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On the Evolution of Crackle in Jet Noise from High-Performance Engines

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Crackle, the impulsive quality sometimes present in supersonic jet noise, has traditionally been defined in terms of the pressure waveform skewness. However, recent work has shown that the pressure waveform time derivative is a better quantifier of the acoustic shocks believed to be responsible for its perception. This paper discusses two definitions of crackle, waveform asymmetry versus shock content, and crackle as a source or propagation-related phenomenon. Data from two static military jet aircraft tests are used to demonstrate that the skewed waveforms radiated from the jet undergo significant nonlinear steepening and shock formation, as evidenced by the skewness of the time derivative. Thus, although skewness is a source phenomenon, crackle's perceived quality is heavily influenced by propagation through the near field and into the far field to the extent that crackle is caused by the presence of shock-like features in the waveform.

Nomenclature

$_{\mathrm{peak}}$ = frequency of maximum radiation
$_{s}$ = sampling frequency
OASPL = overall sound pressure level, dB re 20 µPa
$p(t)$ = pressure waveform, in pascals
$\partial p/\partial t$ = time derivative of pressure, in Pa/s
PDF = Probability density function
$\mathrm{Sk}\{p(t)\}$ = skewness of pressure, "pressure skewness"
$\mathrm{Sk}\{\partial p/\partial t\}$ = skewness of pressure time derivative, "derivative skewness"

I. Introduction

CRACKLE, the supersonic jet noise phenomenon, has been labeled as an annoying and dominant characteristic of the total noise[1,2]. Ffowcs Williams *et al.*[1] described it as "sudden spasmodic bursts of a rasping fricative sound . . . It is a startling staccato of cracks and bangs and its onomatope, 'crackle,' conveys a subjectively accurate impression." One of the main conclusions of Ffowcs Williams *et al.*, a conclusion that has guided many investigations since then, was that the skewness of the pressure waveform can be used to conveniently identify a

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crackling jet. The skewness, Sk, a dimensionless normalization of the third central moment of the waveform probability density function (PDF), is a measure of the PDF's asymmetry. (Note that for a symmetric distribution, e.g. a Gaussian, Sk = 0.) Based on analysis of various jet noise recordings, Ffowcs Williams *et al.* established a threshold of Sk > 0.4 for waveforms as "distinctly" crackling. However, in the same study, the authors concluded that the "physical feature of a sound wave that gives rise to the readily identifiable subjective impression of 'crackle' is shown to be the sharp shocklike compressive waves that sometimes occur in the wave form." However, because pressure waveform skewness, $\mathrm{Sk}\{p(t)\}$, quanitifies only the asymmetric occurrence of pressure values, it is wholly insensitive to temporal rate of pressure changes, $\partial p/\partial t$. It was therefore deemed an "incomplete metric" by Papamoschou and Debiasi.[3] Gee *et al.*[4] showed that the shocklike features were essential in creating a crackling waveform, i.e., that reproduction of the power spectrum and the positively skewed PDF of an F/A-18E waveform were insufficient to create a crackling waveform. They suggested that the quantifying of crackle should be based on the waveform *time derivative*, which could be used to conveniently identify the shocklike features noted by Ffowcs Williams *et al*. Other metric possibilities, including stationary and time-varying loudness,[5,6] have proved to be relatively insensitive to waveform shock content.

The recent findings by Gee *et al*.[4] lead to questions that are the principal focus of this paper. First, should crackle be defined in terms of the asymmetry of the PDF using Ffowcs Williams *et al.*'s threshold, or should it be defined in terms of a waveform's shock content? Second, is crackle a source or propagation phenomenon? Both are important considerations when attempting to reduce crackle in laboratory and full-scale jets. It is noted at the outset that this paper is limited to examining these questions from a physical basis; psychoacoustic connections are not explored at this time. To contexualize our approach to addressing these questions, a review of the crackle-related literature is first given. Following this review, two static full-scale experiments involving the F-35AA Joint Strike Fighter and F-22A Raptor are summarized and analyzed from the perspective of crackle.

II. Crackle, Skewness, and Shocks: A Discussion

The study by Ffowcs Williams *et al.*[1] established a relatively simple criterion for defining crackle. Waveforms for which $\mathrm{Sk}\{p(t)\} > 0.4$ distinctly crackled, and waveforms for which $\mathrm{Sk}\{p(t)\} < 0.3$ did not crackle. In 1975, the same year that Ffowcs Williams *et al*. published their work on crackle, Schlinker[7] reported skewness values in his dissertation on supersonic jet experiments. Since that time, several studies have included $\mathrm{Sk}\{p(t)\}$ as part of the experiment documentation and discussed implications in terms of laboratory and full-scale crackling jets.[2,3,8-18] Crighton[19] sought an explanation for pressure skewness in the context of nonlinear propagation of jet noise, a theme that has been discussed more recently by Petitjean and McLaughlin,[8] Petitjean *et al*.,[9] and by Schlinker *et al*.[13,14] Krothapalli *et al*.[2] described pressure skewness as a source mechanism resulting from Mach wave radiation and proposed that asymmetric wavefoms were caused by "microexplosions" due to rapid expansion of cool, ambient air when entrained in the hot jet. The connection between Mach wave radiation and crackle/skewness has been described by various authors, including the context of crackle reduction.[20,10,11] Recent numerical simulations by Nichols *et al.*[21] and Anderson and Freund[22] have both indicated that pressure skewness originates as a source phenomenon. Schlinker *et al*.[13] applied an eductionmethod to high-power set point data from a full-scale engine and localized impulsive signatures to be around 5 nozzle diameters. Note further that pressure skewness has been documented in military jet flyover data,[15,16] in static rocket motors and launch vehicles,[23,24] and in explosive volcanic eruptions.[25]

Because of the simplicity of Ffowcs Williams *et al*.'s criterion for distinct crackle, $\mathrm{Sk}\{p(t)\} > 0.4$, it is natural that it has seen significant use in analysis of jet data at different scales. However, one issue with using $\mathrm{Sk}\{p(t)\} > 0.4$ as a threshold is that the statistics of the waveform are dependent on the response of the data acquisition system. Ffowcs Williams *et al*. showed how a lack of low-frequency response could transform the waveform shape and noted, "The skewness factors measured are likely therefore to be unique to this type of measurement and analysis equipment, which is in wide use throughout the international aviation community." (Although the equipment was in wide use then, it certainly is not now!) Although they added a footnote in which they indicated they reproduced the results for many of their experiments using a recorder with a flatter low-frequency response, the statement by Ffowcs Williams *et al*. suggests that $\mathrm{Sk}\{p(t)\} > 0.4$ was never intended to be an absolute threshold for crackle. Yet, it has been used as *de facto* criterion ever since. This concern with low-frequency instrumentation response was shared by McInerny *et al*.[26] in their studies of launch vehicle noise, which has very low peak frequencies. They showed that high-pass filtered waveforms meant to simulate loss of low-frequency response resulted in artificially greater skewness values. McInerny *et al*. affirm the potential problem in using the $\mathrm{Sk}\{p(t)\} > 0.4$ as an absolute threshold to define crackle for data acquired using instrumentation with different frequency responses.

