# Modeling the pedestal in a chirped-pulse-amplification laser

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## ABSTRACT

The pedestal (pre-pulse and post-pulse) associated with a chirped-pulse-amplification (CPA) laser is modeled. In this model, the power gain can be treated as a function of instantaneous frequency due to the large chirp of this type of laser. It is found that self-phase-modulation (SPM) can play an important role determining the final shape of the compressed pulse, even at relatively low values of the cumulative B-integral, $B \lesssim 2$.

## 1. INTRODUCTION

The study of high-density plasma physics,[1] and ultrafast x-ray emission,[2,3] using high-power, ultrashort laser pulses is of great current interest. Detailed knowledge of the temporal shape of the pulse is crucial. High-density plasma physics experiments require a laser pulse with high-intensity contrast. Short pulse x-ray conversion efficiency may be improved with an appropriate pre-pulse.[4] A fully controllable pulse shape is desirable for the studies of these fields.

CPA laser systems allow the production of high-power, ultrashort pulses in solid-state lasing media.[5] Before amplification, a laser pulse is frequency chirped and temporally expanded using a fiber,[6] and/or an expansion grating pair.[7] The longer pulse allows more energy to be extracted from the subsequent amplifier system than would be by a short pulse. After amplification, the pulse is compressed by a compression grating pair[8] to picosecond or subpicosecond duration. There is a resulting increase in power equal to the chirp ratio,[9] the chirped-pulse duration divided by the compressed-pulse duration. In this type of laser, the compressed pulse may carry an significant amount of energy which remains uncompressed. This uncompressed portion is commonly referred to as the pedestal.[10]

The purpose of this paper is to provide an intuitive way of understanding the pedestal of a CPA laser. The model is based mainly upon time domain analysis. The instantaneous frequency, defined as the time derivative of the light-field phase, is used as an approximation of Fourier-transform frequency to simplify the analysis. The validity of this approximation is based on the assumption that the time scale of the envelope variation of the laser pulse is much larger than the dephasing time of the gain medium. The details of this treatment are derived in the appendix. Within this model, the following topics are studied: (a) the power spectrum of a linearly chirped pulse, (b) pulse compression and the resulting pedestal, (c) spectral gain-narrowing effect in amplification process, (d) SPM induced from the amplifier, (e) the results of the frequency mis-match between the seeding pulse and the gain profile of the amplifier.

## 2. MODEL

The model considers a laser pulse, $E(z, t) = E_1(z, t)\, e^{i[\omega_0 t + \phi_m(t)]}$, traveling in the +z direction through a gain medium with a small-signal power-gain coefficient $\alpha(\omega)$. The input pulse is $E(0, t) = A_1(t)\, e^{i[\omega_0 t + \phi_m(t)]}$, where $A_1(t)$ is the amplitude of this laser pulse. The time derivative of the initial phase variation $\phi_m(t) = 1/2\, b(t/\tau)^2 + \phi_{NL}(t)$, describes the frequency chirp of the input pulse, where $\tau$ is a scale length of the pulse width, b defines the amount of linear chirp, and $\phi_{NL}(t) = O[(t/\tau)^3]$ represents the possible nonlinear frequency chirp generated from the optical fiber or expansion gratings. Dispersion effects and transverse intensity variations are neglected to simplify the analysis. Since preserving the initial frequency chirp is necessary for a CPA laser, the role of SPM becomes important. The nonlinear dependence of the index of

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refraction on applied signal strength is given by the optical Kerr effect,[9] $n = n_0 + n_{2E}\langle E^2\rangle = n_0 + n_{2E}|E_1|^2/2 = n_0 + n_2 I$. The basic equation governing the light propagation in an amplifier including SPM is (see appendix)

$$\frac{\partial E_1(z,t)}{\partial z} = \frac{\alpha[\omega_i(t)]}{2} E_1(z,t) - i\frac{\beta_2}{2}|E_1|^2 E_1(z,t), \tag{1}$$

where $\beta_2 = 2\pi n_{2E}/\lambda_0$, and $\lambda_0$ is the laser wavelength in vacuum. Note that z and t refer to coordinates in the moving pulse frame, which is defined by the transformation, $z = \hat{z}$ and $t = \hat{t} - \hat{z}/c$, where $\hat{z}$ and $\hat{t}$ are ordinary laboratory coordinates and c is the velocity of light in the amplifier. The instantaneous frequency is defined by $\omega_i(t) \equiv \partial\phi_{tot}/\partial t$, where $\phi_{tot}$ is the total field phase variation. When the SPM is still small, such that the instantaneous frequency $\omega_i(t)$ can be approximated by $\omega_0 + d\phi_m(t)/dt$ with no z dependence, Eq. (1) can be solved to obtain[6]

