

# On the possible role of short-range shock formation and coalescence in jet aeroacoustic source characterization

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## ABSTRACT

The number of jet and rocket noise studies has increased in recent years as researchers have sought to better understand aeroacoustic source and radiation characteristics using predominantly linear reconstruction techniques. While jet and rocket noise is often finite-amplitude in nature, little is known about the existence of shock formation and coalescence close to the source. A numerical experiment determines that significant shock coalescence can occur when finite-amplitude noise is propagated over short distances at amplitudes similar to those expected of jet and rocket noise. Additionally, the errors associated with using linear reconstruction techniques are shown to be large when significant shock coalescence occurs. The results of this experiment point out the need for additional studies targeting shock coalescence and its possible role in near field jet and rocket noise propagation.

## 1. INTRODUCTION

Various methods for characterizing aeroacoustic sources have been developed and many of them are used extensively to study aeroacoustic radiation. Laufer *et al.*<sup>1</sup> pioneered the first in-depth study of jet noise source localization using directional microphones based on a spherical reflector to locate two intense noise generating regions in supersonic jets. Beamforming methods, which utilize microphone arrays to steer a beam over a specified area, have also been developed and applied to imaging jet noise. Near-field acoustical holography techniques, originally developed for imaging structural radiators, have also been effectively adapted for use in aeroacoustic source reconstruction.<sup>2</sup> Additional methods have also been studied with varying success.

Each method employs or assumes some form of linear propagator, typically in the geometric or acoustic near field. This inherently implies that there is no nonlinearity; i.e., no significant interaction between frequency components of the noise near the jet. Although this assumption is likely valid for subsonic jets, most launch vehicles and military jet aircraft have been shown to exhibit nonlinear propagation effects over long-range propagation.<sup>3-5</sup> In the cases where nonlinear interactions do occur, the medium can no longer be considered as source free, since nonlinear propagation can be described in terms of “virtual sources.” Since higher-order wave phenomena can significantly affect the noise spectrum and therefore the accuracy of a source

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characterization, knowledge of the amount of nonlinearity present in the near field can be extremely useful. Particularly important can be the existence and interactions of acoustic shocks.

Pestorius<sup>6</sup> first captured the idea of shocks coalescing in shock tube experiments and simulations, while Lighthill<sup>7</sup> suggested the idea, described in terms of “bunchings” and “unions,” for conical shock waves using analytical and computational methods. However, no experiment to date has concretely determined the role of shock coalescence in high-amplitude jet or rocket noise propagation, in either the far or near fields. The questions of interest are then a) What role does nonlinear propagation play in the near field of high-amplitude aeroacoustic sources? and b) Can linear source characterization or imaging techniques adequately describe the behavior of such sources?

In this paper, a numerical experiment is presented in which noise waveforms at amplitudes similar to those expected in military aircraft and rockets are propagated over short distances to determine whether significant shock formation and coalescence can realistically occur close to a finite-amplitude jet or rocket noise source. Once this is established, a simple linear reconstruction is performed to show whether or not a source can be treated as linear in the near field by determining the reconstruction errors. It is shown that shock coalescence can occur within short propagation distances. When it does occur, treating a source as linear in the near field may lead to significant reconstruction errors.

## 2. METHODS

To create a simple reconstruction scenario for this investigation, a Gaussian-distributed point source in space was used to numerically propagate a finite-amplitude random-noise waveform. Calculated autospectra at different propagation distances were compared to the source autospectrum by removing the inverse-square law effect due to spherical spreading. For accurate waveform propagation that includes nonlinear effects, an algorithm using a weighted essentially non-oscillatory (WENO) scheme<sup>8</sup> in space and a third-order Runge-Kutta scheme in time was used to solve an extended Navier-Stokes equation set,<sup>9</sup> which was composed of the momentum equation, the continuity equation, the entropy-balance equation and two relaxation equations for diatomic nitrogen and oxygen. The one-dimensional form of the equation set was used, which includes spherical spreading.

Input waveforms were created using an array of 216 Gaussian-distributed random numbers, which was filtered to have a power-law dependent spectra (commonly referred to as a “haystack” spectrum), which is characteristic of jet mixing noise. This spectral shape has been shown experimentally to exist for supersonic jet<sup>3,10</sup> and rocket noise<sup>4</sup> measurements and therefore represents a plausible high-speed jet/rocket noise spectrum. Interestingly enough, it is noted that waveform steepening is also known to cause white broadband spectra to take on this same spectral shape.