The discussion thus far has centered on the body of literature describing the use of $\mathrm{Sk}\{p(t)\}$ to chararcterize jet noise data, nearly often in the context of crackle. There appears to be no debate in the jet aeroacoustics community that supersonic jets create skewed waveforms and that these waveforms, being of high amplitude, can contain acoustic shocks. However, because of the Ffowcs Williams *et al.* study, $\mathrm{Sk}\{p(t)\}$ and crackle were made synonymous, a connection that Gee *et al.*[4] rebutted in their 2007 study by drawing, in part, on a separate body of literature related to high-amplitude jet aeroacoustics and shock formation and propagation. Schlinker[7] suggested the use of the time derivative of the waveform to characterize the rapid changes in the pressure waveforms and McInerny and others analyzed the skewness of the time derivative, $\mathrm{Sk}\{\partial p/\partial t\}$, in launch vehicle[24] and military jet data[15,16] because it is a much more sensitive indicator to shock content. Although centered finite-difference methods have been used to obtain derivative estimates,[9] McInerny and Olcmen[27] noted that smoothness requirements are not met for this kind of differentiation, and so a first-order forward or backward estimate is more accurate.[4] Note that an alternate wavelet-based descriptor of shock content and crackle has been recently proposed by Baars and Tinney[28] and may be considered as part of future analyses.

Since the demonstration[4] that the time derivative could be useful in better defining cracke, the statistics of the time derivative have been documented in laboratory-scale experiments by, e.g., Gee *et al.*,[12] Mora *et al.*,[11] and Baars *et al.*[29] However, fundamental questions remain: Do shock-containing, crackling waveforms occur in jets without skewness? Are shocks present at the source or are they formed through the course of nonlinear propagation? Despite suggestions that pressure skewness is a sufficient indicator of N-shaped waveforms (e.g., see Ref. 21) in real jets, Gee and Sparrow[30] showed a measured PDF at 305 m from the F-22A Raptor that is nearly Gaussian ($\mathrm{Sk}\{p(t)\} \approx 0.1$), but where the shock content crackle quality was “distinctly” audible. (In fact, the first author observed this sound quality firsthand near the measurement location while wearing double hearing protection!) This potentially suggests a greater decoupling of skewed pressure values and large time derivatives than previously thought.

A fundamental question remains: Where does the acoustic shock content originate? Although Ffowcs Williams *et al.*[1] discussed nonlinear propagation as a possible reason for crackle, they suggested (probably incorrectly) far-field nonlinear propagation was too weak and, therefore, the nonlinearity and resultant rapid pressure changes occurred at the source. Nichols *et al.*[30] found skewed, steepened waveforms just outside the shear layer in their large-eddy simulations of a heated, supersonic jet and also concluded that, although propagation effects could result in strengthening, the shocks already existed at the source. Finally, Baars *et al.*[29,30] examined the statistics of a Mach 3.0 unheated jet and concluded that, for their experiment, shocks were due to local (source) nonlinearities and not caused by cumulative, propagation effects. All three of these studies suggested that, whether crackle is defined using $\mathrm{Sk}\{p(t)\}$ or as the presence of acoustic shocks (possibly relying on $\mathrm{Sk}\{\partial p/\partial t\}$), it should be defined as a source phenomenon. In the present study, we offer a different viewpoint. If auralization of crackle relies on the presence of acoustic shocks, then it is more appropriately quantified using $\partial p/\partial t$ than $\mathrm{Sk}\{p(t)\}$. In the context of acoustic shocks, we show evidence using full-scale military jet noise data from the F-35AA and F-22A that crackle should be understood as a phenomenon heavily influenced by *cumulative* nonlinear propagation effects.. However, we also use the same data sets to strengthen the current belief by the jet aeroacoustics community that pressure waveform skewness is, in fact, a source phenomenon.

III. Full-Scale Experiment Summmaries

A. F-35AA Joint Strike Fighter

The F-35AA static run-up measurements were conducted 18 October, 2008 at Edwards Air Force Base (EAFB), CA. The measurements were made jointly by the Air Force Research Laboratory, Blue Ridge Research and Consulting, and Brigham Young University. A photograph of the tied-down aircraft is displayed in Figure 1.

Figure 1. Tied-down F-35AA aircraft, along with tripods of the near-field microphone array.

Measurements[18,31,32] were made using 6.35 mm Type 1 free-field and pressure microphones located at a height of 1.5 m (5 ft). The pressure microphones were oriented skyward, for nominally grazing incidence. The free-field microphones were pointed toward the plume, aimed at a point approximately 6.7 m aft of the aircraft. This point, which is about 7-8 nozzle diameters downstream of the engine exit plane (the same scaled distance used for a previous F-22A experiment in 2004)[33], was set as the origin for defining observation angles. During the test, the average wind speed was less than 1 kt and the ambient pressure was virtually constant at 0.914 kPa. Temperature and relative humidity varied from 7 – 16 °C and 21-27%, respectively.

Data acquisition for the array described in this paper (the "near-field" array involving all the microphones closer than 76 m)[18] was carried out using a National Instruments® 8353 RAID server connected to a PXI chassis containing PXI-4462 cards. Analog input ranges for each channel were adjusted (in 10 dB increments) for low and high-power settings, based on the sensitivity of each microphone, in order to maximize the dynamic range of each of the 24-bit cards. The system sampling frequency was varied between 96 and 204.8 kHz. The lower sampling rate was required because of slower hard drive write speeds for the early-morning tests while the system was cold and during afterburner, where system vibration was greater. The system was located forward of the aircraft and to the sideline (about 70°) at an approximate distance of 35 m. Data at 50% Engine Thrust Ratio (ETR) and 100% ETR (military power), both sampled at 96 kHz, are described in this paper. Additional results from this experiment are shown in Refs. 18, 31, and 32.

B. F-22A Raptor

In July 2009, researchers at Brigham Young University and Blue Ridge Research and Consulting took extensive noise measurements near an F-22A Raptor at Holloman Air Force Base. The jet was tied down to a concrete run-up pad and cycled through four engine power conditions: idle, intermediate, military, and afterburner. Data at 150 channels were recorded using a similar recording system using PXI-446x and 449x series cards at 96 kHz for the three lower power conditions and 48 kHz at afterburner, because of slow hard drive write speeds caused by system vibration. (This despite the fact that the system was located inside a concrete building about 25 m to the sideline of the aircraft.) A complete description of the experiment is found in Ref. 34.

The data analyzed in this study were recorded on a rectangular array of microphones shown in Fig. 2. The 90 microphones were 15.2 cm (6.0 in) apart and covered an aperture 0.6 m high by 2.6 m (2 ft x 8.5 ft) long. The rig that held the microphones was positioned at ten locations along a 22.9 m (75 ft)-long track. The rig was also adjusted to three heights during the experiment, with the center of the array at 0.7, 1.3 and 1.9 m (27, 51, and 75 in). When the rig was moved to a different position for a new scan, it was positioned such that several microphones overlapped the previous scans. When the data from the 30 scans are pieced together, they yield a 1.8 m x 23.2 m (6 ft by 76 ft) measurement plane.

The track was moved to the different locations as illustrated in Fig. 3 by the solid black lines. The red triangles along the track indicate the locations of the center of the microphone array for subsequent measurement scans. The set of measurements obtained 4.1 m from the shear layer of the jet plume are referred to as plane 1 data, while plane 2 data comes from measurements taken 5.6 m from the shear layer of the jet plume. Data along both measurement planes 1 and 2 are taken at the three heights and ten horizontal positions described in the previous paragraph. Additionally, measurements were taken along a third plane, parallel to the centerline, and in 10° increments along a 23 m arc referenced from the estimated peak source location.[33] The height of the array center was 1.9 m (75 in) for the arc measurements.

