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# On the Perception of Crackle in Noise Radiated from Military Jet Aircraft

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**Crackle is a phenomenon sometimes found in supersonic jet noise. It has been commonly quantified by the skewness of the time waveform. In this investigation, a simulated waveform with a virtually identical probability density function and power spectrum as an actual F/A-18E afterburner recording has been created by nonlinearly transforming a statistically Gaussian waveform. Although the afterburner waveform crackles noticeably, playback of the non-Gaussian simulated waveform yields no perception of crackle at all, despite its relatively high skewness. Closer examination of the two waveforms reveals that although they have virtually identical statistics, there are considerable differences in their time rates of change in the intense compressive portions of the waveforms. The afterburner waveform is much more shock-like with its more rapid variations in pressure that the non-Gaussian simulated waveform. This results in a significant difference in the probability distributions of the time derivatives of the actual and simulated data and suggests that the perception of crackle in jet noise waveforms may be better quantified with statistics of the time derivative of the waveform, rather than by the skewness of the time waveform itself.**



## Nomenclature

| | | |
|---|---|---|
| $g(x)$ | = | nonlinear transformation function |
| $M$ | = | mean of the time series, $M = \langle x \rangle$ |
| $K$ | = | kurtosis, $K = \left\langle (x-M)^4 \right\rangle / \sigma^4$ |
| $S$ | = | skewness, $S = \left\langle (x-M)^3 \right\rangle / \sigma^3$ |
| $V$ | = | variance, $V = \left\langle (x-M)^2 \right\rangle$ |
| $E$ | = | quadratic cost function to be minimized |
| $x(t)$ | = | Gaussian time series |
| $y(t)$ | = | non-Gaussian time series generated by $y = g(x)$ |
| $\sigma$ | = | standard deviation of the time series |
| $\langle\ \rangle$ | = | expectation operator |

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## I. Introduction

PREVIOUS studies[1,2] of the phenomenon known as "crackle" in supersonic, heated jets have shown that the skewness of the time series is an important parameter in determining when a jet is or is not crackling. The skewness, *S*, which is a normalized form of the third central moment of the probability density function (PDF) of the time waveform, is a measure of the asymmetry of the PDF. A recent study[3] of the propagation of noise from a tied-down military jet aircraft has shown that the F/A-18E time series at 18 m (60 ft) and at afterburner (AB) has a skewness value of $S = 0.60$ at the peak emission angle. According to the threshold suggested by Ffowcs Williams *et al.*[1] that a waveform with $S > 0.3$ crackles, crackle should be readily noticeable in the AB time series. Playback of the AB recording confirms this hypothesis.

A similar perception of crackle has been noted by the authors, however, in the waveforms of certain propagation measurements at fairly large distances where *S* is no longer significant (e.g., $S < 0.15$) and therefore should be crackle-free according to Ffowcs Williams *et al.*[1] It has further been noted that this crackle-like quality of the noise at large distances appears to be linked to cases where nonlinearity plays a significant role in the propagation[4]. These observations have prompted this study of the cause of the *perception* of an impulsive or crackle-like quality in certain jet noise waveforms. The focus of this investigation is not on the physical mechanisms responsible for the generation of a skewed waveform near the source. Rather, the focus is whether significant skewness by itself is sufficient to cause a listener to perceive crackle in a waveform. In the remainder of this paper, a method by which non-Gaussian waveforms may be generated is first described. The discussion on non-Gaussian simulation theory is followed by results of the creation of skewed waveforms and relevant analysis.

## II. Non-Gaussian Waveform Simulation Theory

In order to study the effect of skewness on the perception of crackle, methods by which non-Gaussian (skewed) waveforms may be generated from Gaussian waveforms have been investigated. It is desired that these waveforms have, in particular, the same mean (*M*), variance (*V*), skewness (*S*), and kurtosis (*K*) as the AB F/A-18E waveform, in addition to having the same spectrum. Specification of these four moments, which are defined in the nomenclature, makes possible the generation of a simulated waveform with a virtually identical PDF as the measured waveform. The underlying theory of non-Gaussian waveform generation and its implementation are now discussed.

