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Investigation of a Single-Point Nonlinearity Indicator in the Propagation of High-Amplitude Jet Noise

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There is evidence to suggest that nonlinearity is important in the propagation of high-amplitude jet noise [Gee et al., AIAA J. 43(6), 1398-1401 (2005)]. Typically, the power spectral density (PSD) is used to assess the impact of jet noise on the surrounding environment, but such an assessment requires multiple measurement locations to observe the nonlinear evolution of the PSD. The difference in the PSDs measured at different locations depends on a combination of source level, nozzle diameter, and propagation distance. As a result, full scale measurements have to be extended over large distances, and model scale measurements require high measurement bandwidths. These constraints complicate the measurement and make it difficult to observe nonlinear effects using the PSD. Here a different technique for determining the importance of nonlinearity is investigated. The imaginary part of the cross-spectral density of the pressure and the square of the pressure, also called the quadspectral density (QSD), is related to the rate of nonlinear change of the PSD. Thus, the extent to which the PSD is evolving nonlinearly can be determined at a single measurement location. In the absence of absorption, energy is conserved, and the integration of the product of the QSD with frequency over all frequencies must be zero; when nonlinearity is present, the value of the QSD can be nonzero for many frequencies. Because nonlinearity tends to transfer energy from low frequencies to high frequencies and the QSD is positive at frequency components that are losing energy, using the integral of the QSD over its positive values as a nonlinearity indicator eliminates the need for high bandwidth measurements. Experimental measurements were taken in a plane wave tube with a working length of 9.55 m in which boundary layer losses dominate over atmospheric absorption. Experimental and numerical results show that the ratio of the integral of the QSD over the frequencies for which it is positive (Q_{pos}) to the integral over the frequencies for which it is negative (Q_{neg}) is close to one. Also, because the QSD is third-order in pressure, normalizing its integral by the cube of the rms pressure yields a quantity that is easily compared across experimental conditions. Results for both periodic and broadband signals are presented and the practicality of using the QSD as a single-point indicator of nonlinearity addressed.

Nomenclature

c_0	=	equilibrium sound speed
d	=	experimental jet nozzle diameter
d_0	=	full-scale jet nozzle diameter
p	=	acoustic pressure
Q_{p^2p}	=	imaginary part of the cross-spectral density of the square of the pressure and the pressure (also QSD)
QSD_{neg}	=	all values of the QSD which are negative
QSD_{pos}	=	all values of the QSD which are positive
Q_{neg}	=	summation over frequency of (ω*QSD_{neg})

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Q_{pos}	=	summation over frequency of (ω*QSD$_{pos}$)
r	=	radial propagation distance
S_p	=	power spectral density (also PSD)
x	=	linear propagation distance
α	=	generalized atmospheric and boundary-layer absorption coefficient
β	=	coefficient of nonlinearity
ρ_0	=	equilibrium density
ω	=	angular frequency
ω_0	=	angular frequency of the source

I. Introduction

IN the study of the propagation of jet noise, the power spectral density (PSD) is typically the quantity used to assess impact on the surrounding community. There is evidence[1] to suggest that a correct assessment requires knowledge of whether the propagation is linear or nonlinear. If the propagation is nonlinear, then the nonlinearity must be accurately accounted for in any prediction model.

It has been shown[1] that, under some conditions, a linear model does not accurately predict the propagation of jet noise, especially at higher frequencies. The importance of nonlinearity is usually determined by examining the evolution of the PSD with propagation distance. This method, which requires measurements at multiple locations, has inherent complications for both full-scale and model-scale jet measurements. In full-scale experiments, many effects influence the propagation of the noise, including wind and temperature gradients, ground impedance, and the spatial extent and directivity of the source. The complexity of this environment calls for measurements (acoustic and meteorological) at many locations so that these effects can be quantified and selectively removed during analysis.

