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Damping of Trapped-Particle Asymmetry Modes in Non-Neutral Plasma Columns

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Abstract. Asymmetry modes ($m = 1$, $k_z \neq 0$) are diocotron-like modes in finite-length plasma columns in Malmberg-Penning traps. We have investigated the modes with a detailed 3-d particle-in-cell (PIC) drift-kinetic computer simulation. Although PIC simulations do not employ realistic collisions, the simulations in this case reproduce many of the salient features of the data. Particle transport associated with the damping is seen not to be a direct collisional effect, but rather a feature of orbital dynamics associated with transitions from trapped-to-untrapped or untrapped-to-trapped state relative to the inversion plane of the asymmetry. In the simulations we observe a B^{-1} dependence of the mode frequencies and a $B^{-0.5}$ dependence of the damping constant for large rigidity. We further observe a steepening of the dependence of the decay constant to B^{-2} as the rigidity of the plasma falls below about 2.0. We have also used the simulations to investigate the modes at small seed amplitudes and observe linear flattening in the mode frequency as the seed amplitude becomes small. In contrast, the decay constant does not flatten for small seed amplitude.

Non-neutral plasmas, typically ions or electrons, can be confined for long periods of time in a cylindrical Malmberg-Penning trap. A stiff axial magnetic field confines the particles radially and charged rings at the ends of the otherwise grounded cylinder provide electrostatic longitudinal confinement. Diocotron modes are azimuthal drift waves in the cylindrical plasma that vary spatially as $\exp(im\theta)$. The theory of diocotron modes in non-neutral plasmas has its origins in seminal papers by Briggs, Daugherty and Levy [1] and the comprehensive treatment of non-neutral plasmas by Davidson [2]. Experimental and theoretical work at the University of California San Diego for more than a decade has also contributed particularly to the foundation of understanding of these modes [3].

Here we consider a modification of the diocotron mode when an applied "squeeze voltage" is applied to an additional ring installed at the longitudinal center of the trap and the plasma is offset from the symmetry axis in opposite directions on each side of the center ring such that the parity of $k_z \neq 0$ is odd. The squeeze voltage creates an energy barrier at the longitudinal median plane that gives rise to a population of trapped particles on either side of the divide as well as a population of untrapped particles with sufficient energy to traverse the entire length of the trap. The result is a new mode revolving at a frequency different from the wall value of the rotation frequency profile. These "trapped-particle asymmetry modes" have been experimentally observed and reported by Kabantsev *et al.* [4, 5, 6, 7]. The dependence of the decay constant on magnetic field has been particularly problematic. The most recent data are divided into two regimes, one at lower magnetic fields varying roughly as B^{-1} eventually giving way to a less steep dependence ($B^{-0.5}$) at higher magnetic fields [6]. Hilsabeck and O'Neil have ascribed

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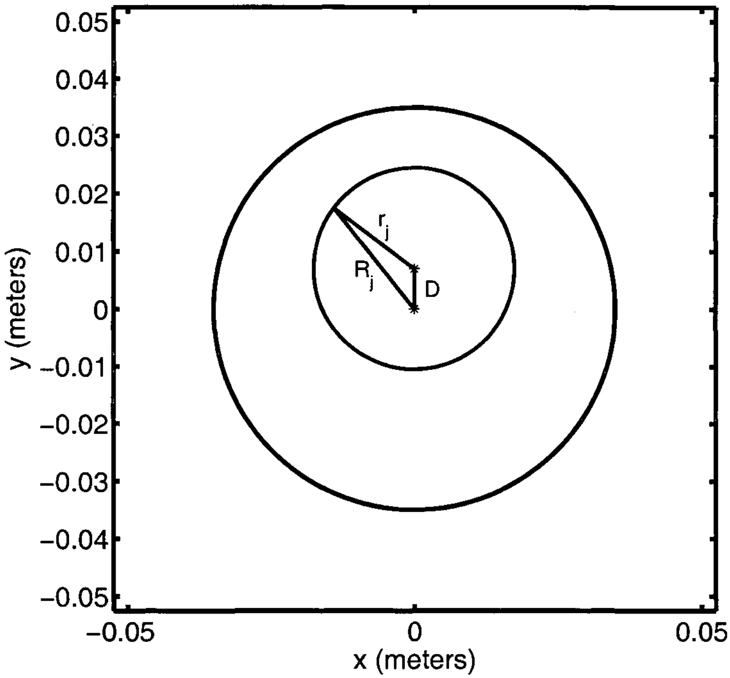