A zero-memory nonlinear system may be defined as a system that acts on the present input in a nonlinear fashion[5]. Correspondingly, a non-Gaussian waveform, $y(t)$, may be generated by passing a statistically Gaussian signal, $x(t)$, through a zero-memory nonlinear transformation function (NTF), denoted as $y = g(x)$. In order to be a zero-memory transformation function, $g(x)$ must be time invariant. As an example, a quadratic zero-memory NTF, similar to that used to model hardening-softening nonlinearities[5], is written as

$$y = g(x) = a + bx + cx^2 . \tag{1}$$

The choice of coefficients *a*, *b*, and *c* result in a non-Gaussian signal, $y(t)$, whose statistics vary according to the values of the coefficients. The output waveform, $y(t)$, will have a PDF that is given by

$$PDF(y) = \frac{PDF(x)}{dg/dx} . \tag{2}$$

For the special case where $c = 0$, $y(t)$ has Gaussian statistics because the transformation function is no longer nonlinear and a linear transformation of a Gaussian signal remains Gaussian.

Equation (1) was chosen *ad hoc* to be the NTF for these jet noise waveform simulations because it was noted that the PDFs for high-amplitude jet noise waveforms in Refs. 1 and 3 are similar to that of a hardening-softening nonlinearity. A wide range of alternative NTFs has also been studied, including a cubic polynomial of the form

$$y = a + bx + cx^2 + dx^3 \tag{3}$$

and a generalized sigmoidal function, expressed as

$$y = a + \frac{b}{1 + e^{-c(x-d)}} \ . \tag{4}$$

The functions found in Eqs. (3) and (4) may initially appear to be more plausible candidates than the quadratic NTF in Eq. (1) because specification of *M*, *V*, *S*, and *K* to find the three coefficients in Eq. (1) represents an overdetermined system with four constraints and only three unknowns. However, it was found that the cubic and sigmoidal NTFs generally did not yield improved results and the outputs were extremely sensitive to initial parameter choices in the iterative scheme used to determine function coefficients. The remainder of this section is dedicated to a description of the coefficient selection process.

Equation (1) appears to closely approximate the statistical behavior of the measured AB jet noise for appropriate choices of coefficients *a*, *b*, and *c*. Before iteratively selecting the coefficients, a Gaussian white noise waveform is first created and filtered so as to possess the same power spectrum as the actual AB data. This input waveform, $x(t)$, is then transformed according to Eq. (1) when the desired coefficients are found. The coefficients are derived via iterative minimization of a quadratic error or cost function based on the desired moments of the probability density function of the simulated data, *PDF*(*y*), which is calculated according to Eq. (2). For example, given a zero-mean process ($M = 0$) and specified desired values for the variance (*V*), skewness (*S*), and kurtosis (*K*), the error function, *E*, may be written as

$$E_i = M_i^2 + (V_i - V)^2 + (S_i - S)^2 + (K_i - K)^2 , \tag{5}$$

where the subscript *i* represents the values of the moments of *PDF*(*y*) for the *i*th iteration. The MATLAB function "fminsearch"[6] is used to find the values of *a*, *b*, and *c* of the NTF that minimizes *E*. The results of the simulation process are now shown and discussed.

## III. Simulation Results and Playback

### A. Results

The preceding theory has been used to create a simulated version of the F/A-18E AB waveform. After the nonlinear transformation, minor differences in *M* and *V* for $y(t)$ are removed by subtracting off the mean and scaling the waveform to match the overall sound pressure level of the measured F/A-18E waveform (150 dB re 20 $\mu$Pa). The NTF used to generate the simulated non-Gaussian waveform is shown in Fig. 1, where *x* and *y* are normalized by their standard deviations. The parabolic shape in Fig. 1 is obtained using $a$ = -0.0959, $b$ = 0.9911, and $c$ = 0.0960 in Eq. (1). The dashed lines are shown as an example of how the transformation from a Gaussian to a non-Gaussian waveform occurs. In the example shown, an input $x$ = 2.5 yields the output $y$ = 3.0. The values of *S* and *K* for $x(t)$, $y(t)$, and the AB data are shown in Table 1. As a result of the nonlinear transformation process, the non-Gaussian simulation waveform has skewness and kurtosis values that are very close to the actual AB data.