In model-scale measurements, the scaling-up of frequency and the constraints of working in an anechoic chamber of finite dimension often make it difficult to detect nonlinear effects by examining the PSD. Nonlinear effects are cumulative, so maximizing the value of r/d of a measurement (where r is radial distance from the source and d is jet diameter) is important to their detection. Because model-scale measurements are usually conducted in an enclosed space, r is limited. Furthermore, decreasing d increases the frequency bandwidth needed for the measurements. Frequency bandwidth well above the peak frequency (one to two decades) is generally necessary to see significant differences between measurements and linear predictions and therefore to detect the presence of nonlinearity. Peak frequencies are usually around a few hundred Hertz in full-scale measurements; in model-scale measurements, they are usually on the order of a few kiloHertz[2]. Frequency is scaled as d_0/d (where d_0 is the full-scale jet diameter), meaning that a greater bandwidth is required for a smaller jet. While it is possible to record data at a sample rate of roughly 200 kHz (thereby obtaining a Nyquist frequency of 100 kHz), amplitude and phase calibrations are not readily available for most microphones at these high frequencies. A typical 1/8" microphone (Brüel & Kjær type 4138) has a flat amplitude response to about 140 kHz; a similar 1/4" microphone (Brüel & Kjær type 4938) is reliable only to 70 kHz. Phase responses are not specified for these microphones. This restriction, combined with the limitation on r posed by the measurement space, makes it very difficult to see nonlinearity in model-scale measurements of the PSD.

In light of these factors (meteorology, measurement space, and frequency bandwidth), it would be beneficial to be able to determine the presence or importance of nonlinearity with a measurement at a single location. This paper explores the use of a single-point nonlinearity indicator based on the work of Morfey and Howell.

II. Theory and Derivation

Morfey and Howell[3] derive an expression containing a quantity that has the potential to serve as a single-point nonlinearity indicator. They begin with the Burgers Equation, a second-order parabolic wave equation that includes the effects of both nonlinearity and absorption. After some manipulation, including transformation to the frequency domain and ensemble-averaging, they obtain an equation similar to

$$\frac{d}{dx}\left(e^{2\alpha x}S_p\right) = -\omega\frac{\beta}{\rho_o c_o^3}e^{2\alpha x}Q_{p^2p}, \tag{1}$$

where x is distance from the source, α a generalized absorption coefficient, S_p the PSD, β the coefficient of nonlinearity, and Q_{p^2p} the imaginary part of the cross spectral density (also known as the quadspectral density, or QSD) of the square of the pressure and the pressure. Equation (1) has been modified slightly from its original form to apply to plane waves. The left-hand side of Eq. (1) is the spatial derivative of the absorption-corrected PSD. The right-hand side can be viewed as a source term for changes in the PSD that are due to nonlinearity. Thus, the QSD could be a valuable tool in determining the presence or importance of nonlinearity in the propagation of a wave.

A normalization of this quantity, often referred to as “Q/S” or “the Morfey-Howell nonlinearity indicator”, has recently been used by several researchers[1,2,4] in the analysis of high-amplitude noise. However, its physical meaning is not well understood and has not yet been thoroughly investigated.

One of the signatures of nonlinearity in sound propagation is the transfer of energy from lower-frequency to higher-frequency spectral components. A propagating sinusoid with sufficient amplitude will generate spectral components at harmonics of the fundamental frequency; as the propagation distance increases, more of the higher harmonics become important. These harmonics gain energy at the expense of the fundamental. Thus, the left-hand side of Eq. (1) for an originally sinusoidal wave should be negative at the fundamental frequency and positive for the harmonic frequencies. Figure 1 shows this quantity for an initially sinusoidal wave at two different propagation distances. These data were taken in a 5.2-cm diameter plane wave tube with a fiberglass anechoic termination and driven by two JBL 2402H drivers. The microphones used were 1/4” Brüel & Kjær type 4938 mounted with the diaphragm approximately flush with the inner wall of the tube. The microphone locations were 0.10 m, 3.25 m, 6.40 m, and 9.55 m from the source, respectively. For a more complete description of the experimental apparatus, please see Falco *et al.*[5]

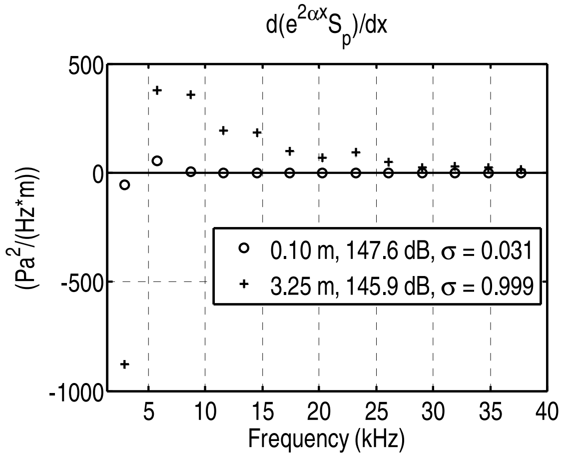

Figure 1. The left-hand side of Eq. (1) for measured waveforms at two different propagation distances.