FIGURE 1. The $m = 1$ mode displaces the charge center of trapped particles relative to the symmetry center of the trap. Trapped particles revolve roughly in circles of radius r_j around the displaced charge center. The orbit of a trapped particle about the charge center creates a radial oscillatory motion whose amplitude is the mode amplitude, D. R_j is the radius of a particle relative to the symmetry center. Untrapped particles revolve roughly in circles around the symmetry center of the trap. Transitions from trapped to untrapped state or from untrapped to trapped state result in radial transport.

the damping mechanism to velocity scattering of marginally trapped particles. They have developed a theory based on this mechanism for which they report good agreement with measurements [5, 8].

We have reported elsewhere the results of detailed 3-dimensional computer simulations of the trapped-particle asymmetry modes [9]. In the simulations the decay mechanism is demonstrated to be a consequence of the orbital dynamics of particles moving back and forth between the trapped and untrapped populations. The interchange between the two populations is a result primarily of slow modulations (diffusion) of the longitudinal velocities of the particles. The simulations (and data) have demonstrated that the mode frequency increases with decreasing squeeze voltage, that the mode frequency varies as B^{-1} and that the decay of the mode varies with magnetic field as $B^{-0.5}$ for a range of magnetic fields $0.02T \leq B \leq 0.32T$. Here we observe that the dependence of the decay constant steepens as the magnetic field decreases below this range (see Fig. 2).

We have not simulated any specific experiment, although we have chosen parameters

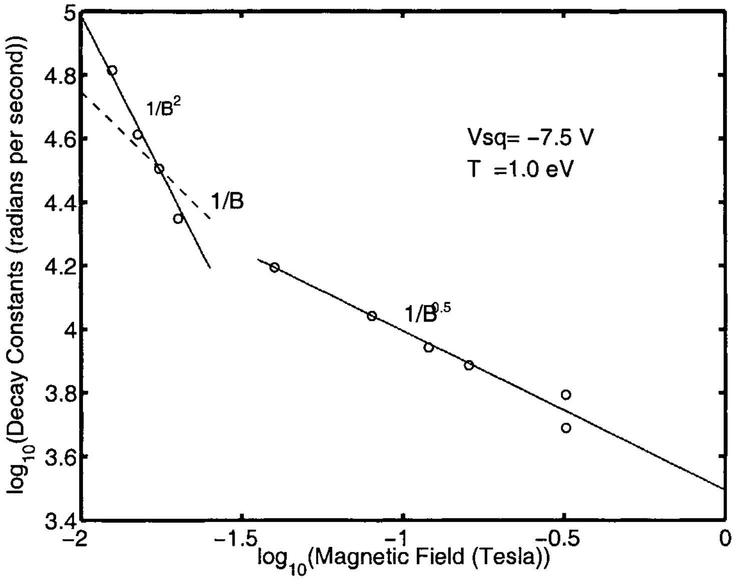

FIGURE 2. Dependence of the decay constant on magnetic field for a squeeze voltage that is about half the central potential of the plasma. The dependence steepens roughly where there are about two particle longitudinal bounce cycles for each mode cycle.

that lie in the general parameter space of the UCSD data [6]. Using Fig. 2 of Kabantsev *et al.* and the formula for rotation frequency on the same page of this latter reference, we get a typical decay constant of $10^3 s^{-1}$ for a typical density of $10^7 cm^{-3}$. The ratio of decay constant mode frequency is then about 10^{-3}. This is for a 1 Tesla field. Taking numbers from our simulations [9] and using 0.8 Tesla (our closest magnetic field for the experimental 1 Tesla), we get a ratio of 25×10^{-3}, i.e. 25x too large. However, the decay constants from the simulations have a rough dependence on the number of simulation particles ($N^{-0.5}$, a collisional effect) that we have determined empirically. In this paper we use $N = 1.5 \times 10^6$ particles to represent 6.8×10^8 electrons. Thus we must divide our result by $\sqrt{450}$ to correct to a 1 : 1 simulation. The resulting value of the decay constant puts us very near the mark, other things being equal. The code accurately computes rotation frequencies where collisionality plays no role, so we conclude that the simulation roughly predicts the salient features of the data.

