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MODAL RESULTS FOR ACTIVE CONTROL OF ENERGY DENSITY IN A RECTANGULAR ENCLOSURE

Scott D. Sommerfeldt[1], John W. Parkins[2], and Jiri Tichy[2]

[1]Dept. of Physics
277 Fletcher Building
Brigham Young University
Provo, UT 84602

[2]Graduate Program in Acoustics
The Pennsylvania State University
P.O. Box 30
State College, PA 16804

INTRODUCTION

There has been considerable research performed relative to applications involving active attenuation of the sound field in enclosures. Much of this work has focused on the optimal number and locations of the control sources and error sensors. Typically, a number of microphones are used as the error sensors, and the sound field is controlled by minimizing the sum of the squared pressures from these microphones. In recent years, an alternative sensing approach has been developed, based on minimizing the acoustic energy density at the error sensor location(s)[1,2]. This new approach has been tested both numerically and experimentally, with the results indicating that one can often achieve improved global attenuation of the field by minimizing the acoustic energy density, rather than the sum of the squared pressures.

Previous results from minimizing the energy density at the error sensors have concentrated on investigating the control that can be achieved by looking at the global energy in the field before and after control, and also by looking at the attenuation that can be achieved as a function of frequency. However, it has also been found that additional insight can be gained by examining the acoustic field in terms of the acoustic modes contributing to the acoustic field. This paper will present some of the numerical results obtained by performing a modal decomposition of the field with no control, as well as with several different control methods. These results provide insight into the control mechanisms and provide indications as to why one can often achieve improved global attenuation by minimizing the acoustic energy density rather than the squared pressure.

MODAL REPRESENTATION OF THE FIELD

The pressure field in a rectangular enclosure can be represented in terms of the modes of the enclosure as

$$p(\vec{x}) = \sum_{N=0}^{\infty} \left(A_N + B_N Q_c\right) \Psi_N(\vec{x}) \ . \quad (1)$$

Here, N denotes a triple sum over the indices (l,m,n) corresponding to the x-, y-, and z- directions. The functions Ψ_N correspond to the eigenfunctions of the enclosure, Q_c designates the complex control source strength, and the coefficients A_N and B_N are the modal coefficients associated with the primary field and the secondary control field, respectively. (The source strength of the primary source is included in the A_N coefficients, and a single primary source and control source are assumed here for simplicity.)

In this paper, three different performance functions are investigated in terms of the modal decomposition of the acoustic field. The first performance function corresponds to the global potential energy in the enclosure, which will correspond to the minimum possible energy in the enclosure for the given source configuration. The second performance function investigated corresponds to the squared pressure at a discrete location. While this approach is very straightforward for experimental implementation, it often leads to the production of localized zones of silence, rather than the broad global attenuation desired. The third performance function investigated corresponds to minimizing the total energy density at a discrete location. This approach also makes use of a local measurement, but prior research has indicated that minimizing the energy density often yields more global attenuation than minimizing the squared pressure.

Using the expression for the squared pressure in Eq. (1), the three performance functions can be minimized to yield the optimal control source strengths for each of the control methods. The results of this minimization can be expressed as

$$
\begin{aligned}
Q_{c,pe} &= -\frac{\sum_{N=0}^{\infty} B_N^* A_N}{\sum_{N=0}^{\infty} B_N^* B_N} \equiv Q_{c,opt} \\
Q_{c,p} &= -\frac{\sum_{i=1}^{I}\sum_{N=0}^{\infty} A_N \Psi_N(\vec{x}_i)}{\sum_{i=1}^{I}\sum_{N=0}^{\infty} B_N \Psi_N(\vec{x}_i)} \\
Q_{c,e} &= -\frac{\sum_{i=1}^{I}\sum_{N=0}^{\infty}\sum_{M=0}^{\infty} A_N B_M^* \left[\Psi_N(\vec{x}_i)\Psi_M(\vec{x}_i) + \frac{1}{k^2}\nabla\Psi_N(\vec{x}_i)\cdot\nabla\Psi_M(\vec{x}_i)\right]}{\sum_{i=1}^{I}\sum_{N=0}^{\infty}\sum_{M=0}^{\infty} B_N B_M^* \left[\Psi_N(\vec{x}_i)\Psi_M(\vec{x}_i) + \frac{1}{k^2}\nabla\Psi_N(\vec{x}_i)\cdot\nabla\Psi_M(\vec{x}_i)\right]}
\end{aligned}
\qquad (2)
$$

