# COHERENT COSMIC-RAY MODEL BASED ON "DIRECT" MUON PRODUCTION*

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(Received 3 September 1968)

We consider a cosmic-ray muon model in which a "directly produced" muon component with the same slope as the primary spectrum is added to the conventional pion-derived muons. It is found to be consistent with recent burst measurements, horizontal-air-shower data, atmospheric gamma-ray fluxes, and muon spectrograph measurements. It may also explain the anomalous mu-less air showers. The model requires that the photonuclear cross section increase at very high energies.

The first observations with the Utah neutrino detector[1] indicated that cosmic ray muons with energies $10^{12}$ eV do not show the full $\sec\theta$ enhancement to be expected if they were the progeny of pions and kaons produced in the upper atmosphere. Instead, they behave as though a significant fraction of them were produced directly or via a short-lived intermediary. Such a process (which we call the $X$ process) would lead to a muon spectrum flatter by one power of $E$ than a spectrum of muons derived from pions and kaons. In this paper, we investigate the consequences of adding a flat component to the usual muon spectrum. The intensity of this component required to fit our angular distributions (about 2% relative to pions) implies a cross section ~100 $\mu$b. The spectrum resulting from this fit is found to be in agreement, from $10^{11}$ to $10^{14}$ eV, with a wide variety of cosmic-ray observations, with one exception we know of[2]; in addition, this model could account for the heretofore unexplained phenomenon of the mu-less air showers at $10^{15}$ eV via $X$-process electrons.

The muon spectrum.—The specific form of the differential muon spectrum assumed was[3]

$$M(E)=C_\pi E^{-\gamma-1} r^\gamma K(E,\theta) B(E\cos\theta^*+B)^{-1} + RC_\pi E^{-\gamma-1} \quad (\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}\ \mathrm{sr}^{-1}\ \mathrm{GeV}^{-1}), \tag{1}$$

where $\gamma=1.7$, $r=0.76$, $B=90$ GeV, and $\theta^*$ is the zenith angle at the top of the atmosphere of a muon trajectory with a zenith angle $\theta$ at the detector. We take $C_\pi=0.225$ to make our $\pi$ and $K$ component (the first term) agree with the Osborne-Wolfendale-Palmer (OWP) spectrum[4] at 300 GeV. $K(E,\theta)$ is a factor given by Osborne[5] to correct for a 20% ratio of kaons of all charges to pions of all charges; $K$ ranges from 1.16 to 1.38 over the range of energies and angles used in our work. The parameter $R$, which is a measure of the relative $X$-process amplitude, is determined by fitting to the underground angular distributions.

Built into this spectrum are the assumptions that both the $\pi$ and $K$ parents and the $X$ process follow the primary differential spectrum $E^{-2.7}$. At low energies the assumption might break down for the $X$ process near threshold, but in this region the $\pi$-$K$ component is dominant; while at high energies the muon spectrum from the $\pi$-$K$ component might be unreliable, but the $X$ process is dominant. Of course, the primary integral spectrum may not have a constant slope; it is known to change from 1.7 to 2.2 at ~$3\times10^{15}$ eV.

Muon interaction in rock.—With the muon spectrum thus prescribed to within the parameter $R$, the absorption of high-energy muons must be adjusted to fit the muon depth-intensity curve. Vertical muon intensities underground were calculated by the Monte Carlo method of Hayman, Palmer, and Wolfendale[6] for three values of $R$ and for six values of the coefficient $b$ in the energy-proportional muon-energy-loss term. Then, for a given $R$, a series of effective $b$ values were obtained as a function of depth, such that the predicted intensities matched the vertical depth-intensity curve adopted by us from a world-wide survey.[7] The resulting effective $b$ values are shown in Fig. 1, where the electromagnetic contribution to the total $b$ is shown by the dashed line. The difference between these curves gives some measure of the photonuclear interaction required by our model, though, of course, only in the asymptotic region can one really compare $b$ and $b_{\mathrm{eff}}$. A photonuclear cross section ~500 $\mu$b is implied—about five times that obtained at accelerator energies.