Figure 2. The tied-down F22A Raptor with one engine at afterburner. Shown also is the 90-microphone rectangular array.

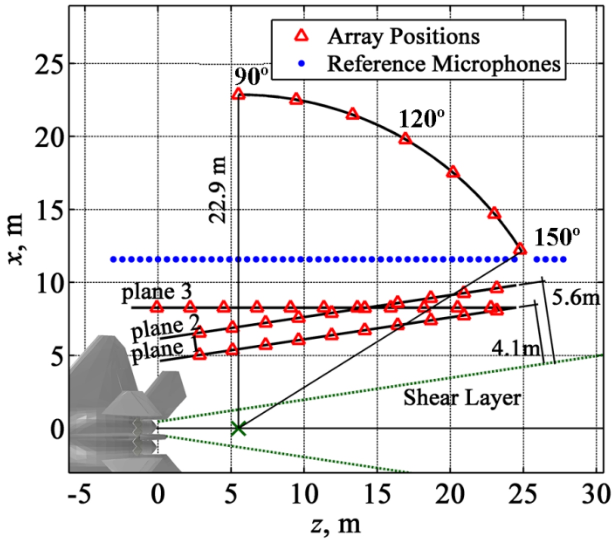

Figure 3. Diagram of the experimental set-up for the acoustical measurements on an F-22A Raptor. The triangles, each 2.3 m apart, mark the center of the microphone array for individual scans. The origin is set at ground level centered below the jet nozzle: x is the distance away from the jet plume's centerline, y is the height off the ground and z is the distance downstream from the nozzle. The green "x" refers to the estimated peak source location and the reference from which the angles are measured.

For the sake of completeness, the data from this 2009 experiment have already been analyzed in a variety of contexts, from level-based,[34,35] similitary spectra,[36-38] autocorrelation,[39,40] and intensity[41] analyses, to near-field acoustical holography[42-44] and equivalent source modeling.[45,46] Note further that the far-field nonlinear propagation results from the 2004 F-22 measurements,[33] which cover a range of 23-305 m, are also germane to the discussion of crackle and acoustic shock formation.

IV. F-35AA Data Analysis

The present F-35AA analysis builds on a previous analysis,[18] and considers analyses of OASPL, $\mathrm{Sk}\{p(t)\}$, and $\mathrm{Sk}\{\partial p/\partial t\}$ for 50% ETR and 100% ETR in order to provide examples of low and high-power engine conditions. A more comprehensive analysis as a function of engine condition will be presented in the future. The three measures are shown in Fig. 4 for 50% ETR and Fig. 5 for 100% ETR, respectively. The microphone locations are denoted by markers, with a cubic interpolation between data points. Because of the nature of the interpolation, the accuracy of the levels or skewness values immediately around the aircraft is likely reduced by aircraft shielding or scattering. In Fig. 4a, the maximum directivity of the OASPL at 38 m occurs near, but possibly at greater angles than, 150° defined relative to the engine inlet and the 6.7 m origin. These level-based data in Fig. 4a provide insight into the spatial properties of the skewness of the acoustic pressure waveform and its time derivative for 50% ETR. Displayed in Fig. 4b and 4c are skewness maps for the pressure and derivative, respectively. In Fig. 4a, the pressure skewness values range between approximately 0.0 and nearly 0.1. Although they are small, they possess a clear trend in directivity that is around approximately 120°, about 30° upstream of the OASPL lobe for this condition. Values are low near the shear layer downstream. The relative constancy (0.075-0.095) in the maximum direction outside the first few nozzle diameters provides evidence that the waveform asymmetry (i.e., $\mathrm{Sk}\{p(t)\}$) is produced as a source phenomenon, or at least very near the shear layer.

To examine the F-35AA data at 50% power for nonlinear steepening effects, the results for $\mathrm{Sk}\{\partial p/\partial t\}$ are displayed in Fig. 4c. Even for 50% engine power, $\mathrm{Sk}\{\partial p/\partial t\}$ exhibits fundamentally different behavior than $\mathrm{Sk}\{p(t)\}$; there is a clear trend in terms of increasing skewness of $\partial p/\partial t$, with larger magnitudes of the derivative occuring at greater distances from the source. The *only* cause for a systematic increase in positive derivative magnitudes near the maximum radiation direction is nonlinear waveform steepening. The relative importance of this steepening at low engine power, a somewhat surprising result, is not considered futher in this paper, but we note that $\mathrm{Sk}\{\partial p/\partial t\}$ demonstrates significantly non-Gaussian behavior and agrees with the propagation trends noted both by Gee *et al.*[12] and Mora *et al.*[11] for laboratory-scale measurements.

The OASPL, $\mathrm{Sk}\{p(t)\}$, and $\mathrm{Sk}\{\partial p/\partial t\}$ for military power are repeated from Ref. 18 in Fig. 5. Note that many of the same trends from 50% ETR are repeated, but with significantly greater values. As expected, the increase in thrust has shifted the main OASPL lobe in Fig. 5a toward the sideline, to approximately 125-130°. The values for $\mathrm{Sk}\{p(t)\}$ in Fig. 5b have increased significantly from 50% ETR, with values of 0.3-0.41 over a very large angular aperture from about 80-140°. In addition, $\mathrm{Sk}\{p(t)\}$ appears to originate relatively close to the nozzle, potentially corroborating the 5 nozzle diameter estimate by Schlinker *et al.*[13] The rapid decrease in $\mathrm{Sk}\{p(t)\}$ with increasing downstream distance near the shear layer agrees with laboratory-scale measurements by Mora *et al.*,[11] large eddy simulations of Nichols *et al.*[21] and with analyses by Gee *et al.*[23] for solid rocket motor noise. It is important to note that the skewness values for the F-35AA at 100% ETR, with the maximum region ranging between 0.3 and 0.4, would be considered "borderline" by the traditional Ffowcs Williams *et al.*[1] crackle criterion. However, despite the qualitative nature of words like "distinctly," there is *absolutely* no question that crackle is readily audible in noise from the F-35AA at military power. Thus, Fig. 4b provides evidence beyond the caution by Papamoschou and Debiasi[3] and by Gee *et al.*[4] that the choice of metrics used to define crackle should be revisited.

Although the spatial trends are similar, Fig. 5c depicts a dramatic increase in $\mathrm{Sk}\{\partial p/\partial t\}$ for 100% ETR relative to 50% ETR, with maximum values exceeding 8.5 near the peak radiation direction. This figure provides unique and significant insight into the nature of the radiation of finite-amplitude noise from military jets. Whereas there is a broad spatial region with relatively constant pressure skewness, the skewness of the derivative—an indicator of the formation of acoustic shocks—has a rapid growth in the aft direction near that of maximum OASPL (see Fig. 5a). The maximum derivative skewness occurs in areas along the principal radiation lobe (~110-140° from Fig. 5a), indicating correlation of nonlinear propagation with Mach wave radiation. However, the apparent origin of the evolving skewness appears to be quite close to the nozzle, upstream of the estimated maximum overall source location. This observation also suggests the importance of the initial high-frequency content in the nonlinear evolution and waveform steepening. The pressure derivative skewness also increases in the sideline and forward directions, suggesting nonlinear wave steepening is occurring, but not nearly as quickly. Note, however, that the values in the forward direction for 100% ETR exceed those in the aft direction for 50% ETR.