$$E_1(z,t) = A_1(t)\exp[\alpha(t)z/2]\exp[-i\phi_{spm}(z,t)], \tag{2}$$

with the phase modulation,

$$\phi_{spm}(z,t) = (1/2)[\beta_2/\alpha(t)]|A_1(t)|^2\{\exp[\alpha(t)z]-1\}, \tag{3}$$

where $\alpha(t)$ is short for $\alpha[\omega_i(t)]$. Equation (3) is nothing but the result of the B-integral with time dependence. For a multiple-pass amplifier with a single-pass power gain equal to $G(t) \equiv \exp[\alpha(t)L]$, where L is the length of the laser amplifier, and power loss equal to 1-R, the laser field is amplified M times to become

$$E_1(t) = A_1(t)[RG(t)]^{M/2}\exp[-i\phi_{spm}(t)], \tag{4}$$

with

$$\phi_{spm}(t) = (1/2)[\beta_2/\alpha(t)]|A_1(t)|^2[G(t)-1]\{[RG(t)]^M - 1\}/\{[RG(t)]-1\}. \tag{5}$$

In a regenerative amplifier, the transverse mode of the laser pulse is confined to the lowest-order mode, or 00 mode. Transverse variations of the laser pulse cause different SPM at each transverse position.[11] Since our purpose is to avoid SPM in amplification process, the peak intensity near the beam-axis is most important.

## 2.1. Power spectrum

The Fourier transform of a linearly chirped pulse $A(t)\exp\{i[\omega_0 t + 1/2\, b(t/\tau)^2]\}$ is

$$\int A(t)\exp\{i[\omega_0 t + 1/2\, b(t/\tau)^2]\}\, e^{-i\omega t}\, dt = \exp[-i\omega'^2\tau^2/(2b)]\int A(t)\exp[i(b/2)(t/\tau - \omega'\tau/b)^2]\, dt\,, \tag{6}$$

where $\omega'$ is short for $\omega - \omega_0$. The integral on the right hand side of Eq. (6) has a similar form to Huygens' integral in the Fresnel approximation. This approximation is commonly used in diffraction theory. For light with wavelength $\lambda$, the diffracted field located at a distance z away from the source is given by[9]

$$E(x') = (\lambda z)^{-1/2}\exp[i(2\pi z/\lambda - \pi/4)]\int E(x)\exp[i\pi(x/a - x'/a)^2 a^2/\lambda z]\, dx \tag{7}$$

where a, the half-aperture width, is added here for comparison. Aside from the multiplicative factors preceding both integrals, the two expressions are similar with the chirp parameter b equal to $2\pi$ times the Fresnel number $a^2/\lambda z$ in the diffraction analogy. Just as the power spectrum of a Fourier-transform-limited optical pulse can be compared to Fraunhofer diffraction, the power spectrum of a linearly chirped pulse can be compared to Fresnel diffraction.



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Following Eq. (6), the power spectrum of the amplified chirped pulse from Eq. (4), including SPM, and nonlinear chirp is proportional to

$$\left| \int A_1(t)[RG(t)]^{M/2} \exp\{ i[\phi_{NL}(t) - \phi_{spm}(t)] \} \exp[i(b/2)(t/\tau - \omega'\tau/b)^2] \, dt \right|^2 . \quad (8)$$

The analogy is that the light field "diffracts" from the time domain to frequency domain. The initial pulse envelope is the light source. The gain function is a soft aperture which shapes the source pulse. The higher order nonlinear chirp $\phi_{NL}(t)$ introduces aberration near edges. The phase variation due to SPM, $\phi_{spm}(t)$, is a phase plate which diffracts the light to the frequency domain and changes the power spectrum. When a sharp change in the pulse envelope occurs, the induced phase variation $\phi_{spm}(t)$ also change sharply. Thus, the power spectrum changes significantly even when the peak phase variation due to SPM is relatively small. This is similar to a phase plate, which diffracts light significantly at a sharp change in thickness, even for a small phase change. The chirp parameter b which behaves like the Fresnel number, is quite large in CPA lasers (for example, b = 300), and thus the power spectrum is analogous to the near-field diffraction pattern of the pulse envelope.