Fits to the angular distributions.—Muon intensities at zenith angles $\theta$ and slant depth $h$, measured to the top of the atmosphere, were derived from our spectrum, using $b_{\mathrm{eff}}$ from Fig. 1. Predictions for the ratio $I(h,\theta)/I(h,0)$ for $R=0.04$,

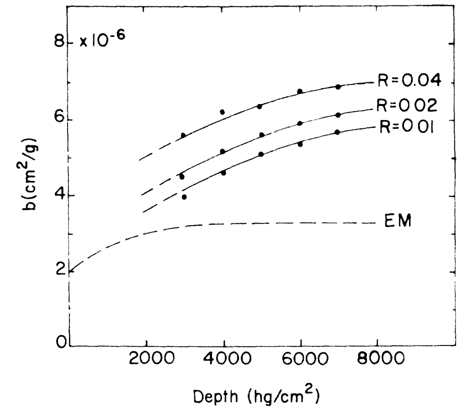

FIG. 1. Effective values of $b$ as a function of depth required to fit our adopted spectrum to the vertical depth-intensity curve for three values of $R$. Also shown is the theoretical $b$ for electromagnetic processes (EM).

0.02, and 0 are plotted in Fig. 2. The prediction $R = 0$ corresponds to a constant $b$ value of 4.0 $\times 10^{-6}$ cm$^2$/g, since that is the value used to infer the conventional OWP muon spectrum from depth measurements. (The OWP spectrum agrees closely with our spectrum with $R = 0$, i.e., the $\pi$-$K$ component, up to 5000 GeV.) A prediction assuming all-kaon parentage and the conventional $b$ value is shown for 75 deg only.

The points plotted in Fig. 2 are the observed intensities, $I_{\text{obs}}(h, \theta)$, divided by $I_V(h)$, the vertical world-wide depth-intensity (WWDI) curve. The 0-deg plot compares the world-survey vertical points with the adopted curve (unity on the ratio plots). In the other angular ranges, the Utah points are shown. Since the mean $\theta$ of the zenith-azimuth bins assembled to produce each point (as described in Ref. 1) is not, in general, the central angle of the 10-deg range, but instead tends toward the smaller angles at the shallower depths and vice versa, we also show the predictions of our model (heavy lines) for the average angle appropriate to each point.

The observed points in Fig. 2, although based on the same data[8] as Ref. 1, differ mainly because we used 2.61 g/cm$^2$ for the density and 5.65 for $Z^2/A$ obtained from an improved rock survey. Confidence in the uniformity of our surrounding rock comes from a detailed study of muon intensities observed in different directions as compared with the number of muons predicted from our model and assuming a constant density. The agreement of our observations with predictions in the 40- to 50-deg region (where the predictions for all values of $R$ converge) anchors the density of our study relative to that of other investigators.

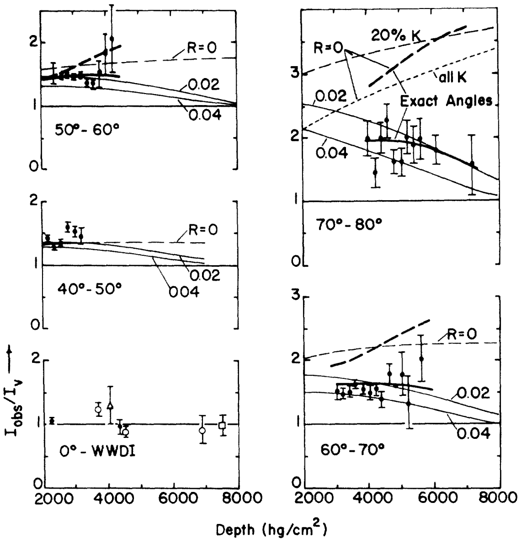

FIG. 2. Observed muon intensities relative to a world-survey vertical-depth–intensity curve, $I_V(h)$, for various angular ranges. The 0° plot shows the experimental points of the world survey (WWDI) relative to the adopted hand-drawn fit; the data shown for other angular ranges are the Utah points. The curves shown are the predictions of our model, $I_{\text{pred}}(h, \theta)/I_{\text{pred}}(h, 0)$, for various values of $R$. This ratio is very insensitive to the value of $b$. Heavy lines are the predictions for the average angles appropriate to the experimental points. The key to the various curves is given in the upper right-hand figure. See text for description of "exact-angles" curves.