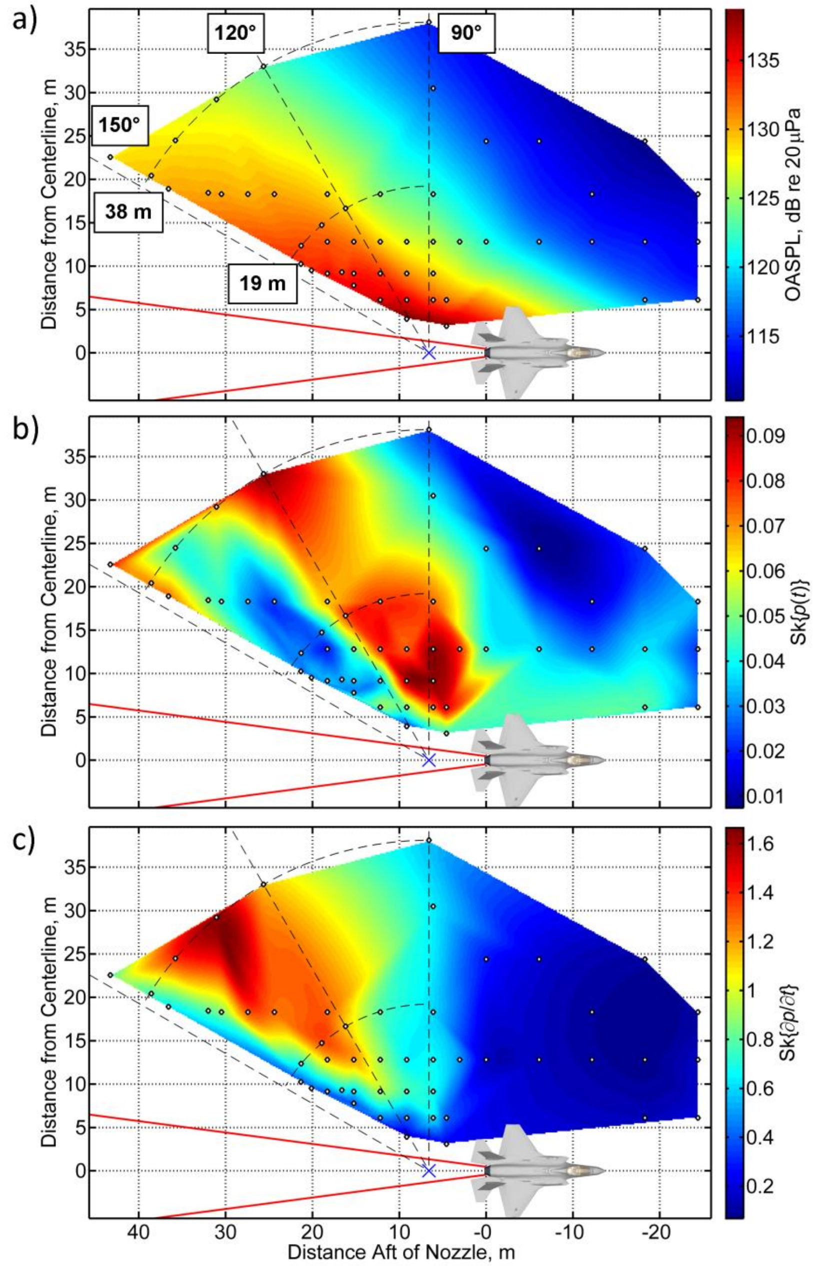

Figure 4. Measurement of a) OASPL, b) $\mathrm{Sk}\{p(t)\}$ and c) $\mathrm{Sk}\{\partial p/\partial t\}$ for the F-35AA at 50% ETR.

Although the documentation of $\mathrm{Sk}\{p(t)\}$ has been moderately common and $\mathrm{Sk}\{\partial p/\partial t\}$ is seeing more use, there has been little quantitative work to describe their behavior for broadband noise transmission. Atmospheric dispersion, which produces a slight rounding of positive waveform peaks more than negative ones, and suppression of outliers due to nonlinear propagation should cause a postive $\mathrm{Sk}\{p(t)\}$ to decrease with distance, at least in the far field. To relate $\mathrm{Sk}\{\partial p/\partial t\}$ to shock formation, Shepherd *et al.*[47] recently calculated $\mathrm{Sk}\{\partial p/\partial t\}$ for a nonlinearly evolving sinusoid in the preshock region. They showed that the skewness increases exponentially from near zero to values greater than 10 as the shock formation distance is approached. Extension of the sine-wave study to broadband noise, where shocks form at varying distances, is not fully known, but preliminary experimental work by Muhlestein and Gee[48] suggests similar behavior of the derivative skewness for noise in the preshock region. A derivative skewness value of ~1.8 near the shear layer increases to a value more than 8.5 by 38 m. With present quantitative understanding, this indicates that slightly steepened waveforms at the shear layer undergo rapid waveform steepening and shock formation. This finding was confirmed by examining an amplitude-normalized, retarded time-aligned waveform segment along 130° at 8 and 38 m in Ref. 18; although both waveforms were skewed, only the 38 m waveform had significant shock content. Further discussion of these results as they relate to crackle is delayed until after presentation of the F-22A results.

