Distorted-Wave Analysis of the Reaction $^{12}C(\pi^+, \pi^+ p)^{11}B$

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A comparison of distorted-wave impulse-approximation calculations with recent $^{12}C(\pi^+, \pi^+ p)^{11}B$ data shows excellent agreement and demonstrates the applicability of the quasifree knockout reaction model. Both the shape and the magnitude of the energy-sharing distributions are well described by the calculations.

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In recent papers distorted-wave impulse-approximation (DWIA) calculations for the proton knockout reaction $A(\pi^+, \pi^+ p)B$ have been reported. The emphasis of these papers was an examination of the importance of distortion effects[1,2] arising from the interaction of the incoming and outgoing pions and the outgoing proton with the residual core of the target nucleus, and the treatment of the π-nucleon interaction in the nuclear medium. These studies were carried out for different angles, bombarding energies, and target nuclei, and in all cases showed important distortion effects. No direct comparison with experimental data was made because of the paucity of published data, and the lack of detail for those data which were published.

Piasetzky *et al.*[3] have published extensive data on the angular correlations measured in $(\pi^\pm, \pi^\pm p)$ reactions at 245 MeV. These data strongly support the interpretation of the $(\pi, \pi p)$ reaction in terms of a quasifree knockout model, particularly for backward angles. Furthermore, the data show that the $(\pi, \pi p)$ reaction represents a large portion of the total reaction cross section. However, since the pion energy spectrum was not measured, it is impossible to generate a missing-mass spectrum and therefore to deduce the relative population of final states in the residual nucleus. A direct comparison between the data of Ref. 3 and DWIA calculations would require a great number of calculations and integration of these calculations over the missing mass (with assumptions concerning the final-state population), outgoing proton energy, and in most cases the azimuthal angle. Such integrations and assumptions would almost certainly reduce the sensitivity of the comparison.

Fortunately, detailed energy-sharing distributions for the reaction $^{12}C(\pi^+, \pi^+ p)^{11}B$ have recently been published by Ziock *et al.*[4] These data allow us to make a direct comparison between theory and experiment, and a first test of the adequacy of the DWIA model for the reaction.

As shown in Ref. 2, in the factorized DWIA the triply differential cross section for the reaction $A(\pi^+, \pi^+ p)B$ resulting from the knockout of a bound proton with quantum numbers (n, l, j) leading to a specific final state in nucleus B can be written schematically as

$$\frac{d^3\sigma}{d\Omega_\pi d\Omega_p dE} = KS\sum_\lambda |T_{l\lambda}{}^{BA}|^2 \left[\frac{d\sigma}{d\Omega}\right]_{\pi^+ - p}. \qquad (1)$$

In this expression K is a known kinematic factor, S is the usual proton spectroscopic factor arising from the overlap of the wave functions of nuclei A and B, and $[d\sigma/d\Omega]_{\pi^+ - p}$ is the $\pi^+ + p$ half-off-the-energy-shell two-body cross section. The quantity $T_{l\lambda}{}^{BA}$ is the distorted-wave integral

$$T_{l\lambda}{}^{BA} = (2l+1)^{-1/2} \int \chi^{(-)*}(\vec{k}_{\pi B}, \vec{r})\chi^{(-)*}(\vec{k}_{pB}, \vec{r}) \times \psi_{nl\lambda}(\vec{r})\chi^{(+)}[k_{\pi A}, (B/A)\vec{r}]\, d^3r, \qquad (2)$$

where the χ's represent the incoming pion and outgoing pion and proton distorted waves, and $\psi_{nl\lambda}$ is the bound-proton single-particle wave function with principal quantum number n, orbital angular momentum l, and z component of orbital angular momentum λ.