The best fit to our data is obtained by inspection, with a value of $R = 0.02$; it is unlikely that $R$ is less than 0.01 or greater than 0.04. Vertical and horizontal spectra obtained with this choice of $R$ are plotted in Fig. 3. We now proceed to compare these spectra with a wide variety of cosmic-ray phenomena.

Direct spectrograph measurements.—The pion component of our spectrum has already been chosen to agree exactly with the OWP spectrum at 300 GeV, thus assuring agreement with spectrograph measurements below, say, 500 GeV. Horizontal spectrograph measurements extend to higher energies, and a potential discrepancy between these measurements and our results had already been pointed out by Nash and Wolfendale.[9] The three highest energy points of the Nash and Wolfendale data, corrected to correspond to intensities at the top of the atmosphere, are plotted in Fig. 3 (lower left), where it is seen that

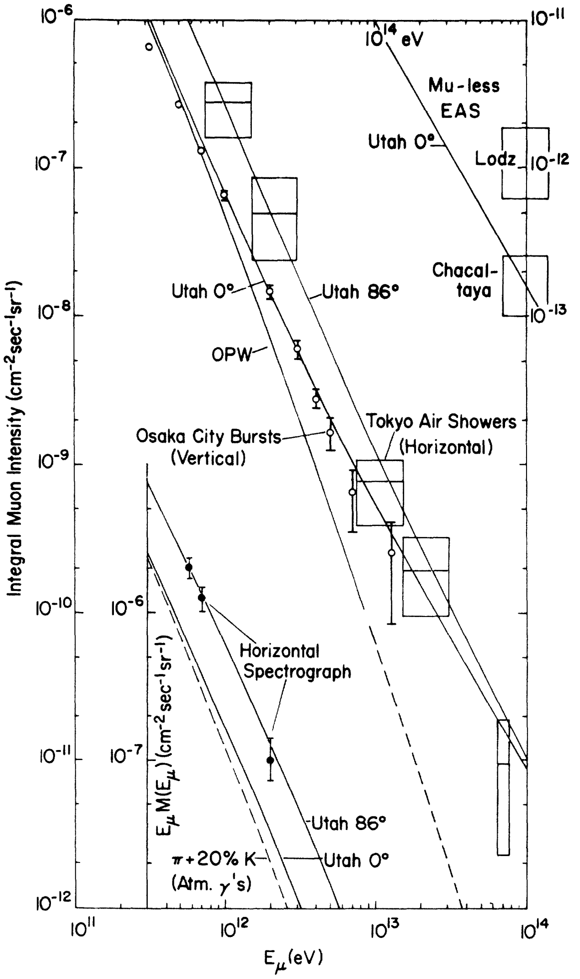

FIG. 3. The Utah muon spectrum compared with pertinent cosmic-ray observations. Lower left: differential spectra multiplied by $E_\mu$. The integral spectrum (center) is continued in the upper right-hand corner.

they are in excellent agreement with the predictions of our model (which still gives a considerable enhancement factor).

Atmospheric $\gamma$ rays.—Measurements of the intensities of high-energy $\gamma$ rays in the upper atmosphere, if interpreted as the product of $\pi^0$ mesons, yield a pion production spectrum. If one then assumes charge independence and a particular $K$-to-$\pi$ ratio, it is possible to predict a muon spectrum. Osborne and Wolfendale,[10] for example, found that they could account for the sea-level muon spectrum in this manner by assuming pions plus 20% kaons. Now, even if equal numbers of electrons and muons are produced by the $X$ process, the contribution of these electrons to the atomospheric $\gamma$-ray component cannot be large; for, although a small contribution of "directly produced" muons can still dominate the $\pi$ and $K$ muons because of nuclear absorption of the parents, a small contribution of "directly produced" electrons cannot compete with the $\pi^0$'s as a source of $\gamma$ rays. Consequently, the atmospheric $\gamma$ rays are to be compared in our model with the $\pi$ and $K$ component only; but this component (Fig. 3, lower left) is, in the energy range relevant here, almost identical to the OWP spectrum, which has already been demonstrated to fit well with the atmospheric $\gamma$ rays.