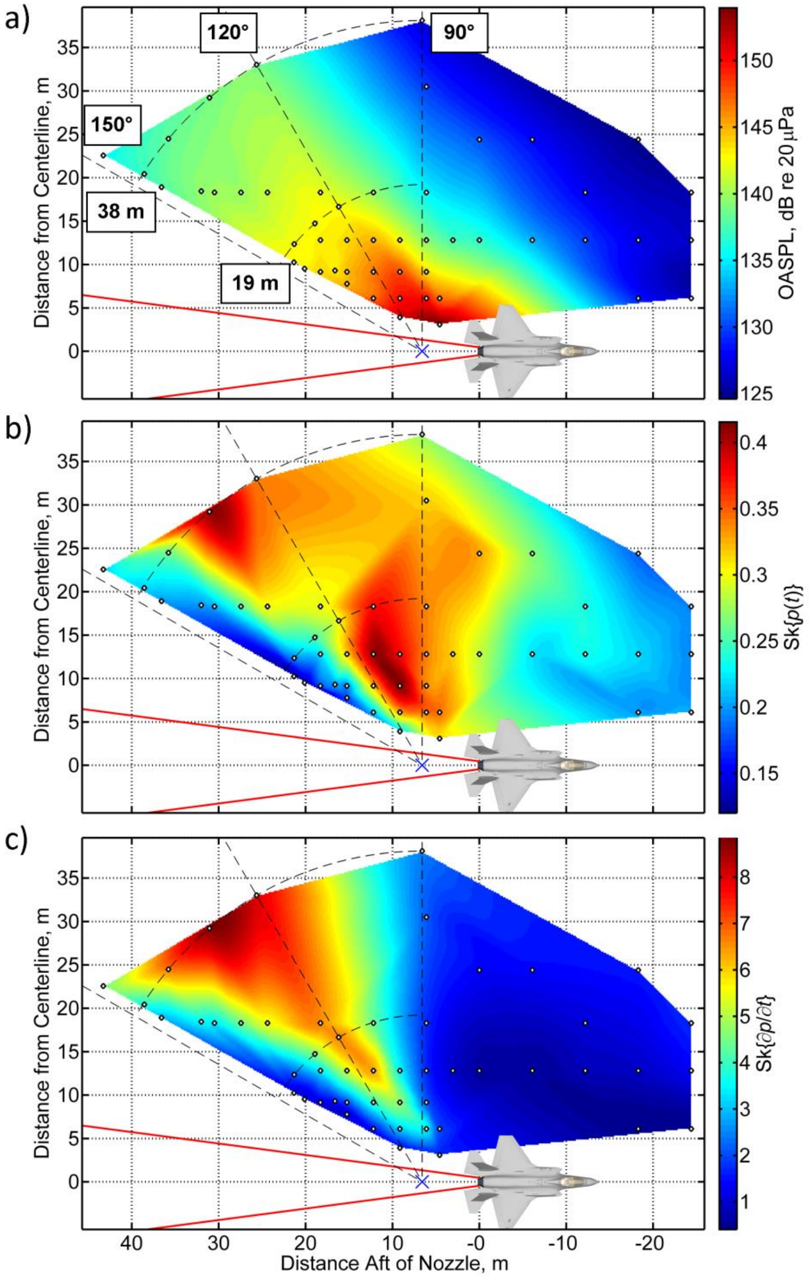

Figure 5. Measurement of a) OASPL, b) Sk$\{p(t)\}$ and c) Sk$\{\partial p/\partial t\}$ for the F-35AA at 100% ETR.[18]

V. F-22A Data Analysis

A. Engine Condition Comparison

As with the F-35AA analysis, the OASPL, Sk$\{p(t)\}$, and Sk$\{\partial p/\partial t\}$ near the F-22A Raptor yield insight into the source versus propagation characteristics of the radiated field. Using the data from the rectangular 90-microphone rig (see Figs. 2 and 3), these measures are shown for four engine conditions (idle, intermediate, military, and afterburner), with idle and intermediate in Fig. 6 and military and afterburner in Fig. 7. Because the afterburner engine condition was measured at a lower sampling frequency (48 kHz vs 96 kHz) than the other three engine conditions, a discussion of the effect of the sampling frequency on skewness estimation is also presented.

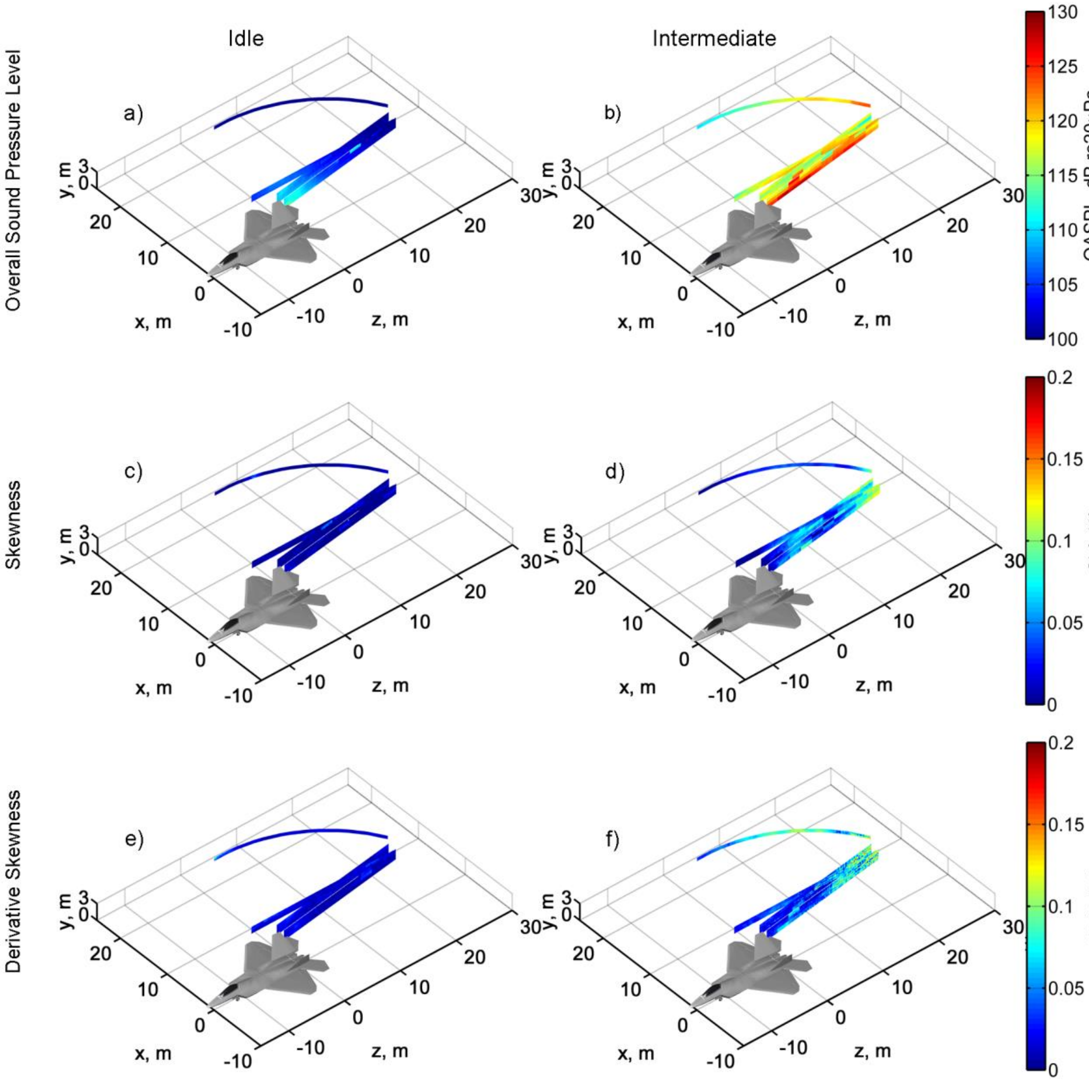

Figure 6. Maps of OASPL, Sk$\{p(t)\}$, and Sk$\{\partial p/\partial t\}$ for the F-22A with one engine at idle and intermediate conditions. (The other engine was held at idle.)

The results in Fig. 6 show that the low-power engine conditions have lower levels, with a maximum OASPL for idle of 114 dB at a distance of 5.5 from the nozzle. The intermediate engine condition is harder to analyze quantitatively, since there is not a set "intermediate" thrust position, and the condition was obtained by the pilot manually adjusting the thrust. This subjectivity led to greater variation in the measurements, and is evidenced by the fact that the maps associated with the intermediate engine condition do not vary continuously. This variability is analyzed further in Ref. 34. In general, it appears that the OASPL has a broad maximum along the shear layer, similar to the F-35AA results for 50% in Fig. 4, with a maximum level of about 130 dB. Note the negligible values of $\mathrm{Sk}\{p(t)\}$, and $\mathrm{Sk}\{\partial p/\partial t\}$ for idle; all measurements are below 0.05 for both quantities. For intermediate power, $\mathrm{Sk}\{p(t)\}$ is similar to that of the F-35AA at 50%, but $\mathrm{Sk}\{\partial p/\partial t\}$ is significantly less over the same range. The reason for the essentially negligible $\mathrm{Sk}\{\partial p/\partial t\}$ is unclear at the present, but could be related to the complex nozzle geometry of the F-22A engine with its deflector paddles that form a nominally rectangular outlet from the round nozzle.