From these expressions, it can be seen that the optimal control source strength for each of the control methods depends on the modal amplitudes of the acoustic field in a different manner. In addition, the methods of minimizing the squared pressure ($Q_{c,p}$) and the energy density ($Q_{c,e}$) both depend on the sensor location, in conjunction with the modal amplitudes, while the method of minimizing the potential energy does not depend on any sensor location. By investigating the control results in terms of the modal amplitudes, it is possible to gain some insights into some of the control effects associated with each of these control methods. This in turn may yield understanding related to why minimizing the energy density may yield improved global attenuation over minimizing the squared pressure.

NUMERICAL RESULTS

The dimensions of the enclosure modeled here are 1.93 m × 1.54 m × 1.22 m, and rigid wall boundary conditions have been assumed. The lowest order modes corresponding to this enclosure are presented in Table 1. For the results presented here, the primary source is located at (0.1,0.4,0.4) and the secondary control source is located at (1.4,1.0,1.0). For the methods of minimizing the squared pressure and the energy density, the error sensor is located at (1.2,0.6,0.6). Two results are presented here to illustrate two of the control mechanisms. The first case is for an excitation frequency of 166.3 Hz, which corresponds to the (1,0,1) mode (sixth mode), while the second case is for an excitation frequency of 125 Hz and lies between the (0,1,0) mode (third mode) and the (0,0,1) mode (fourth mode). The attenuation of the global potential energy in the enclosure achieved using each of the three control methods investigated is shown in Table 2.

The first case, with an excitation frequency of 166.3 Hz, corresponds to an on-resonance excitation condition. The results of the modal decomposition for the uncontrolled field and for the various control methods are shown in Figs. 1 and 2. It can be seen that the sixth mode (1,0,1) is dominant in this case. The method of minimizing the potential energy, using $Q_{c,opt}$, results in a field where the dominant mode has been attenuated with only minor effects on the other modal amplitudes and phases. The method of minimizing the squared pressure leads to poor results in this case, largely due to the fact that the error sensor is near a nodal plane at this frequency. As a result,

Table 1. Mode Frequencies of the Rectangular Enclosure.

Mode Number	Mode Indices	Modal Frequency (Hz)
1	(0,0,0)	0
2	(1,0,0)	88.9
3	(0,1,0)	111.4
4	(0,0,1)	140.6
5	(1,1,0)	142.5
6	(1,0,1)	166.3
7	(2,0,0)	177.7
8	(0,1,1)	179.3
9	(1,1,1)	200.1
10	(2,1,0)	209.7
11	(0,2,0)	222.7
12	(2,0,1)	226.6

Table 2. Attenuation of the Global Potential Energy (dB).

Frequency (Hz)	Potential Energy	Squared Pressure	Energy Density
125	14.3	-13.6	11.2
166.3	0.15	-20.9	0.15

the effect of the control is to increase virtually all of the modal amplitudes shown here. Minimizing the energy density leads to control that is comparable to minimizing the potential energy in terms of the global attenuation. It can be seen, however, that there is some error in achieving the proper phase for the sixth mode, leading to an overall attenuation that is about 3 dB less than when minimizing the potential energy. Thus, in this case where the control mechanism is modal attenuation, the energy density method is able to successfully attenuate the field, while the square pressure method is not successful, due to the poor sensor location. Of course, one could optimize the error sensor location for the given frequency and largely overcome this difficulty, but for complex fields with many frequency components, this could be difficult to achieve for all frequencies.