Sea-level burst experiments.—The muon spectrum can be inferred from measurements of bursts (local showers) at or just below the surface of the earth. The recent results of the Osaka City group[11] are shown in Fig. 3. Energetic showers (~10 cm in diameter) emerging from the rock of a tunnel at a depth of 40 hg/cm$^2$ were observed as they traversed two planar scintillator arrays, 20 m$^2$ in area and separated by 2.4 m. The arrangement permits the selection of showers of known direction in a well-defined aperture. The deduction of a muon spectrum involves the cross sections for bremsstrahlung and pair production by the muons and cascade shower theory. Fluctuations make the estimate of absolute values difficult; Furry fluctuations give approximately the absolute level of the present points. (Without fluctuations, the points would be higher by a factor of ~3.) A normalization was chosen to bring the points near $10^{12}$ eV into agreement with the Utah spectrum. Showers produced by the nuclear interactions of muons (even in our model with $b = 6\times10^{-6}$ cm$^2$/g) produce a relatively small effect, because nuclear showers produce bursts only $\frac{1}{3}$ to $\frac{1}{5}$ as large as electromagnetic showers of the same energy. It is seen that the burst spectrum favors the Utah model over a conventional spectrum, such as the OWP spectrum.[3] Recent burst measurements reported by Erlykin at the Lodz Conference on Air Showers[12] give the same slope for the vertical spectrum as do the Osaka City results.

Horizontal air showers.—The Tokyo group[13] has observed horizontal air showers at atmospheric depths far too great to have been initiated by the nucleonic component of the cosmic rays. These have, in general, been attributed to air showers produced by muons (the analog in the atmosphere of the bursts observed by Osaka City coming out of the rock), but the conventional steep cosmic-ray muon spectrum yields a rate too low by two orders of magnitude. It is seen, in Fig. 3, that our flat muon spectrum accounts nicely for this phenomenon, despite the reduced enhancement of 86° over the conventional picture. Muon intensities have been derived from the Tokyo data in the same way as from the Osaka City bursts (including the reduction of about a factor 3 for fluctuations), except that the frequency corresponding to a given shower number has been corrected for the ratio of bremsstrahlung and pair-production cross sections in air as compared to rock, and the energy per shower particle has been corrected by the ratio of critical energies in air as compared to rock.

Mu-less air showers.—The existence of a qualitatively distinct class of purely electromagnetic air showers, containing no more muons than would be expected on the basis of photonuclear production by the soft component itself, seems to be well established.[14,15] If primary $\gamma$ rays initiate these showers, they are too isotropic to be of galactic origin and too energetic (about $10^{15}$ eV) to penetrate the primordial blackbody radiation in intergalactic space. Explanations invoking abnormal fluctuations, where a primary transfers almost all its energy to $\pi^0$ mesons, have met with little success. However, if the $X$ process involves the transfer of a large fraction of the energy of a primary proton to a massive particle, $X$, which subsequently decays with equal likelihood into muons and electrons, we can compare the rates of electron-induced showers with the muon spectrum at $10^{15}$ eV. Comparisons with the results of the Lodz group at sea level[14] and the Massachusetts Institute of Technology–Tokyo–Bolivian collaboration, working at Chacaltaya,[15] are shown in the upper right-hand corner of Fig. 3. The mu-less shower rate is obtained from the primary cosmic-ray intensity multiplied by the quoted relative abundance of mu-less showers, taking into account the different curves for electron- and nucleon-initiated showers. The flags on the points are very hard to estimate.

Discussion.—It is clear that our spectrum fits a large class of cosmic-ray phenomena rather well, especially in view of the constant slopes assumed for the $\pi$, $K$, and primary components. A slight steepening of the slopes at high energies would be altogether reasonable and would improve the fit, as would a slightly smaller $R$.

To explain a muon spectrum as flat as ours in terms of pions and kaons would appear to require a primary pion spectrum flatter than the primary proton spectrum and would lead to a contradiction in the slope of the atmospheric $\gamma$-ray spectrum. Alternatively, to invoke a large anomalous interaction cross section of the muons to produce the bursts would conflict with the depth-intensity relation.