### 2.2. Pulse compression and pedestal

For a large-chirp pulse, pulse compression is performed by using a pair of gratings to eliminate the phase term, $\exp[-i\omega'^2\tau^2/(2b)]$, in front of the integral at the right-hand side of Eq. (6). Applying the inverse Fourier transform to this spectrum and using the amplified chirped pulse described by Eq. (4), we can obtain the compressed pulse

$$E(t') = (ib/2\pi\tau^2)^{1/2} \exp[-i\ 1/2\ b(t'/\tau)^2] \exp(i\omega_0 t') \cdot \int A_1(t)[RG(t)]^{M/2} \exp\{ i[\phi_{NL}(t) - \phi_{spm}(t)] \} \exp[ibtt'/\tau^2] \, dt . \quad (9)$$

If we neglect the the nonlinear chirp and phase modulation in the integral, the compressed pulse envelope is a Fourier transform of the chirped-pulse envelope.[8] Thus, the chirped-pulse envelope $A_1(t)[RG(t)]^{M/2}$ should be kept near Gaussian, which leads to a Gaussian compressed pulse. The nonlinear chirp $\phi_{NL}(t)$ and phase modulation $\phi_{spm}(t)$ "diffuse" the light in time and form the pedestal of the compressed pulse. This is analogous to focusing a beam (one transverse dimension). When a beam is focused, the focal spot is the Fourier transform of the input beam spatial profile. When phase aberrations occur, the focal spot becomes larger and some low intensity light is diffracted away from focal spot.

### 2.3. Gain narrowing

To explain the pulse-shaping by spectral gain-narrowing effect, we begin from Eq. (4), with an initial laser pulse having a super-Gaussian envelope $A_1(t) = A_1 \exp[-(1/2)(t/\tau)^{2m}]$. The rise time of this super-Gaussian pulse can be estimated by $\tau_r \equiv \tau/m$.[6] In a system which uses expansion gratings after a fiber to obtain large frequency chirp, the laser-pulse shape changes depend on the amount of stretching. The super-Gaussian envelope may no longer be a good model for the pulse seeded into regenerative amplifier, but the heuristic analysis in this paper can still be used. The instantaneous frequency is $\omega_i(t) = \omega_0 + bt/\tau^2 + O[(t/\tau)^2]/\tau$. When the bandwidth of the applied signal $\Delta\omega_L$ is much smaller than the atomic linewidth of the gain medium $\Delta\omega_a$, the gain coefficient can be simplified with a quadratic form, $\alpha[\omega_i(t)] = \alpha_0 - (\alpha''/2)[\omega_i(t) - \omega_a]^2 \approx \alpha_0 - (\alpha''/2)(\omega_0 + bt/\tau^2 - \omega_a)^2$, where $\alpha'' = -d^2\alpha(\omega)/d\omega^2$ evaluated at midband frequency $\omega_a$. In the frequency-matched case, $\omega_0 = \omega_a$, the total power gain after M passes through the amplifier is

$$G(t)^M = G_0^M \exp(-t^2/\tau_{eff}^2) \quad (10)$$

where $G_0 \equiv \exp(\alpha_0 L)$ is the single-pass power gain at midband, and $\tau_{eff} \equiv (2\tau^4/M\alpha''Lb^2)^{1/2}$ represents the shortened half-width of the chirped pulse after M passes.



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A suitable pulse-shaping is accomplished when $\tau > \tau_{eff}$, that is, the sharp edges of the super-Gaussian pulse and the unwanted phase variation near the edges, $\phi_{NL}(t)$, are suppressed by the Gaussian gain curve. With reasonable estimations $b \approx \tau\Delta\omega_L/2$, and $\alpha'' \approx 8\alpha_0/(\Delta\omega_a)^2$, which is exact for a Lorentzian lineshape, the inequality becomes