**Table 1. Skewness (*S*) and kurtosis (*K*) values for the Gaussian, non-Gaussian simulation, and AB waveforms. The Gaussian waveform is very close to that of ideal Gaussian behavior, for which *S* = 0 and *K* = 3.**

| Waveform | *S* | *K* |
|---|---|---|
| Gaussian | 0.02 | 3.01 |
| Simulation | 0.57 | 3.39 |
| AB Data | 0.60 | 3.40 |

In Fig. 2, the initial Gaussian, simulated non-Gaussian, and actual AB waveforms are displayed. Passage of the Gaussian waveform through the NTF yields a simulated waveform that has an appearance that is very similar to the AB data in terms of observable skewness. The Gaussian waveform is symmetric about zero, whereas the non-Gaussian simulation possesses asymmetry similar to the AB waveform. The power spectral densities for these waveforms are displayed in Fig. 3. The power spectral density for the Gaussian noise waveform is essentially equal to that of the measured AB data because of the initial filtering of the white noise signal. The spectral density of the non-Gaussian simulated waveform is slightly distorted at low frequencies as a result of the nonlinear transformation, but generally matches the overall spectrum quite well. In other words, the NTF does not significantly modify the spectral shape in creating $y(t)$ from $x(t)$, but it does significantly impact the waveform statistics. This finding is in accordance with the initial observation by Ffowcs Williams *et al*. that crackle (or waveform skewness) is indiscernible in power spectral calculations[1].

The skewness and kurtosis of the simulated waveform are very close to the actual values of the AB waveform and the mean and the variance are identical. However, this does not necessarily guarantee that the PDFs of the two waveforms will be the same because no constraints have been placed on any of moments greater than order four. To show that the overall statistical behavior of the two waveforms is largely determined by these four moments, the PDFs of the simulated and AB waveforms are shown in Fig. 4 along with the Gaussian PDF of $x(t)$. In order to completely compare the simulated and AB PDFs, Figs. 4a and 4b respectively have linear and logarithmic abscissa. The linear scale emphasizes behavior near the center of the distribution whereas the logarithmic scale is helpful in comparing the tails of the distributions. Examination of Fig. 4a reveals that the simulated PDF matches that of the AB waveform very closely over the high-probability regions of the distribution. As shown in Fig. 4b, there are differences between the two PDFs in both the positive and negative tail regions, but only where the probability of occurrence is very low. Specifically, the simulated PDF only begins to diverge from the AB PDF at the positive and negative tails when the probability is approximately two orders of magnitude below the maximum probability.

### B. Playback of waveforms

The previous study by Ffowcs Williams *et al*.[1] indicates that because $S$ = 0.57 for the simulated waveform, $y(t)$, and $S$ = 0.02 for the Gaussian waveform, $x(t)$, crackle should be present in $y(t)$ but not in $x(t)$. Playback of portions of .wav files created from these waveforms is possible in the electronic version of this paper by clicking on the “Simulated Waveform” and “Gaussian Waveform” hyperlinks below. (Note that Adobe® Acrobat Reader 6.0 or later is needed for the links to function.)

**Simulated Waveform** **Gaussian Waveform**

As playback indicates, there is little difference between the simulated and Gaussian waveforms and nothing in the simulated waveform that has a crackle-like quality. The reasons for the absence of crackle from the highly skewed simulated waveform are now explored.

## IV. Waveform Time Derivative Analysis

Because there are extreme perceptual differences between the non-Gaussian simulated and the recorded AB waveforms, the waveforms themselves merit a closer look. Displayed in Fig. 5 are short segments of the Gaussian, simulated, and AB waveforms. A comparison of the Gaussian and simulated waveforms shows in more detail the nonlinear transformation process that yields the skewed waveform. A comparison between the simulated and AB waveforms reveals clearly a significant difference between the simulated and actual data. The AB waveform tends to “lean” one direction and has very sharp pressure rises at some instances, whereas the simulated waveform does not. These characteristics of the AB waveform are consistent with descriptions of crackling waveforms provided by Ffowcs Williams *et al*.[1] and Krothapalli *et al*.[2] The lack of crackle in the simulated waveform appears to be related to the fact that these features are not present in the simulated waveform.