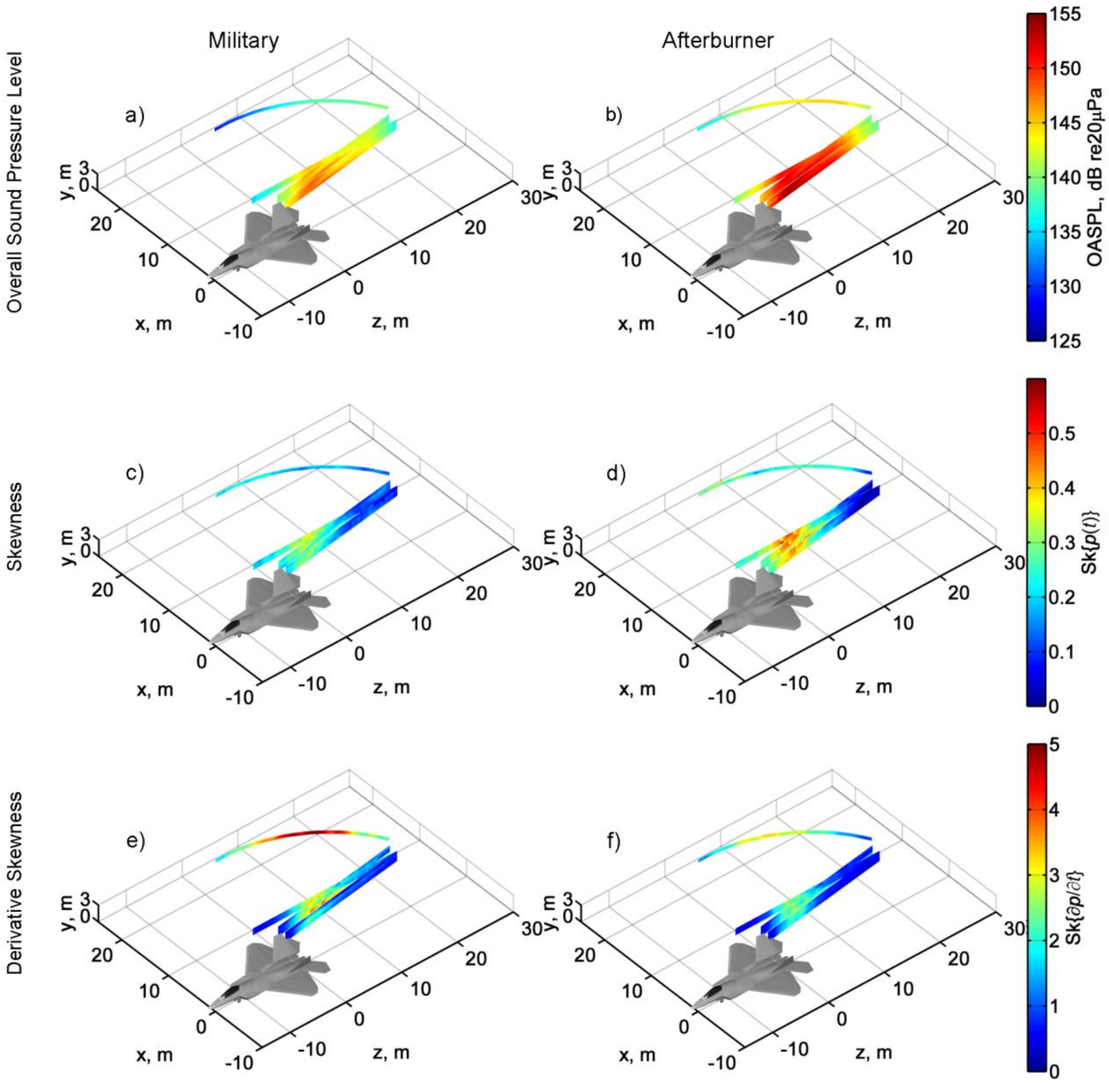

Figure 7. Maps of OASPL, $\mathrm{Sk}\{\boldsymbol{p(t)}\}$, and $\mathrm{Sk}\{\boldsymbol{\partial p/\partial t}\}$ for the F-22A with one engine at military and afterburner conditions. (The other engine was held at idle.)

Figure 7 demonstrates a general trend for the military and afterburner engine conditions of significantly higher OASPL and skewness estimates, relative to the low power conditions. The OASPLs for both engine conditions peak along plane 1 (see Fig. 3) about 10 m downstream from the nozzle. The maximum OASPL is 149 dB for the military condition, and 155 dB for afterburner. For $\mathrm{Sk}\{p(t)\}$, the spatial trend is similar to the OASPL; $\mathrm{Sk}\{p(t)\}$ is greater near, but somewhat upstream of the maximum OASPL region, and decreases with downstream distance. These trends both corroborate the F-35AA data in Figs. 5a and 5b. Similarly, the directivity of $\mathrm{Sk}\{p(t)\}$ in Figs. 7b and 7d for military and afterburner both show a directivity upstream of the OASPL (but note that the proximity of the arc to the shear layer prevents either estimate from being a true “directivity” measurement.) For military power in Fig. 7b, the range of maximum $\mathrm{Sk}\{p(t)\}$ falls between 0.25 and 0.4, but $\mathrm{Sk}\{p(t)\} = 0.5$ was recorded at one location on plane 3. For the afterburner data, $\mathrm{Sk}\{p(t)\}$ approaches 0.6 along planes 1-3, but decreases by the 23 m arc to values of 0.35. Again, we note that the military power data for the F-22A would be "marginally" crackling, which disagrees with our personal observations in the field. For military, as well as afterburner power, the crackle is "distinct."

Values of $\mathrm{Sk}\{\partial p/\partial t\}$ for the F-22A at military power show similar behavior as the F-35AA. The values in Fig. 7e near the shear layer are about 2.0 and increase to 5.3 at 23 m. Comparison with Fig. 5c shows values of 2.5 and 6.2 at comparable distances. For both cases, the trend of systematic, significant growth of nonlinear steepening and shock content away from the shear layer is unmistakable. Comparison of Figs. 7e and 7f for military and afterburner conditions reveals that $\mathrm{Sk}\{\partial p/\partial t\}$ is significantly greater for military than afterburner conditions. Because we expect shock content will be greater for higher levels, this seems counterintuitive. However, the lower 48 kHz sampling rate at afterburner can be used to account for the apparent difference. As is described below, when the data are normalized to the same sampling frequency, the afterburner data exhibit appreciably greater $\mathrm{Sk}\{\partial p/\partial t\}$.

B. Impact of Sampling Frequency on $\mathbf{Sk}\{\partial p/\partial t\}$

Figures 7e and 7f show that the derivative skewness estimate of the afterburner engine condition is lower than the estimate for the military engine condition. This result may be physically misleading, since these two engine conditions were measured with different sampling rates. The probable dependence of $\mathrm{Sk}\{\partial p/\partial t\}$ on sampling rate was recently discussed by Gee *et al.*[12] in analysis of supersonic, laboratory-scale data. In the laboratory-scale jet, the values of $\mathrm{Sk}\{\partial p/\partial t\}$ were significantly less than for the F-35AA in Ref. 18 and in Fig. 5c, yet the waveform still contained significant shocks. It is possible to analyze the effect of sampling rate on the calculation of $\mathrm{Sk}\{p(t)\}$ and $\mathrm{Sk}\{\partial p/\partial t\}$ at military power by numerically downsampling the waveforms of the military power measurement prior to stastical calculations. Figs. 8 and 9 show $\mathrm{Sk}\{p(t)\}$ and $\mathrm{Sk}\{\partial p/\partial t\}$as a function of the ratio between sampling frequency and spectral peak frequency at the measurement location, $f_{\mathrm{s}}/f_{\mathrm{peak}}$, at three locations on the arc array, 90º, 120º, and 150º (see Fig. 3 for definitions of these angles).