$$M\alpha_0 L > (\Delta\omega_a/\Delta\omega_L)^2. \tag{11}$$

This equation shows the condition (lower bound) for a significant gain-narrowing effect as a pulse-shaping tool in the amplifier. In this case we can drop the super-Gaussian term in Eq. (4). If SPM is still negligible, the applied signal becomes a complex Gaussian pulse,[9] $E(t) \approx A_1(RG_0)^{M/2} \exp[-1/2\ (t/\tau_{eff})^2] \exp\{i[\omega_0 t + 1/2\ b(t/\tau)^2]\}$, which can then be compressed to a Fourier-transform-limited pulse with half-width (at 1/e intensity point) approximately equal to $(\tau/\tau_{eff})(\tau/b)$ in the large-chirp limit $b/(\tau/\tau_{eff})^2 \gg 1$. Here the factor $\tau/\tau_{eff} \approx (M\alpha_0 L)^{1/2}/(\Delta\omega_a/\Delta\omega_L)$ is the pulse broadening ratio for the compressed pulse, which is equal to the pulse shortening ratio of the chirped pulse in the amplifier due to the gain-narrowing. The power spectrum of this Gaussian pulse in the large-chirp assumption is proportional to $\exp[-(\tau^4/b^2\tau_{eff}^2)(\omega - \omega_0)^2] = \exp[-1/2\ M\alpha''L(\omega - \omega_0)^2]$, which is exactly the lineshape $G(\omega)^M$, as expected.[10]

To illustrate the gain-narrowing effect, Fig. 1(a) shows the power spectra for different gain-narrowing conditions, $\tau/\tau_{eff} = 0$, 1, 2, and 3, with parameters $\lambda_0 = 10530$ Å, $\tau = 75$ ps, $m = 25$, $b = 236$, and a small amount of phase variation $\phi_{NL}(t) = -(t/\tau)^3 - 4(t/\tau)^4$. The bandwidth for the initial pulse ($\tau/\tau_{eff} = 0$) is 37 Å (FWHM), and this power spectrum looks like a diffraction pattern from a slit. The term $-(t/\tau)^3$ makes the spectrum asymmetric, and the term $-4(t/\tau)^4$ makes the spectrum more peaked near the two edges. Figure 1(b) shows the corresponding intensity profiles of the compressed pulses. For $\tau/\tau_{eff} = 0$ case, the compressed pulsewidth is 1.6 ps (FWHM) and the gross pedestal is the result of its original chirped pulse profile (nearly square top) and the nonlinear chirp. For other cases, the pedestal is significantly suppressed by spectral gain-narrowing as was found in Ref. 10.

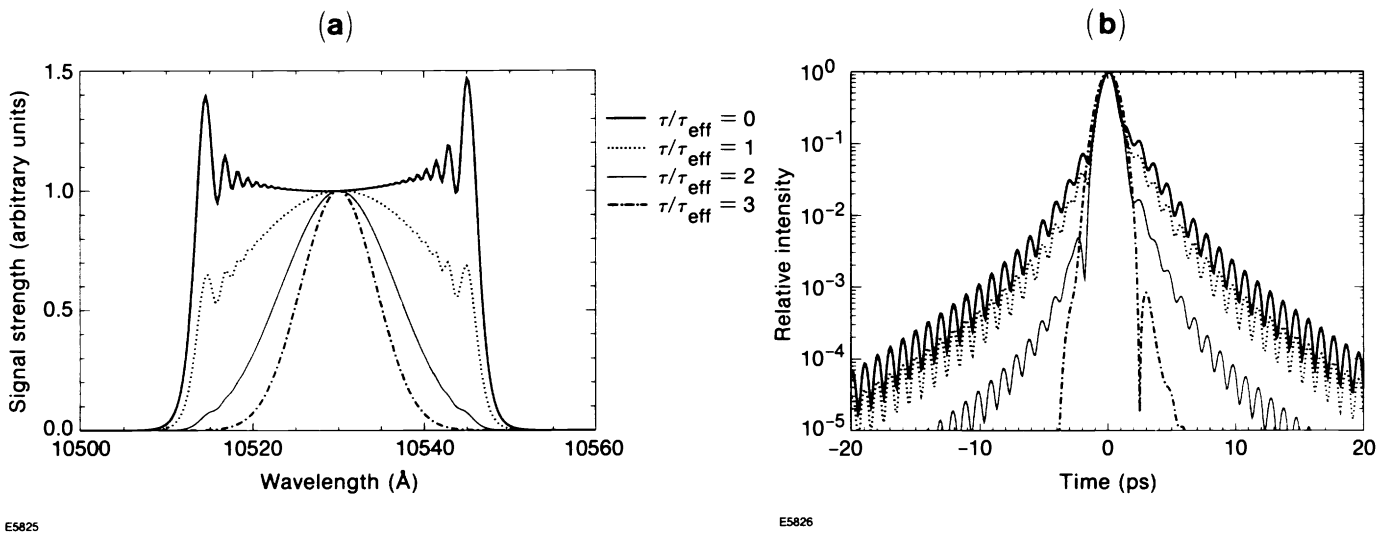



Fig. 1 (a) Power spectra for different spectral gain-narrowing conditions, $\tau/\tau_{eff} = 0$, 1, 2, and 3. They approach Gaussian in large gain narrowing limit. (b) The intensity profiles of compressed pulses. Pedestal suppression by spectral gain narrowing is shown.