The simulation code being used is a drift-kinetic, particle-in-cell (PIC) code [9]. In such a code, all motion transverse to the longitudinal axis of the trap is handled as $\mathbf{E} \times \mathbf{B}$ drift. Thus, the only aspect of collisions that is handled in anything near realistic fashion is the longitudinal motion and even that has a built-in de-emphasis of small impact parameter collisions. Therefore, a conclusion to be drawn is that the decay of the asymmetry modes, insofar as the PIC code reproduces the features of the decay correctly, is virtually independent of transverse momentum transfer in the collisions.

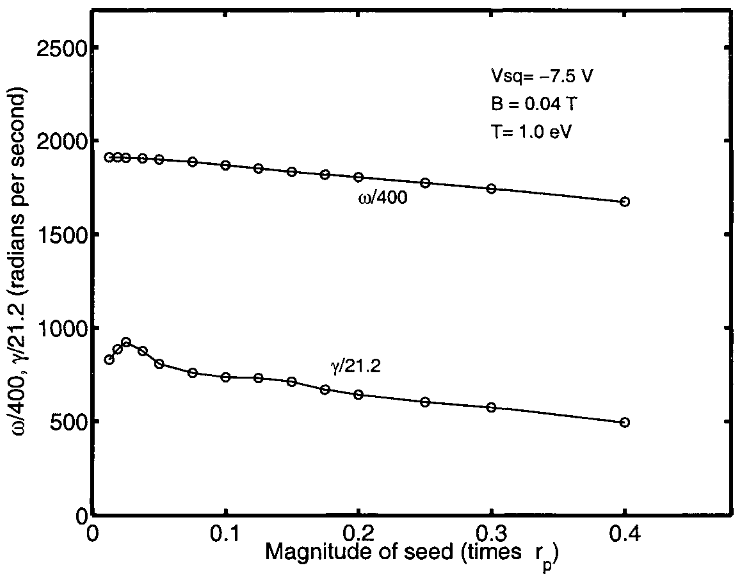

FIGURE 3. Both the mode frequency ω and the decay constant γ depend on the magnitude of the seeded amplitude. However, γ shows variation at small mode amplitude, indicating a nonlinearity in its behavior. The mode frequency has been divided by a factor of 400 merely to allow comparison of the dependences on the same graph. The decay constant has been divided by an empirically determined correction factor of $\sqrt{450}$ that is necessary to correct for the difference between the ratio of actual particles (electrons) to simulation particles in the simulated trap. The ratio is 450 in this case. A similar correction factor is necessary whenever comparisons to actual data are attempted.

The canonical angular momentum of the system is given approximately by [10]

$$P_\theta \approx \frac{eB}{2}\sum_{j=1}^{N} R_j^2 \approx \frac{eB}{2} N \left(D^2 + \frac{1}{N}\sum_{j=1}^{N} r_j^2 \right), \tag{1}$$

where N is the number of particles, $\mathbf{R}_j$ is the position of the jth particle measured from the symmetry axis of the cylinder, $\mathbf{D}$ is the position of the $m = 1$ mode (charge) center, and $\mathbf{r}_j$ is the position of the particle relative to the mode center, i.e. $\mathbf{R}_j = \mathbf{D} + \mathbf{r}_j$. Since the simulation code conserves the angular momentum, any decrease in the mode amplitude D must be accompanied by an overall adjustment in $\sum r_j^2$, i.e. some transport relative to the charge center of the mode. However, in the region of parameter space where we are working, $\sum r_j^2 << ND^2$. In the relatively short simulations times considered here (75 μsec), the net amount of radial transport needed to account for the mode decay is almost imperceptible.

The mode frequency ω and the decay constant γ both depend mildly on the magnitude of the seed of the mode (created by moving a subset of particles radially) [9]. Linear behavior requires that the dependence of frequency and decay constant be independent of the (perturbed) amplitude of the mode for small amplitude. In Figure 3 we observe