Analysis is simplified by temporarily assuming lossless propagation of plane waves. This simplification aids in the understanding of the problem; losses can be added later. Rewriting Eq. (1) for a lossless case ($\alpha = 0$) gives

$$\frac{d}{dx}S_p = -\omega\frac{\beta}{\rho_o c_o^3}Q_{p^2p}\,. \tag{2}$$

Integrating both sides over frequency yields

$$\frac{d}{dx}\int_0^\infty S_p d\omega = -\frac{\beta}{\rho_o c_o^3}\int_0^\infty \omega Q_{p^2p} d\omega\,. \tag{3}$$

The integral on the left-hand side of Eq. (3) is the mean-square pressure of the signal, which is proportional to energy. In the absence of losses and shocks, the total energy of the wave should not change with propagation distance. It follows that

$$\int_0^\infty \omega Q_{p^2p} d\omega = 0\,. \tag{4}$$

If we define QSD_{pos} as all values of the QSD which are positive (frequencies at which energy is being lost) and QSD_{neg} as all values of the QSD which are negative (frequencies at which energy is being gained), conservation of energy and Eq. (4) dictate that

$$\int_0^{\infty} \omega QSD_{pos}(\omega) d\omega = -\int_0^{\infty} \omega QSD_{neg}(\omega) d\omega \,. \tag{5}$$

For the linear case, both of these integrals should equal zero. For the nonlinear case, both integrals should have nonzero values equal in magnitude. Once shocks have formed, however, Eq. (5) apparently no longer holds. Even in the absence of explicit atmospheric and boundary-layer losses, the wave will experience losses at the shocks which will decrease its total energy[6]. Thus, the magnitude of the left-hand side of Eq. (5) should be larger than that of the right-had side when shocks are present. As a result, the ratio of these integrals can be used as a simple test for the presence of shocks (and therefore of nonlinearity).

Given the restrictions on *r/d* mentioned above in the context of model-scale jet measurements, the ability to detect nonlinearity in the pre-shock region (at smaller values of *r/d*) and using a limited bandwidth would be particularly beneficial. Either integral from Eq. (5) could be used as a nonlinearity indicator in the pre-shock region, but since positive values of Q_p2_p tend to occur at lower frequencies, using the left-hand side of Eq. (5) decreases the need for a high-bandwidth measurement.

III. Analysis

Measurements were made using the plane wave tube described above with two JBL 2426H drivers as the sound source. The drivers were supplied with a sinusoidal signal ranging from 500 Hz to 3 kHz in frequency, and the resulting sound waves had amplitudes from 105 dB to 145 dB re 20μPa at the microphone closest to the source. Waveforms were captured at all four microphone locations. Numerical predictions for the second, third, and fourth microphone locations were generated by using the measured waveform at the first microphone as the input to an Anderson-type propagation algorithm[7]. Analytical predictions were made using the experimental conditions as inputs to Blackstock's solution[8] for the harmonic amplitudes of an initially sinusoidal wave in the absence of absorption. All predictions were generated with the same sample rate as the measurements. PSDs and QSDs were calculated by breaking the time signal into shorter records with a 50% overlap, applying a Hanning window to each record, taking a Fourier transform, and averaging.

A. Single-Frequency Source Data

The ratio Q_{neg}/Q_{pos}, where Q_{neg} is the integral on the right-hand side of Eq. (5) and Q_{pos} is the integral on the left-hand side, was plotted as a function of σ, or propagation distance normalized by shock formation distance, given by