DWIA calculations were carried out with the code THREEDEE for the reaction $^{12}C(\pi^+, \pi^+ p)^{11}B$ at a bombarding energy of 199 MeV. Energy-sharing distributions were calculated for both the $1p_{3/2}$ transition to the ground state of ^{11}B and the $1s_{1/2}$ transition to the broad excited state at approximately 22-MeV excitation. The optical model potentials were those used in Ref. 2. The pion distorted waves were obtained from a modified Klein-Gordon equation[5] with a Kisslinger-type[6] potential. The parameters (b_0, b_1) for the incident channel were taken from the analysis of Cottingame and Holtkamp.[7] For the exiting pion we have used both the Cottingame and Holtkamp[7] parameters and those of Aman *et al.*[8] The former provide poor fits to the elastic scattering below about 120 MeV, while the latter fit data up to 90 MeV. The energy-sharing distributions calculated cover pion kinetic energies from 45 to 140 MeV. We have used the parameters of Ref. 7 for $T_\pi > 120$ MeV and those of Ref. 8 for $T_\pi < 90$ MeV. For the intermediate range we have calculated with both sets of parameters and have chosen cross-section values lying between the two, smoothly changing from one set to the other in going from 90 to 120 MeV. Since differences between the calculations carried out with these two pion potentials are relatively small ($\lesssim 20\%$), this procedure has little effect on the cross section. The proton optical-model parameters were taken from the global analysis of proton elastic scattering by Nadasen *et al.*[9]

The proton bound-state wave function was taken as the eigenfunction of a Woods-Saxon potential with an eigenvalue equal to the separation energy. The parameters of the Woods-Saxon well were obtained from the work of Elton and Swift[10] who fitted single-particle binding energies and elastic electron scattering for a series of nuclei.

The half-shell π^+-p cross section $[d\sigma/d\Omega]_{\pi^+-p}$ was approximated by an on-shell cross section. Two possible choices were considered. In the final-energy prescription (FEP) the two-body energy and scattering angle are evaluated in the rest system of the emitted π^+ and proton. The initial-energy prescription (IEP) uses a two-body energy consistent with the initial $\pi^+ + p$ relative momentum (the angle is the same in both prescriptions). The corresponding $\pi^+ + p$ cross sections were calculated using the phase-shift analysis of Rowe, Salomon, and Landau.[11]

In Figs. 1 and 2 we present the energy-sharing distributions for specific regions of excitation energy corresponding to p-state and s-state nucleon knockout. The data are those of Ziock *et al.*[4] The missing-mass spectra of Ref. 4 show that the excitation region $E_{ex} < 9.75$ MeV is predominantly due to the ground state with only about a 20% contribution from the 2.1- and 5.0-MeV states of ^{11}B. Since these states also correspond to p-shell proton knockout, their effect on the analysis is small. The region of excitation energy $E_{ex} > 9.75$ should be dominated by $1s_{1/2}$ knockout, the centroid of which is known to lie at $E_{ex} \simeq 22$ MeV with a width of about 20 MeV.

The curves shown in Figs. 1 and 2 are DWIA calculations normalized to the experimental data for the two choices of the $\pi^+ + p$ two-body cross section discussed. The normalization factor is the spectroscopic factor S of Eq. (1) and is listed in Table I. Also shown are normalized plane-wave calculations using the final-energy prescription. For the plane-wave calculations the normalization factor, or spectroscopic factor if this model were valid, is indicated in the figures.

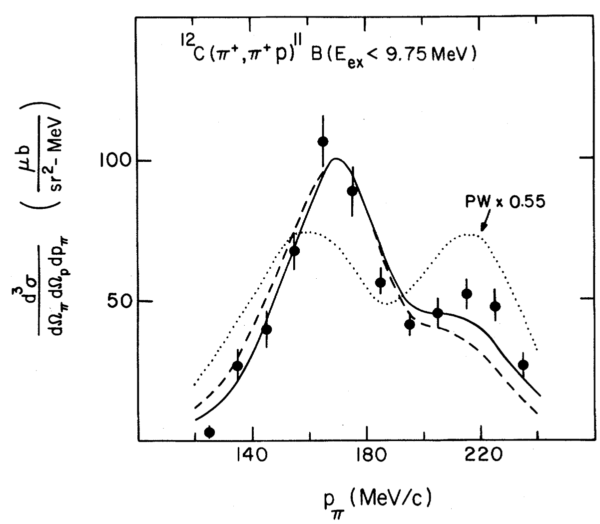

FIG. 1. Energy-sharing cross sections for $p_{3/2}$ knockout at an incident energy of $T_\pi = 199$ MeV and angles of $\theta_\pi = -117.5°$ and $\theta_p = 30.0°$. The data are from Ref. 4. The dotted curve is a PWIA calculation using the final-energy prescription (FEP) for $[d\sigma/d\Omega]_{\pi^+-p}$ normalized as shown. The full (dashed) curve is a DWIA calculation using the FEP (IEP) normalized by the spectroscopic factors listed in Table I.