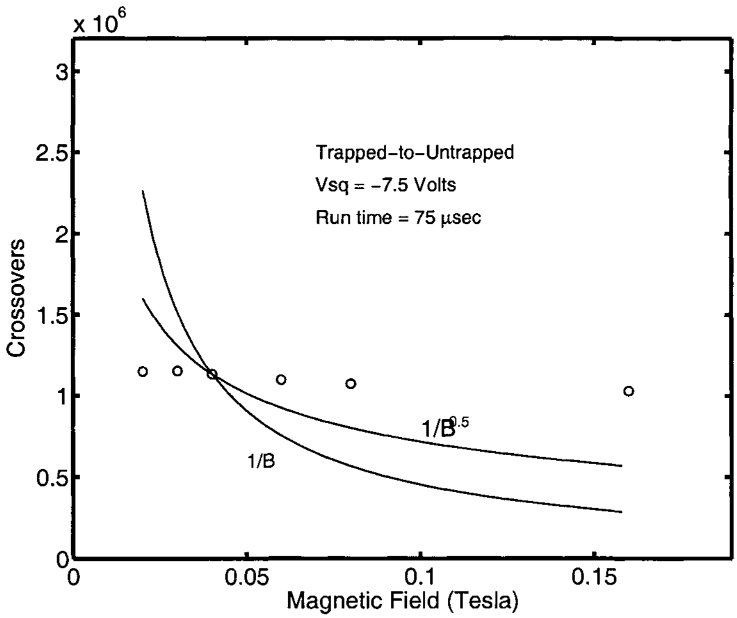

FIGURE 4. The number of simulation particles that change from trapped to untrapped status within a 75 *μsec* simulation as a function of magnetic field. The magnetic field has only a weak influence on the transition rate associated with the mode decay.

that the dependence of γ violates this condition.

Within the parameter space considered here, the code reproduces the expected B^{-1} behavior of the mode frequency (which takes its value from the $\mathbf{E} \times \mathbf{B}$ rotation profile). However, the decay constant varies as $B^{-0.5}$ for large rigidity. To understand why this is so, we have monitored the total number of particle transitions from trapped-to-untrapped state during a simulation as a function of magnetic field (see Fig. 4). The rather mild dependence on magnetic field suggests that the dependence of γ on magnetic field must be sought elsewhere. Since the "radial diffusion" (small variation in $\sum r_j^2$) apparently does not depend on the transition rate, we look at the radial step size as a function of magnetic field (see Fig. 5). Although the connection to the $B^{-0.5}$ dependence of γ is not readily evident, the marked dependence of the radial step size on magnetic field suggests that the answer is to be found in the step size.

In conclusion, our simulations indicate that the decay of asymmetry modes is accompanied by minimal net particle transport that is a consequence of adjustments in orbital motion when a trapped particle becomes untrapped (relative to the full length of the trap), or vice versa. The decay constant of the mode is seen to vary as $B^{-0.5}$, steepening to B^{-2} for smaller rigidity. Unlike the mode frequency, the mode decay constant depends on amplitude, particularly at small amplitude. There is only a small dependence on magnetic field for the rate of transitions from trapped to untrapped status associated

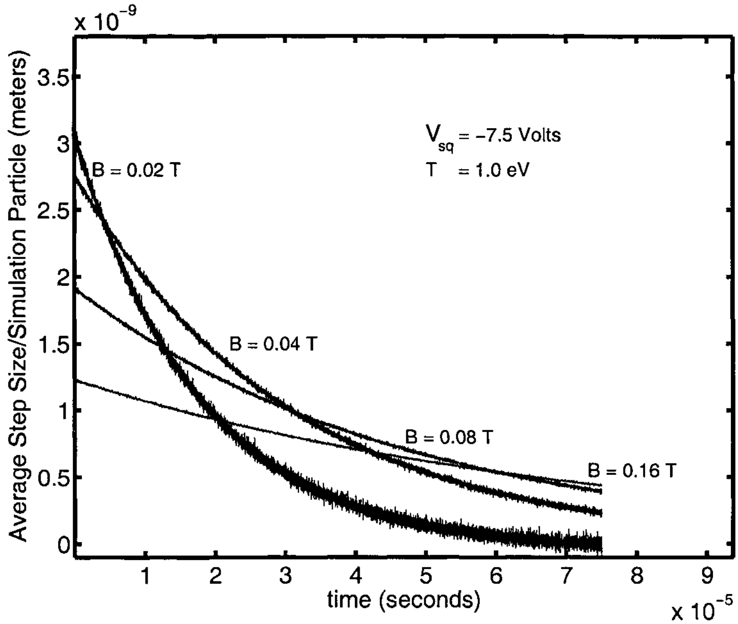

FIGURE 5. The average radial step size per simulation particle as a function of time for different choices of magnetic field. Since the transition rate for the diffusion associated with the mode decay is roughly independent of magnetic field, the $B^{-0.5}$ behavior of the decay constant likely stems from the dependence of the radial step size on magnetic field.

with the decay mechanism. The radial stepsize shows a much stronger dependence on magnetic field. Thus, it is likely that the dependence of the decay constant on magnetic field is to be found in the dependence of the radial stepsize on magnetic field.

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