The AB and non-Gaussian simulated waveforms have essentially the same spectrum (see Fig. 3) and PDF (see Fig. 4) but are perceived to be dramatically different because of the sharp shock-like pressure rises that are present in the AB waveform but not in the simulated waveform. This difference may be better quantified in calculations of the time derivatives of the waveforms, an analysis technique that is largely due to McInerny (e.g., see Refs. 7 and 8). The PDFs of the time derivatives are shown on both linear and logarithmic scales in Figs. 6a and 6b. These figures reveal a dramatic disparity between the time derivatives of the AB and simulated waveforms. The simulated waveform differs only slightly from Gaussian behavior, whereas the AB waveform is significantly non-Gaussian and has a large positive tail that is formed by the steep rise portions of the waveform and continues out to approximately 28 standard deviations. These differences are quantified with $S$ and $K$ calculations for the Gaussian, non-Gaussian simulation, and AB waveforms in Table 1. The derivative of the simulated waveform has zero skewness and somewhat non-Gaussian kurtosis, whereas the AB data reveals extreme non-Gaussian behavior that appears to be related to the perception of crackle.

**Table 2. Skewness (*S*) and kurtosis (*K*) values for the time derivatives of the Gaussian, non-Gaussian simulation, and AB waveforms. The Gaussian waveform is essentially that of ideal Gaussian behavior, for which *S* = 0 and *K* = 3.**

| Waveform | *S* | *K* |
|---|---|---|
| Gaussian | 0.00 | 3.00 |
| Simulation | 0.00 | 3.40 |
| AB Data | 5.59 | 67.24 |

## V. Discussion and Conclusion

This investigation of the perception of crackle gives rise to a number of points of discussion and conclusions. First of all, the results of the simulation demonstrate that skewness is not sufficient by itself to cause a listener to perceive crackle. The analysis of the time derivatives of the waveforms indicates that shock-like features must be present in the time waveform. As discussed previously, however, in actual jet noise recordings, the intense compressive portions of the waveform usually have short rise times, which led Ffowcs Williams *et al*.[1] to the conclusion that the "physical feature of a sound wave that gives rise to the readily identifiable subjective impression of 'crackle' is shown to be the sharp shock-like compressive waves that sometimes occur in the wave form." Although this physical interpretation does, in fact, appear to be correct, positive waveform skewness does not account for how quickly or how slowly pressure fluctuations occur, but rather only that pressure values occur in a waveform with some asymmetric probability. This leads to the conclusion that if the sharp pressure changes account for the subjective impression of crackle, use of the time derivative of the waveform to quantify the crackle phenomenon is merited.

This conclusion regarding an appropriate metric for quantifying crackle leads to another point of discussion, that of similarity between perceived "nonlinearity" and perceived crackle. As discussed previously, a recent study by Gee and Sparrow[4] shows that a waveform that is nonlinearly propagated numerically takes on an impulsive quality that could be potentially perceived as crackle-like. This occurs despite the fact that the waveform's skewness may be essentially zero. Furthermore, the perceived difference between linearly and nonlinearly propagated signals is significant in that the linear waveform does not have this impulsive quality, but this difference is not evident in the nearly identical waveform PDFs. There is, however, a remarkable difference in the PDFs of the waveforms' time derivatives, which indicates that the impulsive quality to the nonlinearly propagated waveform appears to be linked to shock formation in the course of propagation. A crackling waveform in the near field of a jet also possesses shock-like structures that are evident in the PDF of the waveform time derivative and appear to be critical to the perception of crackle. Any further connection between the two phenomena remains to be explored.