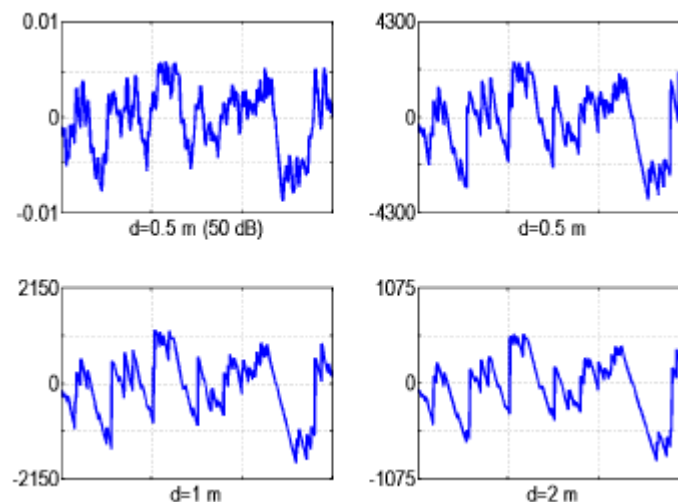
Although the effect of varying both frequency and amplitude were investigated,<sup>11</sup> only the results using two center frequencies are presented here. The first center frequency of 100 Hz was chosen to be in the upper range of rockets and the lower range of military jet aircraft while the second frequency of 500 Hz was chosen to be in the middle-to-high range of military jet aircraft. These two waveforms will be referred to throughout the remainder of this paper as waveform 1 and waveform 2, respectively. Both time series were scaled to have an OASPL of 166 dB re 20 $\mu$ Pa, which is equivalent to an RMS pressure of approximately 4 kPa. These time series loosely represent waveforms produced by high-speed jets. The source was located 0.3 meters away from the origin.

The WENO scheme allows for numerically stable propagation of discontinuities (i.e. acoustic shocks), but necessarily reduces the bandwidth of the input waveform due to its finite number of

points per length. For this investigation, the maximum usable frequency was set at 2 kHz, which is above the primary frequencies of interest while allowing sufficient frequency resolution and propagation distance in a reasonable amount of computation time. Because of the compact nature of the source, the linear reconstruction was performed by removing the  $1/r$  magnitude losses due to spherical spreading, as previously mentioned. This simple reconstruction scheme serves as a method to study the physics of the propagation, while conservatively estimating its effects on spectral characteristics and reconstruction. For this research, the reconstruction was performed on the waveforms after propagating to 0.15, 1 and 2 m.

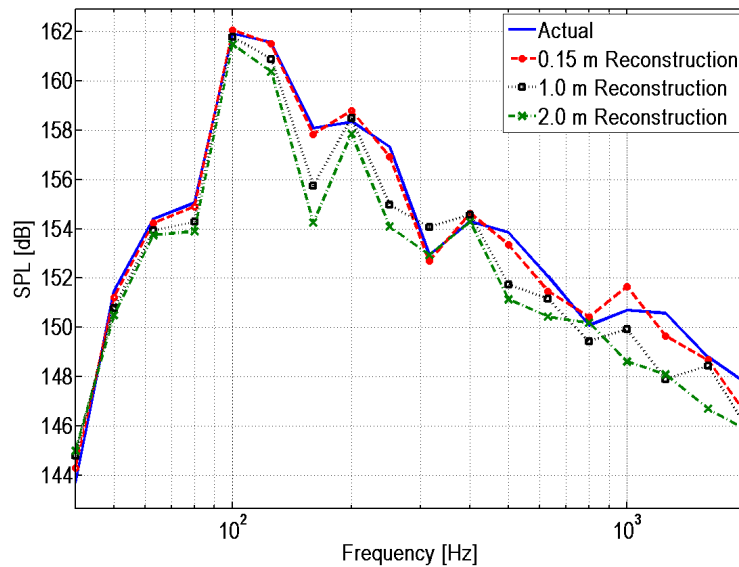
### 3. RESULTS

Figure 1 shows a small portion of waveform 1 after propagating 0.5, 1.0, and 2.0 m. Also displayed in Fig. 1 is that same portion of waveform 1 propagated at a low amplitude (OASPL=50 dB) to show the waveform's shape after 0.5 m of essentially linear propagation. Nonlinear propagation effects are clearly visible as portions of the waveform have become steepened. There also is some evidence of shock coalescence as the number of zero crossings has decreased. In the small portion shown in Fig. 1, the number of zero-crossings has dropped from 33 at 0.5 m to 21 at 2.0 m. For the entire waveform, the number of zero-crossings experienced a 32% decrease after propagating 2.0 meters.