The results for the off resonance case (125 Hz) are shown in Figs. 3 and 4. By examining the results obtained when minimizing the potential energy, it can be seen that the control mechanism involved is primarily that of modal redistribution. i.e. the modal amplitudes are not changed significantly, but the modal phases are adjusted to lead to some destructive interference between the modes. Given the off resonance condition, it is also not possible to achieve significant attenuation for this source configuration. When controlling the energy density, it can be seen that the controlled field is very similar in terms of the amplitudes and phases of the modes of the enclosures. Thus, this approach is able to redistribute the modes in an optimum fashion to yield maximum attenuation of the potential energy. However, when controlling the squared pressure, the resulting amplitudes and phases of the modes are significantly different, and lead to a control solution that greatly increases the global energy in the enclosure.

CONCLUSIONS

It has been demonstrated that in controlling an enclosed field, the desired mechanism of control sometimes corresponds to modal attenuation and at other times corresponds to modal redistribution. The results presented here indicate how the method of controlling the energy density is able to often more faithfully achieve the desired control condition in terms of the modal characteristics of the field. If one is minimizing the squared pressure, one can achieve the desired control results by carefully optimizing the number of sensors and their locations. However, when minimizing the energy density, the attenuation achieved is generally much less sensitive to the sensor locations, and thus the method is often able to achieve the desired control without the careful sensor optimization required for minimizing the squared pressure.

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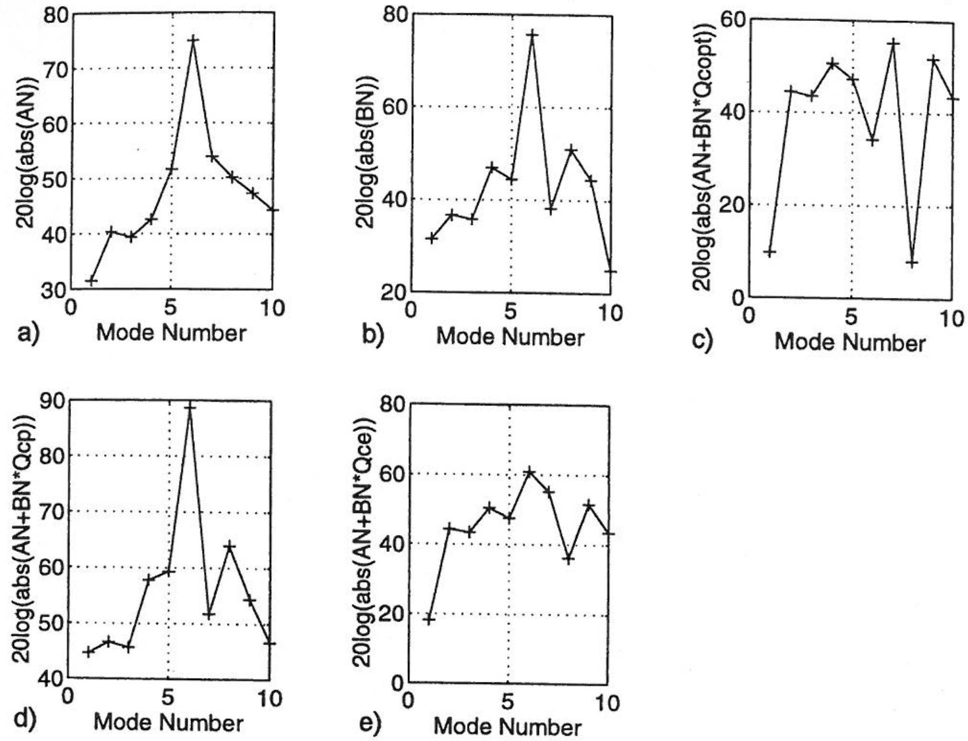

Figure 1. Modal amplitudes for an excitation frequency of 166.3 Hz. a) uncontrolled, b) B_N coefficients, c) minimization of potential energy, d) minimization of squared pressure, e) minimization of energy density.