An additional study that merits mention is that by Downing *et al*.[8], in which high-bandwidth measurements of military jet aircraft flyovers were analyzed. The results revealed a clear relationship between overall sound pressure level (OASPL) and skewness of the time derivative waveform. As OASPL increased, so did the derivative's skewness. On the other hand, only a weak relationship was found between OAPSL and skewness of the waveform itself. An extension of the study by Downing *et al*. would be to perform listening tests with the flyover waveforms to determine if the relative contribution of the crackle-like phenomenon to overall annoyance also increases as a function of OASPL or if it remains relatively constant.

One final point of discussion regards the ability to generate a simulated jet noise waveform that does, in fact, create the perception of crackle. This may be possible with the nonlinear transformation process described in Section II for an appropriate choice of a nonlinear transformation function that can yield an extremely non-Gaussian PDF as is shown in Fig. 6. Work is ongoing to identify and implement that function. Refinement of such a simulation process could greatly reduce the data storage requirements for source characterization from entire waveforms to power spectra and a few statistical calculations that could then be used to create equivalent waveforms.

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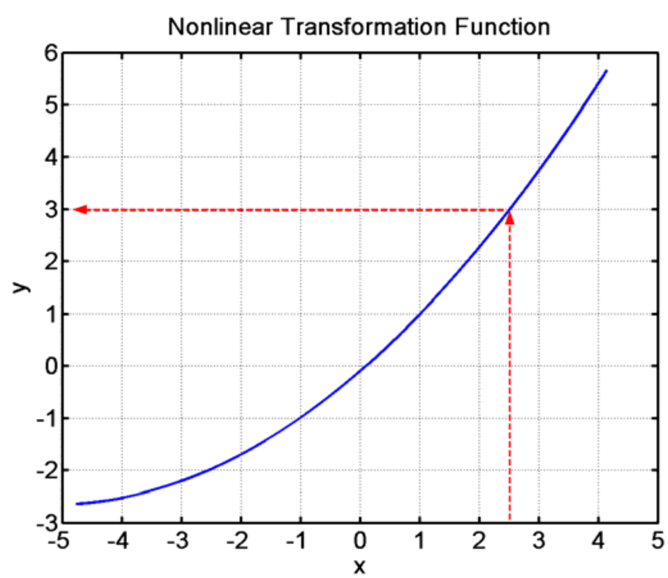


**Figure 1. Quadratic nonlinear transformation function used to transform a Gaussian waveform to a non-Gaussian waveform with positive skewness. The red dashed lines demonstrate how the nonlinear transformation occurs. In this case, *x* = 2.5 results in *y* = 3.0.**