## 2.4. Self-phase modulation

Self-phase modulation in CPA lasers is an important issue when the laser operates near gain saturation. Here the laser pulse has its highest intensity. The laser pulse experiences a time-varying phase modulation, $\phi_{spm}(t)$, produced by the intensity variation of the pulse itself. The new frequency chirp, $\Delta\omega_{spm} = -\partial\phi_{spm}/\partial t$, adds to the original linear frequency chirp of the laser pulse and changes the laser power spectrum. In the case where a pair of compression gratings is used to



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eliminate the linear frequency chirp and compress the laser pulse, this additional frequency chirp is still preserved in the compressed pulse. Since the additional frequency chirp has different group velocity delay than that of the linear chirp in the compression stage, it introduces a new pedestal to the compressed pulse.

Before saturation, the calculated $\phi_{spm}$ can be obtained from Eq. (5). Near saturation we can use the small gain approximation, z = L, $\exp[\alpha(t)L] \approx 1+\alpha(t)L$, and $1/2\ n_{2E}\ |A_1(t)|^2 = n_2 I$ in Eq. (3). The induced SPM in the amplifier (one trip) is $\phi_{spm}(t) = 2\pi n_2 I(t)L/\lambda_0$. The added frequency chirp is $\Delta\omega_{spm}(t) = -(2\pi n_2 L/\lambda_0)\partial I(t)/\partial t$. The maximum frequency shift will occur at the points of maximum slope of intensity I(t) or maximum dI(t)/dt. The frequency shift is strongly enhanced when the initial pulse has a sharp change in its temporal envelope. To amplify an optical pulse which is spectrally broadened and chirped in an optical fiber leading to a square top in time, one should be very carefully to avoid SPM occurring near rising or falling edges of the chirped pulse. For a super-Gaussian pulse $I(t) = I_0\exp[-(t/\tau)^{2m}]$, the maximum frequency chirp, which occurs at $t = [1 - 1/(2m)]^{1/(2m)}\ \tau$, is[6]

$$\Delta\omega_{spm} = fm\ (\phi_{max}/\tau) = f\ \phi_{max}/\tau_r \tag{12}$$

where $\phi_{max} \equiv 2\pi n_2 I_0 L/\lambda_0$ and f is a constant given by the form $2s^s \exp(-s)$ with s = 1 - 1/(2m). The numerical value of f depends on m only slightly; f = 0.86 for m = 1 and tends toward 0.74 for large values of m. Thus, the added frequency chirp for a super-Gaussian pulse is m times higher than for a Gaussian pulse (m = 1).

Gain-narrowing in the amplifier which reshapes the pulse can also reduce $\Delta\omega_{spm}$. The laser intensity given by Eq. (4) is $I(t) = I_0\exp[-(t/\tau)^{2m} - (t/\tau_{eff})^2]$, where $I_0$ is the peak laser intensity after M trips through the amplifier. With this intensity profile, the new frequency chirp after one additional trip through the amplifier, evaluated at $t = \tau$, is

$$\Delta\omega_{spm} = 0.74[m + (\tau/\tau_{eff})^2]\ \exp[-(\tau/\tau_{eff})^2](\phi_{max}/\tau). \tag{13}$$

Equation (13) compared to Eq. (12) shows that the frequency chirp is reduced a factor $\exp[-(\tau/\tau_{eff})^2]$ from large m and small gain-narrowing limits.

In the large gain-narrowing limit, the pulse becomes Gaussian after M trips through the amplifier. The points of maximum slope of intensity move to $t = \pm t_{eff}/\sqrt{2}$. Instead of Eq. (13), the new maximum frequency chirp after one more trip through the amplifier is $\Delta\omega_{spm} = 0.86(\phi_{max}/\tau_{eff})$.