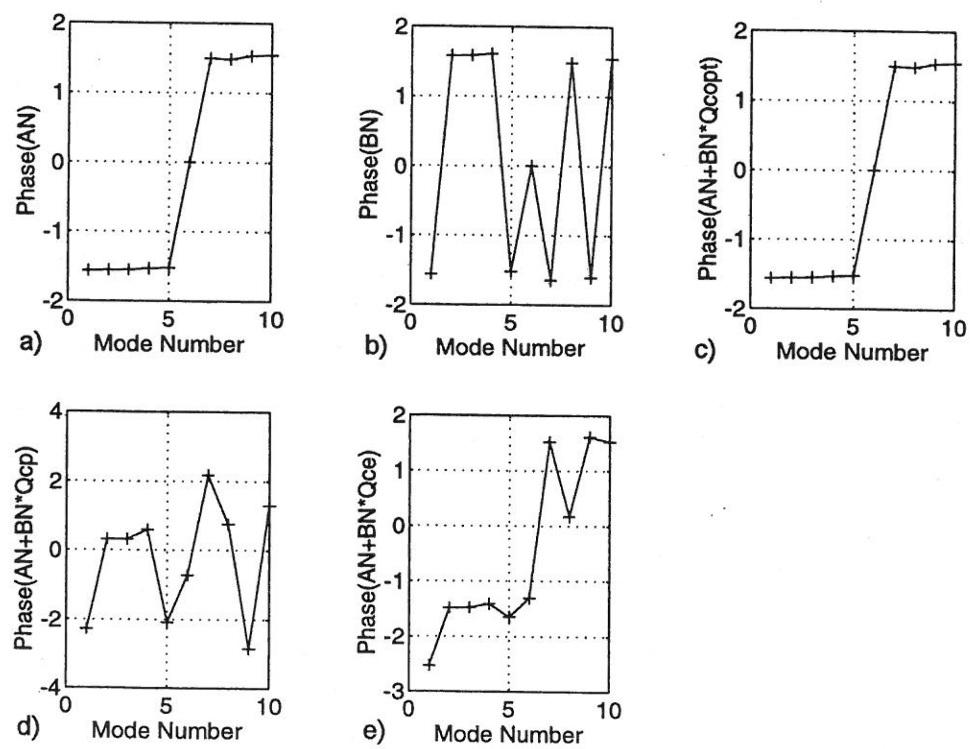

Figure 2. Modal phases for an excitation frequency of 166.3 Hz. a) uncontrolled, b) B_N coefficients, c) minimization of potential energy, d) minimization of squared pressure, e) minimization of energy density.