Intuitively, if the sampling rate is too low there will not be enough information to give an accurate estimate of any statistical measure. However, it also reasonable to expect a lower $_{\mathrm{s}}/f_{\mathrm{peak}}$ requirement for estimating $\mathrm{Sk}\{p(t)\}$ than $\mathrm{Sk}\{\partial p/\partial t\}$, as the quantization of rise times would be more sensitive than peak amplitude values to f_s. These hypotheses can be seen as correct in both Figs. 8 and 9 for all three angles. Below a certain $_{\mathrm{s}}/f_{\mathrm{peak}}$ threshold, the skewnss estimates at any of the measurement locations become random and meaningless, but the threshold is waveform-dependent. Above that value of $f_{\mathrm{s}}/f_{\mathrm{peak}}$ the estimates for both quantities steadily increase. However, in the case of $\mathrm{Sk}\{p(t)\}$ in Fig. 8, it appears that the skewness estimate for the 150º measurement has reached an asymptotic value, and that the 90º and 120º measurements are rapidly approaching an asymptotic value. It is likely that any sampling frequency larger than those that give the asymptotic value of the pressure skewness will yield inconsequentially more accurate values. While two of the measurements considered here do not appear to have reached the asymptotic skewness estimate, they appear to be approaching it, and so increasing or decreasing the sampling rate by factors of two would not change $\mathrm{Sk}\{p(t)\}$ significantly. Thus, despite the sampling frequency during the afterburner engine condition being half of the sampling frequency used during the other three engine conditions, it is likely that the $\mathrm{Sk}\{p(t)\}$ estimates reported in Fig. 5d represent the waveform values accurately. Note that the results also suggest that the sampling frequency must be a factor of 100 greater than the peak frequency for accurate calculations of $\mathrm{Sk}\{p(t)\}$. Although the corner frequency ratio appears dependent on angle, given the varied characteristics of over the measurement span,[38] this rule-of-thumb might be used to assess data sufficiency in laboratory-scale experiments for characterizing $\mathrm{Sk}\{p(t)\}$.

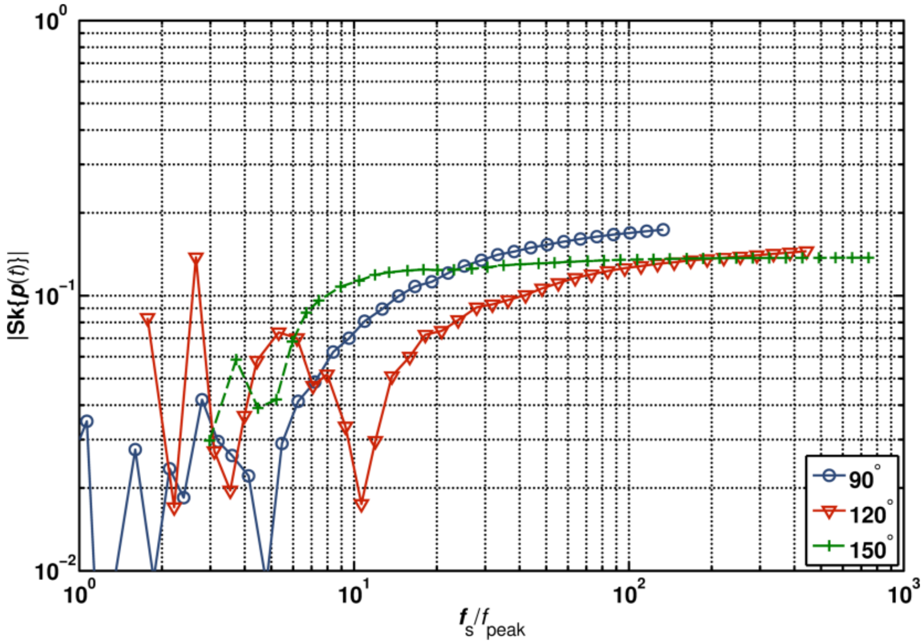

Figure 8. Sk{$p(t)$} as a function of f_s/f_{peak} at 90°, 120°, and 150° for the F-22A experiment along the arc at military power.

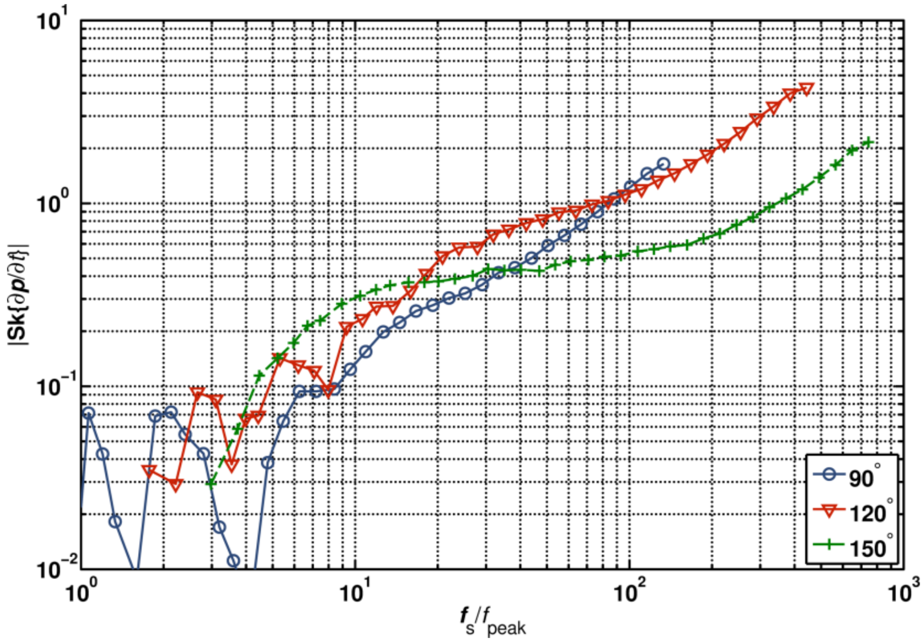

Figure 9. Sk{$\partial p/\partial t$} as a function of f_s/f_{peak} at 90°, 120°, and 150° for the F-22A experiment along the arc at military power.

A different scenario exists for the dependence of Sk{$\partial p/\partial t$} on f_s/f_{peak}, as shown in Fig. 9 for same three waveforms used previously in Fig. 8. Above $f_s/\ _{\text{peak}} \approx 5$, the estimates for the three different angles all increase with f_s/f_{peak}. Figure 9 also shows that, unlike Sk{$p(t)$}, the values for Sk{$\partial p/\partial t$} have not plateaued for the maxium $\ _s/f_{\text{peak}}$ represented by the experimental conditions here. Since shocks in a lossy medium are, in fact, continuous functions with finite thickness, we presume that the curves will eventually approach an asymptotic value for some greater f_s/f_{peak}. With this in mind, the data in Fig. 9 suggest that a sampling frequency of 96 kHz is insufficient to accurately capture the actual shock rise times for full-scale engine data, much less a sampling frequency of 48 kHz! Figure 9 speaks to the importance of sampling rate requirements to capture the high-frequency content present in military jet data at high frequencies, and the problem of making comparable measurements at laboratory scale. We believe the wave-like nature of the curves in Fig. 9 are caused by the inclusion or exclusion of ground-reflected shocks in the waveform, which varies with experiment geometry and $f_\mathrm{s}/f_\mathrm{peak}$. This behavior is still under investigation.