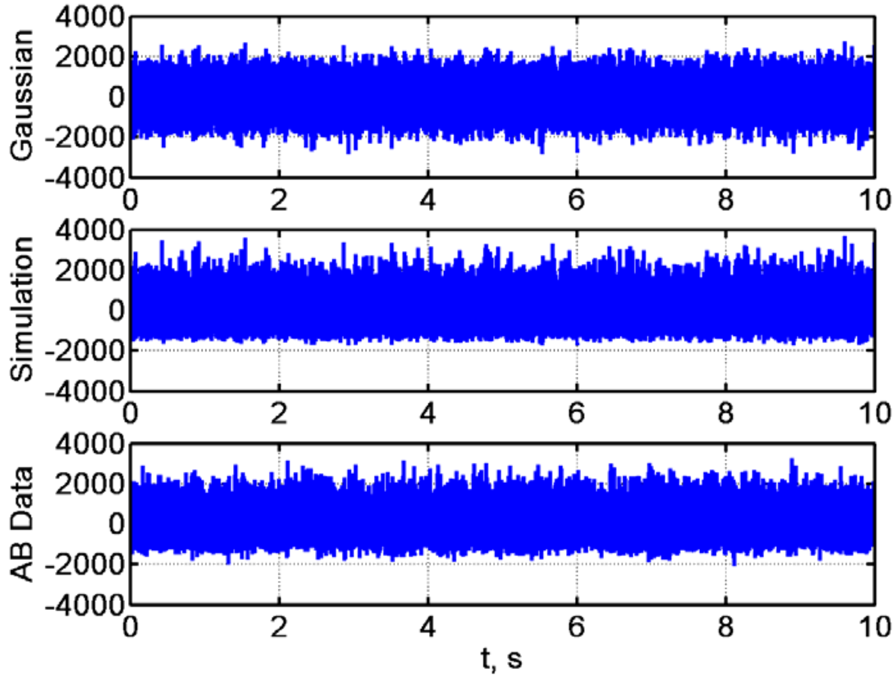


**Figure 2. Gaussian, simulated non-Gaussian, and AB waveforms. All three waveforms are zero-mean signals and have the same variance. The Gaussian waveform has zero skewness, whereas the simulated and AB waveforms have significant positive skewness.**