**Figure 1:** Snapshots of waveform 1 propagated at several distances (with axes scaled according to spherical spreading for ease of comparison). A reference low-amplitude 50 dB waveform is also shown (top left). Nonlinear propagation has caused shocks to form and some shock coalescence to occur.

After waveform 1 was numerically propagated to 0.15, 1.0, and 2.0 m, its autospectrum was calculated and the magnitude reconstruction back to the source location was applied. The original and reconstructed autospectra are compared on a one-third octave scale in Fig. 2. The reconstruction from 0.15 m matches the source spectra very well, while the reconstruction from 1.0 and 2.0 m begins to deviate slightly from the exact spectrum with frequency-averaged error magnitudes of approximately 0.8 dB and 1.2 dB, respectively. The deviations come from changes in the time waveform due to waveform steepening, though the slopes of the high frequency decays for all cases are very similar. Additionally, the center frequency did not experience any shift, indicating that the coalescence was not significant enough to appreciably alter the spectra over this propagation range.

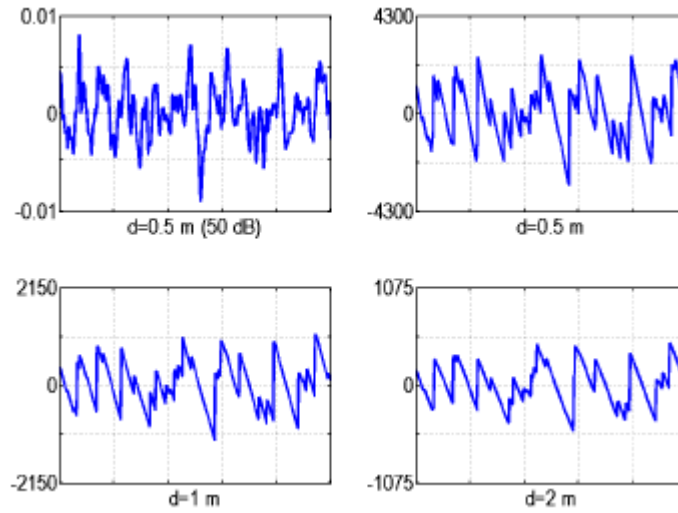


**Figure 2:** The reconstructed one-third octave spectra of waveform 1 initially at 166 dB OASPL after propagating 0.15, 1.0 and 2.0 m along with the actual spectrum at the source. The reconstruction errors for all three distances are less than 1.5 dB since the spectral shape is generally maintained.

Waveform 2 was also propagated a distance of 2.0 m at an initial OASPL of 166 dB. Snapshots of a small portion of waveform 2 are shown in Fig. 3 at the distances of 0.5, 1.0 and 2.0 m. A visual comparison reveals that the nonlinear distortion is significantly greater for waveform 2 (Fig. 3) than waveform 1 (Fig. 1) over the same distances. This is to be expected given that the shock formation distance decreases as the peak frequency of the spectrum increases. Note that the nonlinear propagation is significant enough that the shocks appear to dominate waveform 2 after only propagating 1.0 m. The number of zero-crossings has also decreased much more than waveform 1, falling from 52 to 23 after 2.0 m of propagation. The entire waveform underwent a 45% decrease in zero crossings, whereas the decrease for waveform 1 was only 32%.

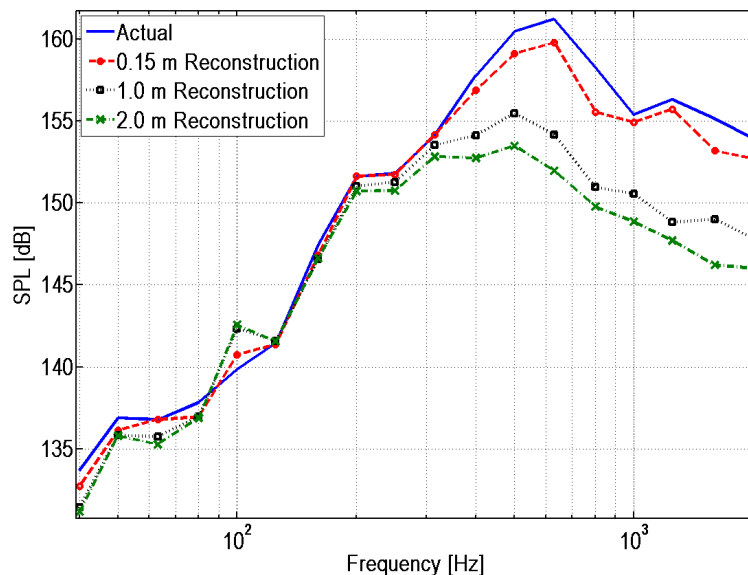
Figure 4 shows the reconstructed one-third octave spectra for waveform 2 from distances of 0.15, 1.0 and 2.0 m. The reconstruction is accurate only below the center frequency while the reconstruction at and above the center frequency has errors approximately -7.0 dB and -8.0 dB for the 1.0 and 2.0 m cases respectively. Shock coalescence has resulted in a downward shift in the center frequency causing high reconstruction errors for the latter two cases. This result demonstrates that shock coalescence can cause very significant errors in linear reconstruction methods over a propagation distance of only 1 m.