According to Bjorken et al.,[16] the most promising models for the $X$ process involve the strong production of a massive particle $X$ which is stable under strong and electromagnetic interactions and decays with high probability into a state containing a muon. $X$ could be the intermediate boson, produced strongly in pairs, or it could be an integral-charge SU(3) triplet. If, as seems likely, the $X$ process transfers a large fraction (say, $\frac{1}{2}$) of the primary energy to the muon, the cross section can be estimated from the fact that the $X$-muon flux is about $10^{-3}$ times the primary flux at the same energy. The cross section is ~100 $\mu$b.

We thank J. D. Bjorken and A. W. Wolfendale for extensive discussions and correspondence, S. Ozaki for communicating his latest burst results before publication and for his great help in relating these results to the muon spectrum, and G. W. Carlson for extensive computer programming.

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*Work supported by the National Science Foundation.

†This is an essential portion of a Ph. D. dissertation by Grant W. Mason, submitted to the Physics Department, University of Utah (1968).

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[6] P. J. Hayman et al., Proc. Roy. Soc. (London), Ser. A 275, 391 (1963).

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# POTENTIALS ON ROTOR SURFACES*

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(Received 1 August 1968)

A small radial electrical potential gradient has been observed to exist across a Duraluminum rotor spinning at high speed in air at pressures from $10^{-5}$ to $10^{-6}$ Torr.

Several investigators have searched without success for axial potential gradients produced in metal spinning rotors. Lodge[1] and Nichols[2] looked for the effect in metal discs spinning in air at atmospheric pressure, but the comparatively large and variable electromotive forces introduced by the rubbing electrical contacts on the axis and on the periphery made their results inconclusive. More recently Freedman[3] and his associates also were unable to find the phenomenon with more reliable equipment. In this paper, experiments are described in which a small radial potential difference is observed across a rapidly spinning Duraluminum (ST14) rotor surrounded by air at pressures between $10^{-5}$ and $10^{-6}$ Torr. Robl,[4] Schiff and Barnhill,[5] Dessler, Michel, Rorschach, and Trammell,[6] and Herring[7] have shown theoretically that a gravitationally induced electrical field should exist outside of a vertical conductor. In a most ingenious experiment, Witteborn and Fairbank[8] have recorded a vertical field of $5.6\times10^{-11}$ V/m inside of a copper cylinder, a result which is in agreement with the theory of Schiff and Barnhill. Since centrifugal fields can be made very much largen than the earth's gravitational field, the effect should be correspondingly larger in a rapidly spinning rotor.[9]

Figure 1(a) shows a schematic diagram of the experiment. The rotor $R$ was spun inside the evacuated metal chamber $V$ by an air-supported, air-driven turbine $T$ situated below $V$. The thin flexible shaft $S$ which connects the turbine to the rotor passes through the electrically insulated vacuum-tight oil glands $G_1$ and $G_2$. This scheme for supporting and driving high-speed rotors in a vacuum has been previously described; so the mechanical details will not be given.[10] Electrical connection with the rotor was made through a water-cooled liquid-mercury contact with the shaft at $M$. The rotor, the vacuum chamber, bearings, turbine, etc. were all nonferromagnetic. This simplified the compensation of the earth's magnetic field at the rotor by a large Helmholtz coil which surrounded the apparatus. The turbine drive was so designed that the direction of rotation could be reversed. The vacuum chamber and the electrical and oil shields were made of metal and were grounded. Figures 1(a) and 1(b) show a schematic cross section (not to scale) and top view of the rotor which is machined in the form of a cross. The rotors were 15 cm in diameter and 2 cm thick, and the cross arms were 2 cm wide. The rotating parts were electrically insulated from the stationary parts by vacuum-pump oil and neoprene O rings in $G_1$ and $G_2$ and the Bakelite-supported air cushion $H$ beneath the turbine. A thin, light metal disc $D$ insured that the small quantity of vacuum-pump oil leaking through $G_1$ could not reach the rotor. The chamber $V$ was evacuated by an oil diffusion pump and forepump through a liquid-nitrogen trap. Variable emf's generated in the liquid-mercury contact with the shaft at first gave considerable trouble, but the problem was finally solved by using a very small needle of copper or