To illustrate the roles of SPM in a CPA laser, Figure 2(a) shows the intensity profiles of ultrashort pulses compressed from chirped Gaussian pulses (m = 1, $\tau$ = 100, b = 200) under a variety of SPM conditions, i.e., peak B-integral values B = 0, 2, 4, 6. Figure 2(b) shows the intensity profiles of ultrashort pulses compressed from chirped super-Gaussian pulses (m = 5, $\tau$ = 100, b = 200) under the same SPM conditions as for Fig. 2(a). In both figures changes in the pulse shape are clearly shown. Note that because of the group velocity delay in grating pairs, the new frequency chirp $\Delta\omega_{spm}$ determines the width of the pedestal. The energy associated with the pedestal is determined by the fraction of the pulse energy near the maximum of $\partial I/\partial t$. In the case of a super-Gaussian pulse of high order, only a small fraction of the pulse energy is in the region of large $\partial I/\partial t$, whereas for a Gaussian pulse (m = 1), a much larger fraction of the pulse energy is contained within the region of large $\partial I/\partial t$. Thus, for a Guassian pulse, while the added frequency chirp may be smaller than for a super-Gaussian pulse, the effect may be as important because of the larger amount of energy involved. These B-integral values can occur quite easily in a multiple-pass regenerative amplifier operating near gain saturation.[11] For example, a laser pulse at $\lambda_0$ = 1053 nm with peak intensity equal to 1 GW/cm$^2$ traveling through an amplifier with the nonlinear index of refraction $n_{2E} = 1.2 \times 10^{-13}$ esu (for $Nd^{+3}$-doped phosphate-glass Q98), and with the amplifier length L = 115 mm, the peak SPM is equal to 0.22. Thus, at this intensity level, a laser pulse can accumulate significant amount of SPM in several passes through the amplifier, and the resulting new frequency chirp will increase the pedestal of the compressed pulse.



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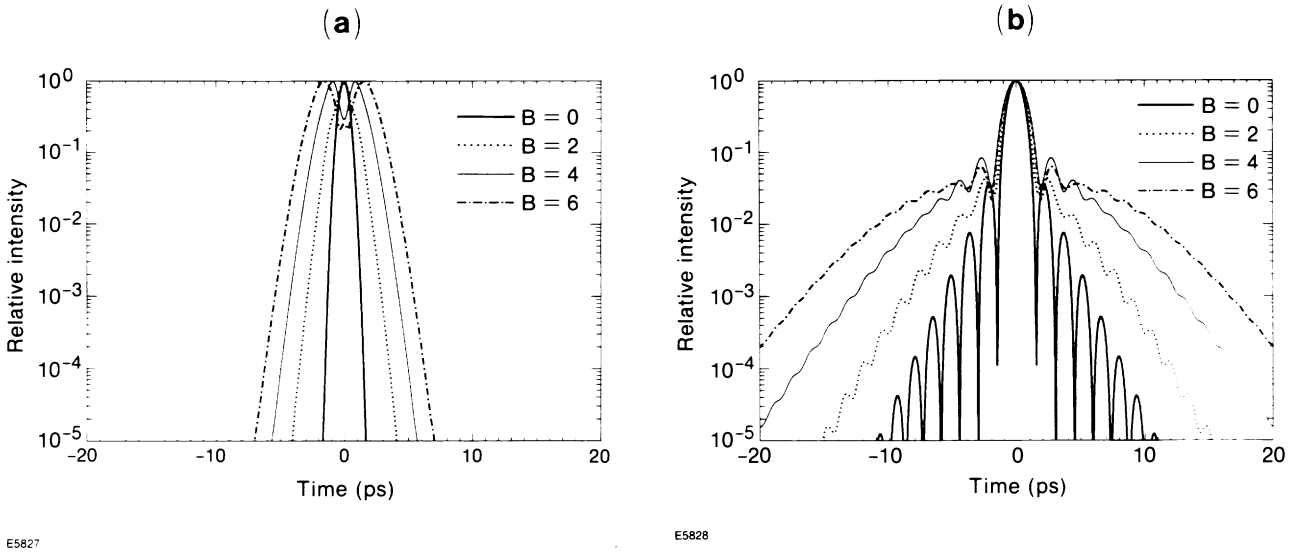



Fig. 2 (a) Intensity profiles of ultrashort pulses compressed from chirped Gaussian pulses (m = 1, $\tau$ = 100, b = 200) under a variety of SPM conditions, i.e., peak B-integral values B = 0, 2, 4, 6. (b) Intensity profiles of ultrashort pulses compressed from chirped super-Gaussian pulses (m = 5, $\tau$ = 100, b = 200). In both figures changes in pulse shape are clearly shown.

