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Two Gravity-Wave Detectors: A Comparison*

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The sensitivity of a dumbbell gravity-wave antenna is compared with that for a cylindrical detector. It is concluded that a Weber cylindrical antenna is decidedly the more sensitive at mutually accessible frequencies, particularly if the detector is to operate at higher frequencies in addition to the fundamental. A dumbbell antenna does offer the possibility of sampling the very low-frequency end of the spectrum which is inaccessible to cylinders.

In the current period of activity following the pioneering work of Weber[1,2] in gravity-wave detection, considerable discussion is being generated concerning antenna design.[3-10] As a possible alternative to a cylinder, a dumbbell (two large masses connected by a rod) is considered here as a possible gravity-wave detector.[6] We compare this alternative antenna to the more familiar cylinder without idealizing either of these detectors as two masses connected by a spring. Analysis of longitudinal elastic vibrations resulting from gravity pulses and thermal noise shows superior sensitivity for cylinders, particularly if a single detector is to operate at several frequencies. A dumbbell may, however, provide a way to observe low-frequency (~100-Hz) gravitational radiation which is in practice inaccessible to cylinders.

The dimensional parameters for the detectors are depicted in Fig. 1. All dependence of the elastic oscillations on directions perpendicular to the horizontal symmetry axes of the detectors is ignored. This is equivalent to setting the Poisson ratio equal to zero. Any attempt at a realistic description of the elastic modes in such de-

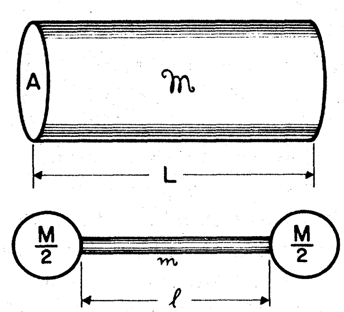

FIG. 1. Two gravity-wave detectors with their relevant parameters.

tectors must take into account these other directions as well as the detector suspension. For the present purposes of comparison this further detail is unnecessary. The end masses $(M/2)$ of the dumbbell in Fig. 1 are also assumed to be point masses without internal degrees of freedom.

The boundary conditions which supplement the standard wave equation of elasticity neglecting damping are as follows: The center of mass remains stationary. For the cylinder the stress vanishes at the ends. For the dumbbell the stress at an end must equal the inertial force on the end mass. These conditions lead to the normal frequencies of the detectors in the usual way. By using the eigenfunction solutions thereby obtained, the total energy in these detectors may then be expressed as a sum over the energies in the individual modes. For purposes of comparison we may choose the initial conditions as is convenient and thus take the time derivative of the displacement to be initially zero. With v denoting the longitudinal sound velocity, E denoting the energy in the nth mode, the superscript C (D) denoting cylinder (dumbbell), $n=0, 1, 2, \cdots$ for the cylinder and $n=1, 2, \cdots$ for the dumbbell, we obtain for the displacement amplitudes

$$A_n^{\,C} = Lv^{-1}(E/\mathfrak{M})^{1/2}2\pi^{-1}(2n+1)^{-1}, \tag{1a}$$

$$A_0^{\,D} \simeq lv^{-1}(E/m)^{1/2}(M/2m)^{1/2}, \tag{1b}$$

$$A_n^{\,D} \simeq lv^{-1}(E/m)^{1/2}\pi^{-1}n^{-1}. \tag{1c}$$

The strain amplitude in each of the detectors at the center is

$$S_n^{\,C} = 2v^{-1}(E/\mathfrak{M})^{1/2}, \tag{2a}$$

$$S_0^{\,D} = 2v^{-1}(E/m)^{1/2}2^{-1/2}, \tag{2b}$$

$$S_n^{\,D} = 2v^{-1}(E/m)^{1/2}. \tag{2c}$$

One notes that for equal amounts of energy deposited, the strain in a dumbbell exceeds that in a cylinder by essentially the factor $(\mathfrak{M}/m)^{1/2}$, where $\mathfrak{M}$ is the same order as M. This fact is of course a two-edged sword. For a given energy deposited from a gravity wave the strains developed in the dumbbell are larger, but then again so are those strains resulting from thermal-noise energy. We return to the thermal-noise question in a later paragraph and consider first the energy deposited from a gravity-wave pulse.