To return to the original goal of making a more accurate quantitative comparison between military and afterburner conditions, reduction of the sampling rate by a factor of two in Figs. 8 and 9 reveals that only marginal changes in $\mathrm{Sk}\{p(t)\}$, but much larger changes (approaching a factor of 1.5) in $\mathrm{Sk}\{\partial p/\partial t\}$ are expected. The downsampling has been carried out for the military data in Fig. 7e, and a new comparison between $\mathrm{Sk}\{\partial p/\partial t\}$ in Figs. 7e and 7f is made in Fig. 10. The comparison between $\mathrm{Sk}\{\partial p/\partial t\}$ for the same sampling frequency reveals what was anticipated on physical grounds previously – the maximum $\mathrm{Sk}\{\partial p/\partial t\}$ for the downsampled military data is 2.7, significantly less than the 3.4 for afterburner. If an extrapolation of the afterburner data in the form of a doubling of sampling frequency is allowed, Fig. 9 suggests that for a sampling rate of 96 kHz, the maximum value of $\mathrm{Sk}\{\partial p/\partial t\}$ will increase from 3.4 to approximately 7. The maps further reveal greater $\mathrm{Sk}\{\partial p/\partial t\}$ at the measurement planes close to the jet are larger for afterburner than for military power, but that increases are expected with propagation distance for both conditions. Overall, these findings emphasize the range and amplitude-dependent evolution of $\mathrm{Sk}\{\partial p/\partial t\}$ of the pressure waveforms of high-amplitude jet noise.

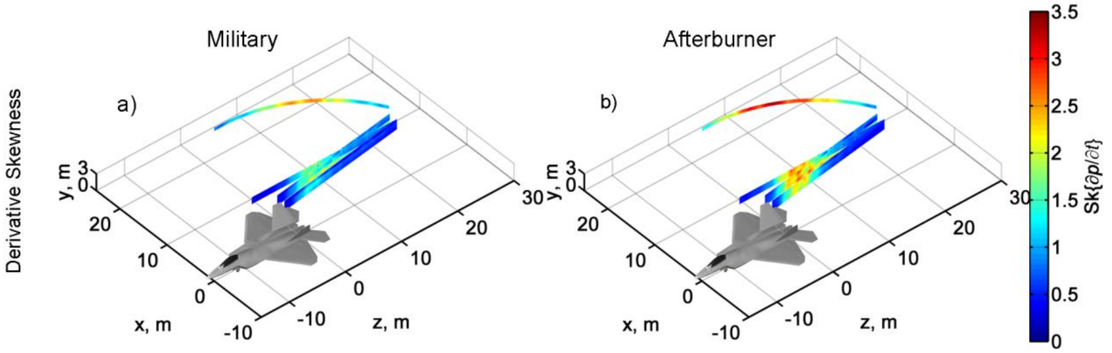

Figure 10. Maps of $\mathrm{Sk}\{\partial p/\partial t\}$ for a) downsampled (96 kHz to 48 kHz) military power data and b) 48 kHz-sampled afterburner data.

VI. Concluding Discussion

Thus far, this paper has presented our view of the body of literature related to crackle, posed key questions, and presented level and statistical analyses of full-scale data from two high-performance military aircraft at high engine power set points. To repose the questions originally asked: First, is crackle more appropriately viewed in terms of waveform asymmetry, which is quantified by the pressure skewness, or the presence of acoustic shocks, which may be quantified by the statistics of the time derivative? Second, should crackle be defined as a source or propagation phenomenon? To answer the first, we contend that crackle, as discussed originally by Ffowcs Williams *et al.*,[1] should be defined as the presence of acoustic shocks in the waveform. It is therefore unfortunate that the original authors chose the waveform skewness, $\mathrm{Sk}\{p(t)\}$, as a measure to quantify crackle, because of its inability to quantify the presence of acoustic shocks in any sense, as pointed out by Papamoschou and Debiasi[3] and Gee *et al.*[4] Consequently, the jet aeroacoustics community has been focused on documenting an inadequate measure for nearly four decades. It is *true* that jets that produce shock-containing waveforms also appear to produce skewed waveforms near the source, but it is *not* true that a jet waveform must be positively skewed with $\mathrm{Sk}\{p(t)\} > 0.4$ for the waveform to crackle. In addition, skewness of pressure is *not* sufficient by itself to perceive crackle, as documented previously by Gee *et al.*[4] Modern high-performance military jet aircraft, which crackle "distinctly" in the near and far fields over large angular apertures, have $\mathrm{Sk}\{p(t)\}$ at or below the previously defined threshold values, particularly in the far field. Thus, it is our belief, as proposed previously by Gee *et al.*[4] that crackle is better defined using the statistics of the time derivative, rather than the pressure waveform itself.

Regarding the question of crackle being a source versus propagation phenomenon, we return momentarily to the origin of pressure skewness. Both full-scale military jet datasets, as well as near-field laboratory-scale data presented by Gee *et al.*[12] and Mora *et al.*[11] show values of $\mathrm{Sk}\{p(t)\}$ that appear to be positive sufficiently close to the source for pressure skewness in a jet to be regarded as source phenomenon. As discussed previously, this assertion is strengthed by numerical simulations by Nichols *et al.*[21] and Anderson and Freund[22]. But, because pressure skewness is *not* crackle, we must consider the spatial behavior of the acoustic shocks to determine whether crackle is a source or propagation phenomenon.

Despite arguments by various authors that regard crackle as a source effect, possibly caused by "local" nonlinearities[1,28], the analysis of both military jet data sets *and* calculations of derivative statistics by Gee *et al.*[12] and Mora *et al.*[11] for laboratory-scale data show a significant increase in $\mathrm{Sk}\{\partial p/\partial t\}$ away from the source. As Gee *et al.*[12,18] show for both laboratory and full-scale data, shocks present in the transition to the far field are *not* present in near-field signatures. Consequently, the noise radiated from the shear layer of a heated, supersonic jet engine exhaust does not have well developed shocks, but rather skewed pressure waveforms that rapidly evolve into shock-rich signals in the dominant radiation direction. Furthermore, in a far-field study of nonlinearity from the F-22A, Gee *et al.*[33] showed a comparison of time-retarded measured waveforms along the maximum radiation direction at afterburner. The continued strengthening of the waveform shock content (lessening of rise times and/or creation of additional shocks) between 23 m and 152 m (more than 250 nozzle diameters!) is unmistakable.

Because of recent efforts tying crackle to jet noise reduction,[10,11,20] this analysis points to a further need to understand crackle as a perceptual phenomenon. To this point, the jet aeroacoustics community predominantly has viewed crackle as a source phenomenon. If defined, as was previously done, as waveform asymmetry, this is true, as the data in this and other studies have indicated. However, if perception is linked to the prevalence and steepness of the acoustic shocks, this study concludes that the crackle will be more pronounced farther from the jet, despite geometric-spreading-induced reductions in overall level. Consequently, it is noted that jet engine noise reduction measures that target a reduction in pressure skewness in an attempt to reduce crackle, without effecting a significant change in overall level may not function as anticipated, because near-field nonlinear propagation could still produce a strong crackle-like perception.

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