### 2.5. Frequency mismatch

If a frequency mismatch exists between the seed pulse and the gain profile of the amplifier, i.e., $\omega_0 \neq \omega_a$, one of the sharp edges of the super-Gaussian pulse (which also carries higher order nonlinear chirp) cannot be suppressed efficiently through gain narrowing. Thus, the pulse-shaping is poor and SPM remains strong at the sharp edge. To avoid SPM, the energy extraction efficiency in the amplifier must be lowered. The total power gain in this case is given by

$$G(t)^M = G_0^M \exp[-(t - t_0)^2/\tau_{eff}^2] \tag{14}$$

where $t_0 = \tau^2(\omega_a - \omega_0)/b \approx 2(\omega_a - \omega_0)\tau/\Delta\omega_L$. If we set a criterion $\tau$-$\tau_{eff} > |t_0|$ for the allowed frequency mismatch for pulse shaping, then Eq. (11) becomes

$$M\alpha_0 L > [\Delta\omega_a/(\Delta\omega_L - 2|\omega_a - \omega_0|)]^2. \tag{15}$$

Equation (15) compared to Eq. (11) shows that due to frequency mismatch, the required power gain $\exp(M\alpha_0 L)$ for pulse shaping becomes higher. The final power spectrum will be narrower as well.

The degree of $\Delta\omega_{spm}$ is affected by frequency mismatch. The laser intensity given by Eq. (4) becomes $I(t) = I_0 \exp[-(t/\tau)^{2m} - (t - t_0)^2/\tau_{eff}^2]$. With this intensity profile, the new frequency chirp after one more trip through the amplifier evaluated at $t = \tau$ is

$$\Delta\omega_{spm} = 0.74[m + (\tau - t_0)\tau/\tau_{eff}^2] \exp\{-[(\tau - t_0)/\tau_{eff}]^2\} \ (\phi_{max}/\tau). \tag{16}$$

In an extreme case where one spectral edge of the seed-pulse is located at the line-center of the amplifier, which is equivalent to $t_0 = \tau$, Eq. (16) becomes $\Delta\omega_{spm} = 0.74m(\phi_{max}/\tau)$. This shows that the amount of frequency-chirp due to SPM is not suppressed by gain-narrowing effect, and, it is nearly m times higher than that of a Gaussian pulse at same intensity level.



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## 4. APPENDIX

In this appendix we derive Eq. (1) through the use of the instantaneous frequency approximation. The applied signal and the resulting electric polarization can be written in a general form,

$$E(z,t) = E_0(z,t)\, e^{i\omega_0 t}$$

$$P(z,t) = P_0(z,t)\, e^{i\omega_0 t} \tag{A1}$$

where $\omega_0$ is the center frequency (carrier frequency) of the laser light, and z, t refer to coordinates in the moving pulse frame. The three basic equations[9] which govern the light propagation in the amplifier are the resonant-dipole equation

$$\frac{\partial^2 P(z,t)}{\partial t^2} + \Delta\omega_a \frac{\partial P(z,t)}{\partial t} + \omega_a^2 P(z,t) = -KN(z,t)E(z,t), \tag{A2}$$

the population-inversion equation

$$\frac{\partial N(z,t)}{\partial t} = i\frac{2^*}{4\hbar}\left[E_0^*(z,t)P_0(z,t) - E_0(z,t)P_0^*(z,t)\right], \tag{A3}$$

and the radiation transport equation

$$\frac{\partial}{\partial z}E_0(z,t) = -i\frac{\omega_0}{2\varepsilon c}P_0(z,t) - i\frac{\beta_2}{2}|E_0(z,t)|^2 E_0(z,t), \tag{A4}$$

where $\omega_a$ and $\Delta\omega_a$ are atomic transition frequency and atomic linewidth, N(z,t) is the number density of the inverted population, K is a constant, 2* is a dimensionless population saturation factor with values between 1 and 2, depending on the gain medium.[9] We have used the slowly varying envelope approximation (SVEA) to simplify Eqs. (A3) and (A4). In Eq. (A3), pumping effects and spontaneous emission are neglected due to the short time scale of amplification. The nonlinear term in the radiation transport Eq. (A4) is added due to optical Kerr effect in the host medium (for example, glass of Nd:glass amplifier). We have neglected dispersion ($\partial^2 E_0/\partial t^2$) in Eq. (A4).