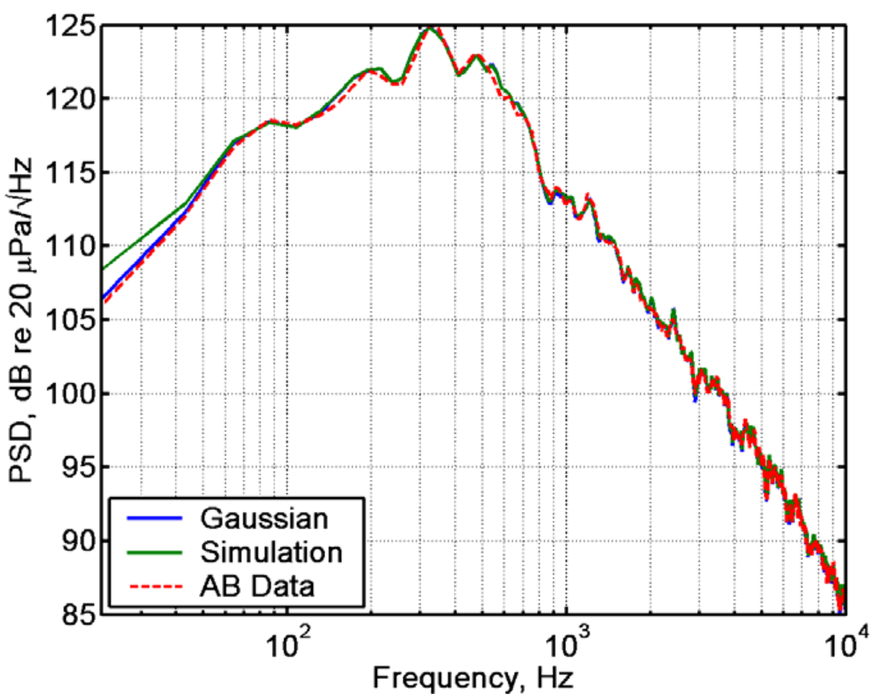


**Figure 3. Power spectral densities of the Gaussian, non-Gaussian simulation, and AB waveforms.**

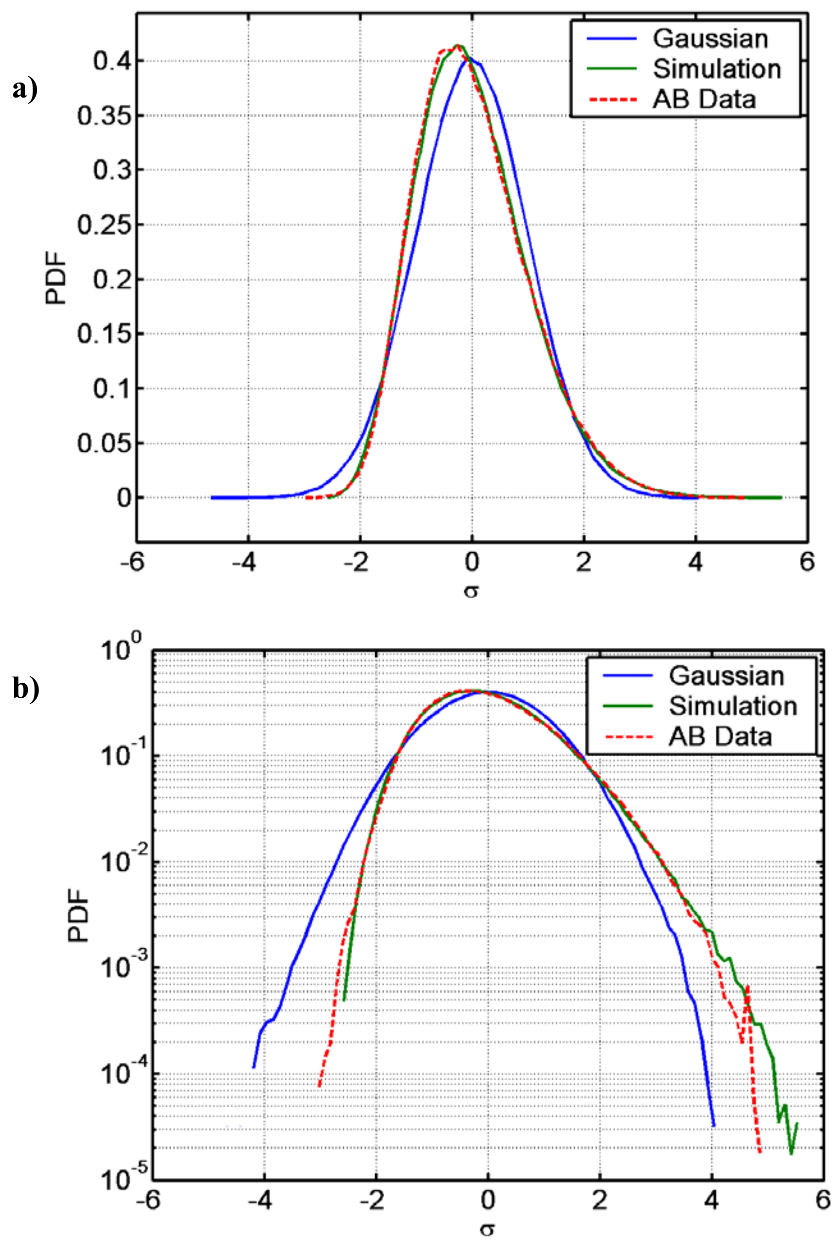


**Figure 4. Probability density functions of the Gaussian, non-Gaussian simulated, and AB waveforms. The PDFs are represented on a linear scale in Fig. 4a and on a logarithmic scale Fig. 4b.**