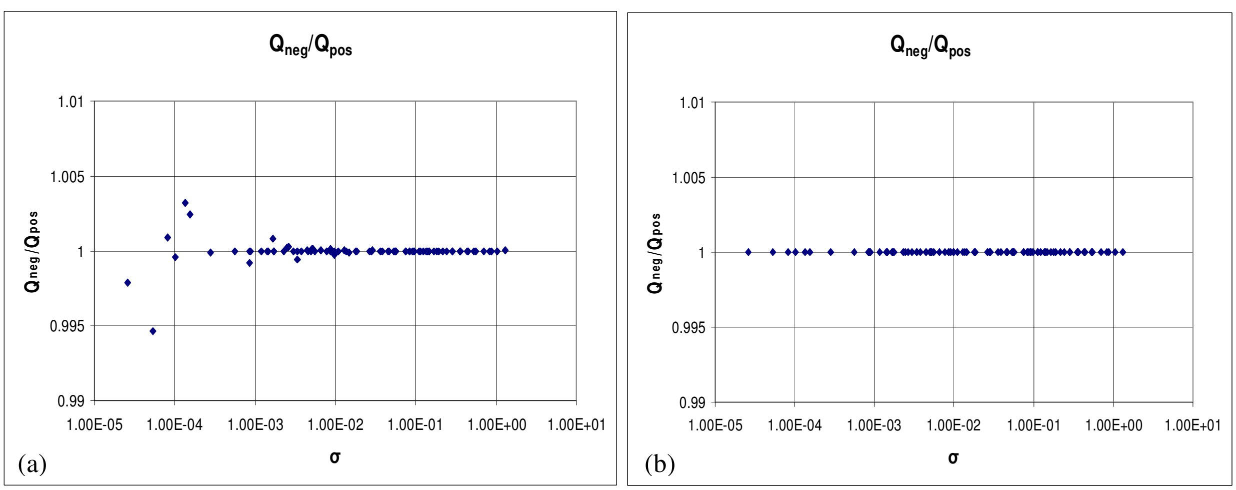

Figure 2. Ratio of the integrals in Eq. (5) as a function of normalized propagation distance for measured data (a) and analytical data (b).

$$\sigma = \frac{x\beta\omega_0 p}{\rho_0 c_0^3} \,. \tag{6}$$

Figure 2 shows these plots for the measured waveforms and for the analytically generated waveforms.

As expected according to Eq. (5), the value of the ratio is close to one at all propagation distances. Many of the data points at smaller propagation distances represent low-amplitude source conditions for which the QSD is small. In these cases the signal-to-noise ratio is also small, resulting in the deviations from unity at these points.

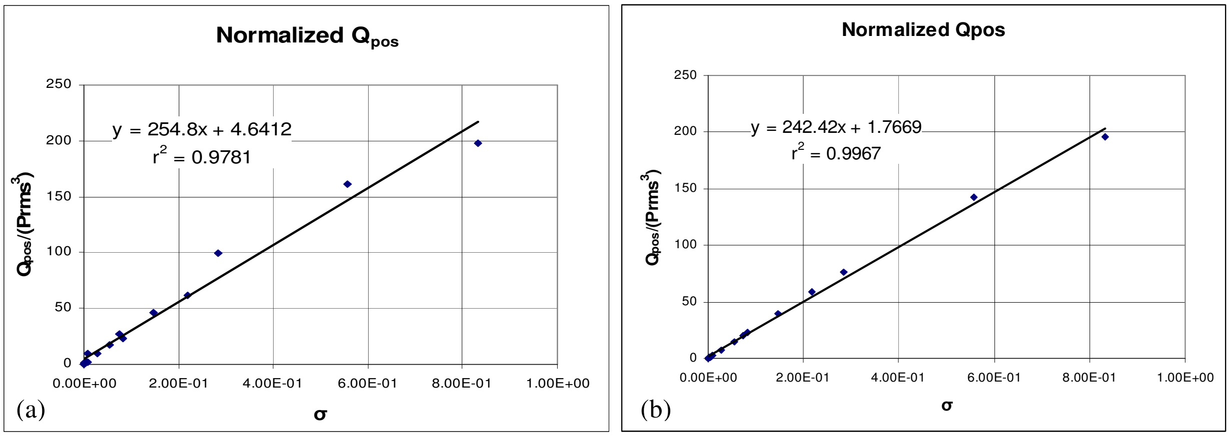

Figure 3. A normalization of Q_{pos} as a function of normalized distance for measured data (a) and analytical data (b) for a 1500 Hz source.

Each value of Q_{pos} was normalized by the cube of the rms pressure of the signal and plotted as a function of normalized distance, and a line was fitted to each plot. Results are shown in Fig. 3 for the measured and analytical data; the numerically propagated data yields results so similar to those for the measurements that it is not shown here. The data in Fig. 3 were generated using a 1500 Hz source and amplitudes ranging from 105 dB to 145 dB re 20µPa. The equations for the two lines are similar and $r^2 > 0.97$ for both cases. This result suggests that, for an arbitrary single-frequency source signal, knowledge of the QSD could be used to obtain the shock formation distance of the signal and therefore information about the source.