By using SVEA, the resonant-dipole Eq. (A2) can be simplified to

$$\frac{\partial P_0(z,t)}{\partial t} + \frac{\Delta\omega_a}{2}\left[1 + i\frac{2(\omega_0 - \omega_a)}{\Delta\omega_a}\right]P_0(z,t) = i\frac{K}{2\omega_0}N(z,t)E_0(z,t). \tag{A5}$$

Due to the frequency chirp, with

$$E_0(z,t) = E_1(z,t)\, e^{i\phi m(t)}$$

and

$$P_0(z,t) = P_1(z,t)\, e^{i\phi m(t)}, \tag{A6}$$



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the resonant-dipole equation becomes

$$\frac{\partial P_1(z,t)}{\partial t} + \frac{\Delta\omega_a}{2}\left\{1+i\frac{2[\omega_i(t)-\omega_a]}{\Delta\omega_a}\right\}P_1(z,t) = i\frac{K}{2\omega_0}N(z,t)E_1(z,t), \tag{A7}$$

where $\omega_i(t) = \omega_0 + \partial\phi_m(t)/\partial t$ is the instantaneous frequency. If

$$\frac{\partial P_1(z,t)}{\partial t} \ll \frac{\Delta\omega_a}{2} P_1(z,t) \approx \frac{P_1(z,t)}{T_2}, \tag{A8}$$

where $T_2 \approx 2/\Delta\omega_a$ is the dephasing time of the resonant dipoles,[9] then, we can simplify Eq. (A7) as

$$P_1(z,t) = i\left(\frac{K}{\omega_0\Delta\omega_a}\right)\left\{\frac{1}{1+i\frac{2[\omega_i(t)-\omega_a]}{\Delta\omega_a}}\right\}N(z,t)E_1(z,t). \tag{A9}$$

The limitation (A8) sets the validity for using the instantaneous frequency as the Fourier-transform frequency. The time scale of the change of the laser pulse envelope should be larger than the dephasing time of the resonant dipoles. The physical explanation is that the dephasing time of the resonant dipole is so short compared to the laser pulse duration that the oscillating dipole tends to "forget" the history of the laser field, and only resonates with the instantaneous frequency of the field. The existence of phase modulation $\phi_m$ does not change the basic forms of Eqs. (A3) and (A4). We can simply replace $E_0(z,t)$ and $P_0(z,t)$ by $E_1(z,t)$ and $P_1(z,t)$, respectively. If the gain medium is homogeneous, by substituting Eq. (A9) into the radiation transport Eq. (A4) with $E_0(z,t)$ replaced by $E_1(z, t)$, we obtain Eq. (1), with

$$\alpha(\omega_i) = \frac{K}{\varepsilon c\Delta\omega_a}\left\{\frac{1-i\frac{2(\omega_i-\omega_a)}{\Delta\omega_a}}{1+\left[\frac{2(\omega_i-\omega_a)}{\Delta\omega_a}\right]^2}\right\}N(z,t) = \alpha_r + i\alpha_i . \tag{A10}$$

The imaginary part of the gain coefficient $\alpha_i$ describes the phase added (per distance) by the resonant dipoles. This phase shift has a linear relation with the instantaneous frequency near the gain center $\omega_a$. If the laser spectrum is narrower than the bandwidth of the gain profile, then the phase shift induced by the resonant dipoles in the amplifier is

$$\phi_{amp}(t) = \frac{\alpha_i L}{2} \approx -\frac{\omega_i(t)-\omega_a}{\Delta\omega_a}\alpha_0 L . \tag{A11}$$

For a linearly chirped pulse, the reactive frequency shift compared to the laser bandwidth is $[\partial\phi_{amp}(t)/\partial t]/\Delta\omega_L \approx (\Delta\omega_L/\Delta\omega_a)\alpha_0 L/(4b)$. Since the parameter b is large in a CPA laser, we can neglect this frequency shift.

By substituting Eq. (A9) into the population Eq. (A3), with $E_0(z,t)$ replaced by $E_1(z, t)$, we obtain

$$\frac{\partial N(z,t)}{\partial t} = -\frac{2^*}{\hbar\omega_0}\sigma[\omega_i(t)]N(z,t)I(z,t), \tag{A12}$$

where $I(z,t) = \varepsilon c|E_1(z,t)|^2/2$, is the laser intensity, and



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$$\sigma[\omega_i(t)] = \frac{K}{c\varepsilon\Delta\omega_a} \frac{1}{1+\left\{\frac{2[\omega_i(t)-\omega_a]}{\Delta\omega_a}\right\}^2} \tag{A13}$$

is the transition cross section as a function of the instantaneous frequency $\omega_i(t)$. To describe the gain saturation, one needs to incorporate Eq. (A12) with Eq. (1), with $\alpha = N\sigma$ for homogeneous gain media, or $\alpha = \int N(\xi)\sigma(\xi)d\xi$ for inhomogeneous gain media with site parameter $\xi$.

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