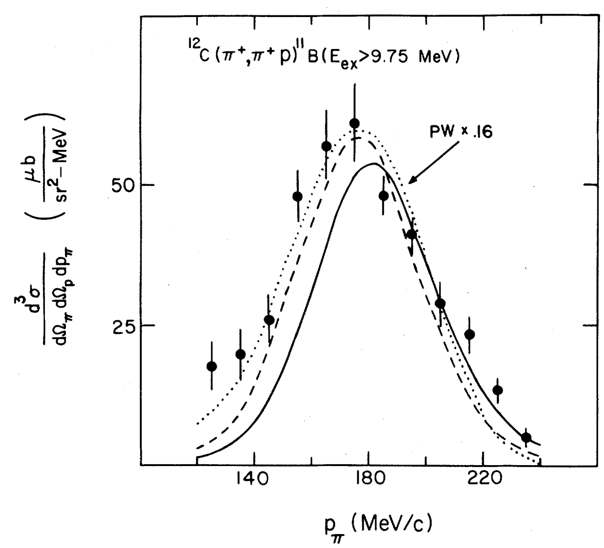

FIG. 2. Energy-sharing cross sections for $1s_{1/2}$ knockout at the same energies and angles as in Fig. 1. The curves are as indicated in the caption of Fig. 1.

TABLE I. Proton single-particle spectroscopic factors.

Levels of ^{11}B Energy	j^{π}	^{12}C(π^+,π^+p)^{11}B FEP	IEP	C^2S ^{12}C(e,ep)^{11}B [a]	Theory
0.0	$3/2^-$				2.85 [b]
2.1	$1/2^-$	2.9	3.6	2.5	0.75 [b]
5.0	$3/2^-$				0.38 [b]
18.0	$1/2^+$	1.8	3.2	1.0	2.0 [c]

[a] Ref. 13.
[b] Ref. 12.
[c] Shell-model limit.

The DWIA calculations reproduce the shape of the data quite well. Slightly better agreement is obtained with the IEP in the case of $1s_{1/2}$ knockout, whereas the $1p_{3/2}$ knockout data are slightly better described by the FEP calculations. However, in either case the fits are quite good considering current uncertainties in pion optical potentials and the range of excitation energy summed over in the experiment. The spectroscopic factors extracted from the normalization are listed in Table I. These are in very good agreement with the shell-model predictions of Cohen and Kurath,[12] as well as with measurements of the (e, ep) reaction.[13]

Several additional observations can be made with respect to the calculations presented in Figs. 1 and 2. Firstly, distortion effects are significant, reducing the plane-wave calculation by about a factor of 5 for the ground-state transition as well as skewing the energy-sharing distribution. The reduction for the $1s_{1/2}$ transition is even greater. Secondly, the PWIA calculation presented in Ref. 4 appears to be in error by about an order of magnitude. Lastly, at this bombarding energy and angle pair the difference between the initial- and final-energy prescriptions is primarily a change in overall magnitude. In contrast, calculations presented in Ref. 2 for a bombarding energy of 160 MeV show a pronounced difference between IEP and FEP calculations of the relative peak heights of the p-knockout energy-sharing distribution due to the change in placing the contribution of the (3, 3) resonance in the distribution. For the kinematics of the present analysis the resonance contributes at fairly low emitted pion energies where the cross section is small.

Overall, the agreement between theory and experiment is excellent, providing the first evidence of the basic applicability to reactions $A(\pi^+, \pi^+p)B$ of the DWIA description of quasifree knockout reactions. More detailed comparisons in the future require improvements in both experiment and theory. One needs better systematic pion potentials and theoretical guidance on the importance of off-energy-shell effects. Probably more importantly one needs more extensive data which allow one to test the off-energy-shell treatment, and the factorization approximation itself. However, the present comparison strongly supports the use of the DWIA, or some variation thereof, in the analysis of these knockout reactions.

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