B. Broadband Noise Data

Broadband noise data were generated by using source conditions similar to those of the measurements as inputs to the Anderson-type algorithm. The frequency content of the source was 500 Hz to 3000 Hz, the amplitude was between 105 dB and 145 dB re 20µPa, and the signal was saved at propagation distances corresponding to the microphone locations in the plane wave tube.

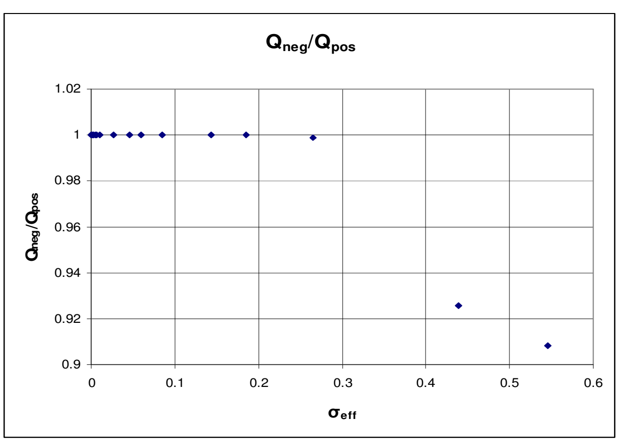

Figure 4. The ratio Q_{neg}/Q_{pos} for numerically-generated broadband noise data.

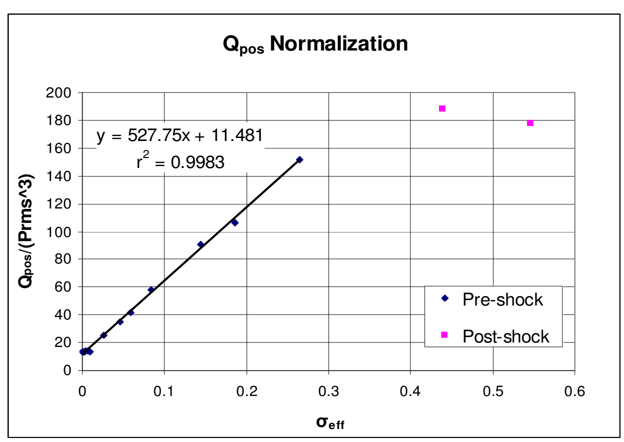

Figure 5. A normalization of Q_{pos} vs. normalized distance for numerically-generated broadband noise.

The same analysis was applied to the noise data as to the single-frequency source data. An effective shock formation distance was calculated using the center frequency and the rms pressure of the source[9]. Because noise tends to steepen and shock at smaller values of σ than single-frequency sources[10], shock formation can happen for $\sigma_{eff} < 1$.

Figure 4 contains a plot of the ratio Q_{neg}/Q_{pos} for the broadband noise data. The ratio is nearly equal to 1 for all but the largest normalized propagation distances. Based on the analysis above, it is likely that shocks have formed in these signals and that energy is being lost at the shocks rather than being transferred to other frequency components.

The normalization of Q_{pos} mentioned above is shown in Fig. 5. A line has been fitted to all but the last two data points because Fig. 4 indicates that shocks are likely present in these signals. The correlation coefficient for this line is very close to one, and the equation of the line is similar to the equations obtained for the single-frequency source data. Its slope is larger, but this is to be expected as shock formation seems to occur at smaller values of σ_{eff} for the noise data. This figure suggests that the preceding analysis is equally valid for single-frequency source signals and broadband noise signals (and therefore potentially for jet noise), given that an appropriate normalized propagation distance is used for the noise signals.

IV. Conclusion

The QSD has been shown to provide information about nonlinearities in the propagation of single-frequency source signals and broadband noise signals. The value of the ratio Q_{neg}/Q_{pos} indicates whether shocks are present in a signal. A normalization of Q_{pos} could be used to obtain the shock formation distance and source conditions of a signal of unknown origin. Because these methods of analysis are valid for broadband noise, they could be used in the evaluation of jet noise provided a suitable normalized propagation distance is employed.

Acknowledgments

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