In summary, for the same spectral shapes and OASPL but different center frequencies, the errors associated with linear reconstruction can be drastically different. The relatively low reconstruction errors for waveform 1 indicate that although the time waveform is changing, the spectral shape can generally be maintained despite the nonlinear effects. This occurs since the asymptotic dependence of low frequencies is proportional to  $f^2$ ,<sup>12</sup> while the high frequency roll off resembles that of a sawtooth wave spectrum, which decay according to the inverse of the harmonic number (i.e.  $1/f^2$ ). This implies that magnitude reconstructions may be “blind” to nonlinear propagation of haystack spectra in the absence of significant diffraction effects when



**Figure 3:** Snapshots of waveform 2 propagated to several distances (with axes scaled according to spherical spreading for ease of comparison). A reference low-amplitude waveform is also shown (top left). Nonlinear effects have caused shocks to form and shock coalescence to occur.

significant shock coalescence has not occurred. However, the results from waveform 2 indicate that when shock coalescence is more significant, the center frequency will shift downward and cause the high and low frequencies to be offset from the original spectrum, even though the slopes are similar. For noise waveforms with spectral shapes other than the “haystack” spectrum used in this study, reconstruction errors would likely be greater as shocks form and the spectral slopes change.



**Figure 4:** The reconstructed one-third octave spectra of waveform 2 initially at 166 dB OASPL after propagating 0.15, 1.0 and 2.0 m along with the actual spectrum at the source. The reconstructed spectra from 1.0 and 2.0 m have significant errors above the center frequency due to shock coalescence.

## 4. CONCLUSION

At amplitudes and center frequencies similar to those of military jet aircraft and rockets, this numerical experiment shows that nonlinear propagation effects can be significant enough to induce nonlinear propagation and shock formation in less than two meters of propagation. Additionally, it is feasible for significant shock coalescence to occur. Although these results include the assumptions of spherical spreading and a “haystack” spectral shape, the outcome of the experiment reveals the importance of considering the finite-amplitude nature of jet or rocket noise sources in the near-field of the source. Furthermore, as nonlinear effects vary with amplitude, distance and frequency, every source characterization technique performed on high-amplitude noise sources must carefully consider the relative importance of waveform steepening and shock formation before determining the accuracy of the results.

As the assumptions made in the study do not represent actual nonlinear propagation from an extended, partially correlated source (i.e., a jet), several cases can be made for and against the existence of shock coalescence close to actual jet and rocket noise sources. First, the amplitudes used in this study are realistic amplitudes close to military aircraft and rockets. Second, the extended nature of the jet source will result in less than spherical decay near the plume, which would cause nonlinearity to happen relatively more quickly. In addition, high-speed jet and rocket noise exhibits significant positive skewness, resulting in large compressive outliers that will form shocks more quickly than the Gaussian distribution presented here. These ideas suggest the existence of more nonlinearity than shown in this study and hence more shock coalescence.

Other reasoning suggests that shock coalescence may not be significant near the jet source. Center frequencies can be much lower than 500 or even 100 Hz, especially for rocket noise. This would result in larger propagation distances required for shock formation. Additionally, partial source coherence, diffraction, and the turbulent nature of the source could prevent the waveform from developing shocks strong enough to rapidly coalesce.

Since the exact role of shock coalescence in jet and rocket noise propagation is currently unknown, the results of this study suggest the need for further investigations that specifically target the existence and role of shock coalescence in the near field of finite-amplitude noise sources. Studies of the amount of shock coalescence required to appreciably alter a spectrum may help determine thresholds for a significant shock coalescence level or distance, or other useful metrics. Additionally, knowledge of the specific effects of shock coalescence and the scalability of shock formation and coalescence may help refine efforts in jet noise reduction and aid other advancements in vibroacoustic modeling of aircraft and launch vehicle structural fatigue prediction.

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