquid has relied primarily on linearized equations, and has largely been conducted in the frequency domain, or has neglected the effects of propagation delays. A subclustering algorithm by which delayed interactions might be included in simulations of large bubble systems was presented. The approximate equations of motion presented here (Eq. (6)) are intended to be used within subclusters, while average pressure quantities for each subcluster will be explicitly delayed.

It was shown that even for closely spaced bubbles (small delays) the delay in bubble interaction can strongly impact the predicted dynamics over a range of parameters. Thus, for large amplitude oscillations of closely spaced bubbles, the delays should not be neglected. This implies that Eqs. (2) and (6) are to be preferred over Eq. (1) when modeling bubble systems.

Future work will require a numerical solver to integrate Eq. (2). Numerical solution of Eq. (2) requires a solver specialized for delay differential equations. The results of numerical integration of Eq. (2) will then be used to benchmark the model equations discussed here (Eq. (6)), and eventually, the subclustering algorithm presented here.

Bibliography

[1] Yu. A. Pishchalnikov, O. A. Sapozhnikov, M. R. Bailey, I. V. Pishchalnikova, J. James C. Williams, and J. A. McAteer, “Cavitation selectively reduces the negative-pressure phase of lithotripter shock pulses,” *Acoustics Research Letters Online*, vol. 6, no. 4, pp. 280–286, 2005.

[2] Yu. A. Ilinskii, M. F. Hamilton, and E. A. Zabolotskaya, “Bubble interaction dynamics in Lagrangian and Hamiltonian mechanics,” *The Journal of the Acoustical Society of America*, vol. 121, no. 2, pp. 786–795, 2007.

[3] Yu. A. Ilinskii and E. A. Zabolotskaya, “Cooperative radiation and scattering of acoustic waves by gas bubbles in liquids,” *The Journal of the Acoustical Society of America*, vol. 92, no. 5, pp. 2837–2841, 1992.

[4] C. Feuillade, “Scattering from collective modes of air bubbles in water and the physical mechanism of superresonances,” *The Journal of the Acoustical Society of America*, vol. 98, no. 2, pp. 1178–1190, 1995.