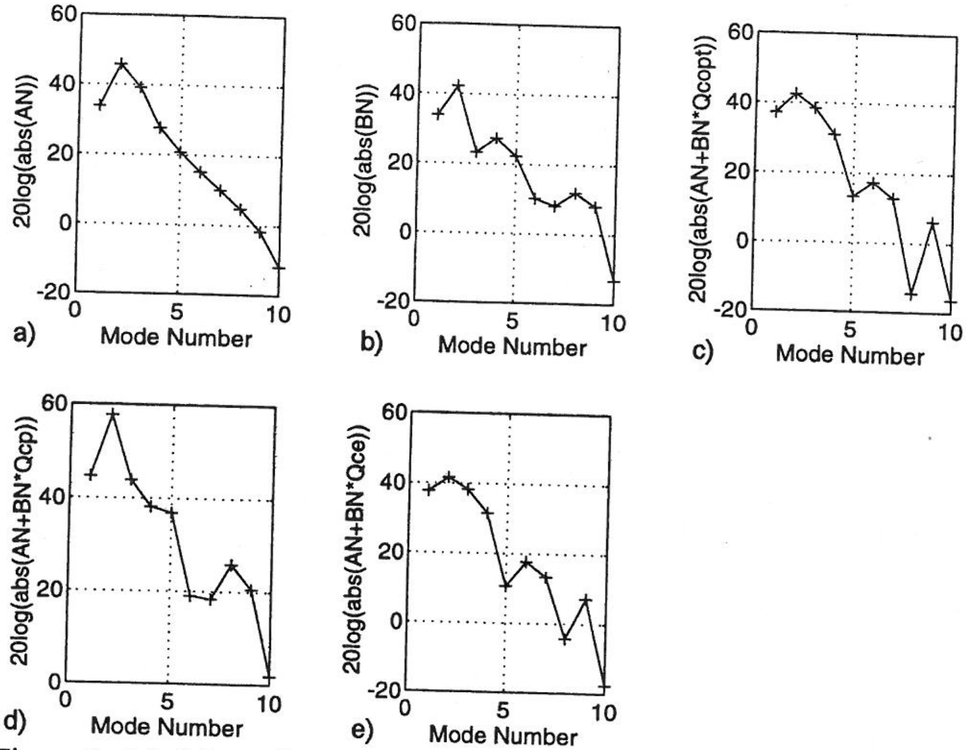

Figure 3. Modal amplitudes for an excitation frequency of 125 Hz. a) uncontrolled, b) B_N coefficients, c) minimization of potential energy, d) minimization of squared pressure, e) minimization of energy density.

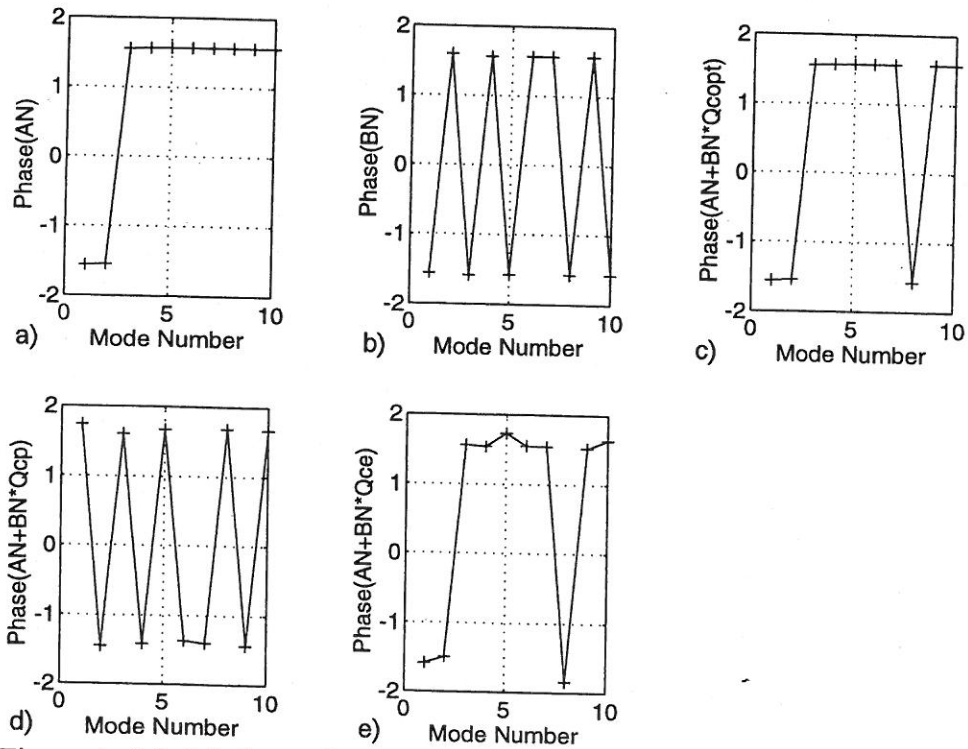

Figure 4. Modal phases for an excitation frequency of 125 Hz. a) uncontrolled, b) B_N coefficients, c) minimization of potential energy, d) minimization of squared pressure, e) minimization of energy density.