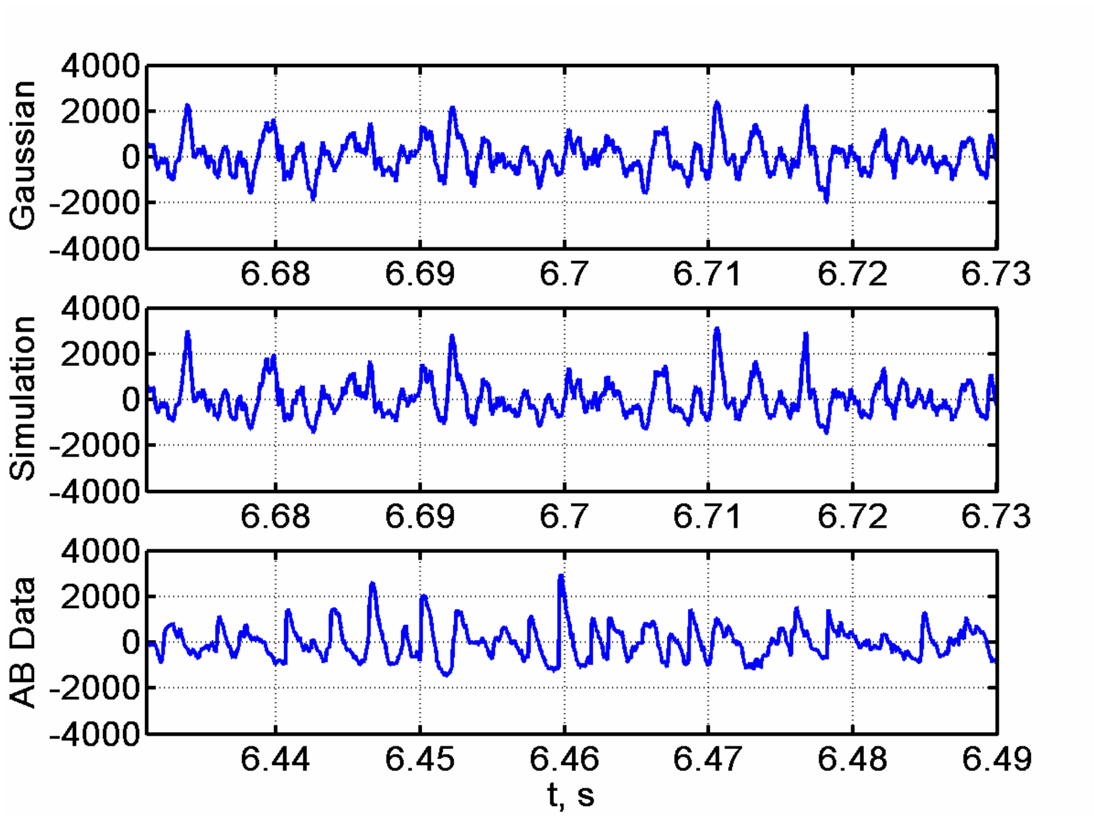


**Figure 5. Segments of the Gaussian, non-Gaussian simulation, and AB waveforms. The compressive portions of the AB waveform are generally much more shock-like than the non-Gaussian simulation. See the text in Section IV for further explanation and discussion.**

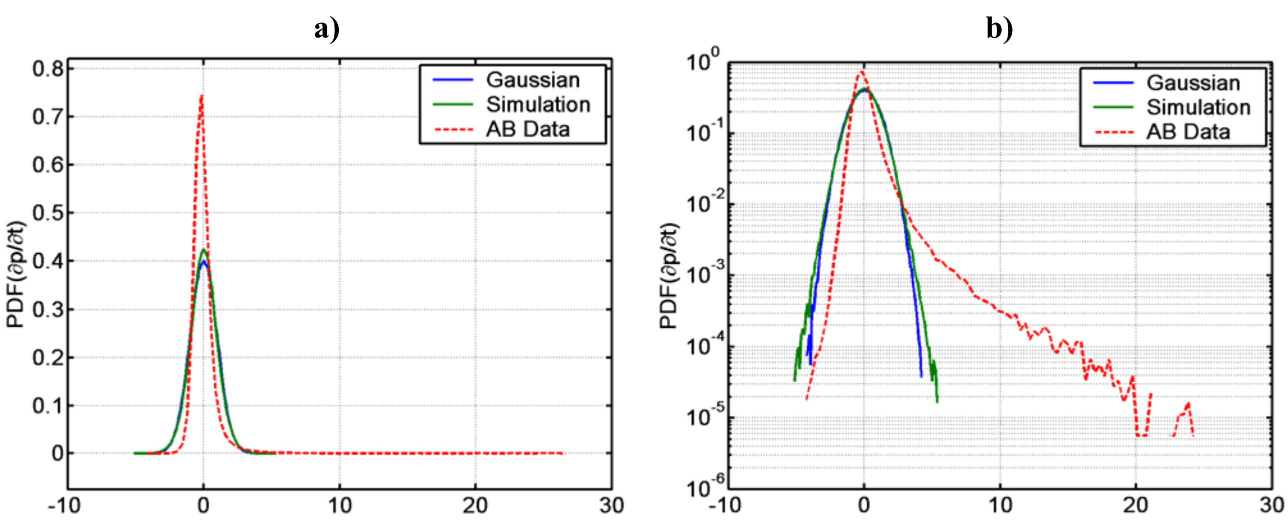


**Figure 6. Probability density functions of the time derivatives of the Gaussian, non-Gaussian simulated, and AB waveforms. The PDFs are represented on a linear scale in Fig. 6a and on a logarithmic scale Fig. 6b. Note that discrete occurrences in the AB waveform's PDF continue out to 28 standard deviations (σ).**