The average energy deposited in the nth mode in a time Δt is given by $E=\Delta t\int I(\nu)\sigma_n\,d\nu$, where $I(\nu)$ is the intensity of the gravity wave in ergs $\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}\ \mathrm{Hz}^{-1}$ and σ_n is the detector cross section for the nth mode averaged over direction. We assume that $I(\nu)$ is approximately constant over any bandwidth where the cross section is appreciable and compute the frequency-averaged cross section following the outline of Refs. 7 and 8. Proceeding in this fashion and using Eqs. (1) we obtain a ratio of the signal displacement amplitude in a dumbbell to that in a cylinder which depends only on detector parameters and varies essentially as $(l/L)(M/m)^{1/2}$ for comparison of lowest modes. Although the signal displacement amplitude of the dumbbell for $l>L$ and $(M/m) \gg 1$ can be larger for the dumbbell than for a cylinder, the displacement amplitude due to thermal noise makes the total displacement amplitude signal-to-noise ratio less for the dumbbell than for the cylinders as is shown below. The signal strain amplitudes for the lowest mode of the dumbbell and the cylinder are independent of the lengths and masses and are comparable in magnitude. In the higher modes both the strain and the displacement amplitudes are considerably reduced in the dumbbell as compared to the cylinder by the ratio m/M as well as a decrease due to mode number dependence.

Using Eqs. (1) and (2) with the equipartition theorem, one obtains expressions for the displacement and strain fluctuation due to thermal noise. Defining the signal-to-noise ratio as S/N, we obtain for detectors at the same temperature the following comparisons valid for *both* displacement and strain amplitudes:

$$\frac{(S/N)_0^{\,D}}{(S/N)_0^{\,C}} \simeq \frac{\pi}{4}\left(\frac{2m}{\mathfrak{M}}\right)^{1/2}, \tag{3a}$$

$$\frac{(S/N)_n^{\,D}}{(S/N)_{n'}^{\,C}} \simeq \frac{1}{2\pi}\frac{m}{M}\left(\frac{m}{\mathfrak{M}}\right)^{1/2}\frac{2n'+1}{n^2}. \tag{3b}$$

For the situation considered here where $m \ll M \simeq \mathfrak{M}$, one readily sees that in all cases the signal-to-noise ratio of a dumbbell is less than that of a cylinder. In particular, when the higher modes of a dumbbell are compared with the lower modes of a cylinder the signal-to-noise ratio of the dumbbell compared to the cylinder is even worse because of the mode number factors in Eq. (3b). That the higher mode response of a dumbbell should be severely reduced is readily understood when one views the dumbbell as approximated by two masses connected by a massless spring. Such a system has only a fundamental mode with frequency $\tilde{\omega}$ determined by the spring and no higher modes. The right-hand side of Eq. (3a) may in fact be written in terms of the parameters of such an equivalent mass-spring system as

$$\pi^2 4^{-1}(M/2\mathfrak{M})^{1/2}(l/L)(\tilde{\omega}/\omega_c^{\,0}),$$

where $\omega_c^{\,0}$ is the fundamental frequency of the cylinder. As m/M becomes smaller one would expect such an approximation to be even more meaningful.[11]

In making these comparisons of signal to noise in various modes it has been assumed, for reasons of simplicity and lack of observational information, that the gravity-wave intensities at the corresponding frequencies are equal. For comparisons corresponding to quite different frequencies the signal-to-noise ratios in Eqs. (3) are multiplied by the ratio of intensities for any real spectrum. If this ratio strongly favors low frequencies, we see from Eq. (3a) that it is possible for a dumbbell operating in its lowest mode to have sensitivity comparable to a cylinder operating at a much higher frequency.

Since both displacement[9,10] and strain[1-5] detection have been proposed, both have been considered here. The question of Q dependence, phase relationship between signal and noise, time resolution, and transducer effects have not been included in the present work since the point of interest here is in a sensitivity comparison between two types of gravitational-wave detectors and not in the calculation of absolute sensitivities. A clever detection system could conceivably improve the relative sensitivity of a dumbbell to a cylinder.[12]

We conclude that cylinders are likely to be superior to dumbbells as gravity-wave antennas in any frequency range which is mutually accessible.

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