AIAA Aviation
16-20 June 2014, Atlanta, GA
20th AIAA/CEAS Aeroacoustics Conference

AIAA 2014-3199

Comparison of Two Time-domain Measures of Nonlinearity in Near-field Propagation of High-power Jet Noise

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Time-domain metrics are used to investigate the nonlinearity of the sound in the vicinity of the F-35 AA-1. The first measure considered is the average steepening factor (ASF), which we define as the inverse of the wave steepening factor and is a ratio of the expectation value of the positive slopes in the waveform to the expectation value of the negative slopes. The second nonlinearity metric is the skewness of the time derivative of the pressure waveform (derivative skewness), which describes the asymmetry of the distribution of slopes in the waveform. Spatial maps of both metrics applied to the F-35 AA-1 data reveal that regions of increasing derivative skewness correspond more closely to the maximum sound radiation area, whereas the largest values for the ASF seem aligned with the regions where the waveform amplitude distributions are most asymmetric. It is proposed that these two metrics reveal different characteristics of the nonlinear propagation of jet noise. The ASF is more representative of the average slopes, which are dominated by high frequencies. Conversely, the derivative skewness identifies of large positive slopes and hence relates to the shock content in the noise.

Nomenclature

ASF	=	Average steepening factor
$E[\cdot]$	=	Expectation operator
OASPL	=	overall sound pressure level, dB re 20 μPa
$p(t)$	=	pressure waveform, in Pa
$\partial p/\partial t$	=	time derivative of pressure, in Pa/s
$\mathrm{Sk}\{p(t)\}$	=	skewness of pressure, "pressure skewness"
$\mathrm{Sk}\{\partial p/\partial t\}$	=	skewness of pressure time derivative, "derivative skewness," see Eq. (3)
σ	=	distance relative to a shock formation distance
WSF	=	wave steepening factor, see Eq. (1)

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I. Introduction

NONLINEAR acoustic propagation is inherently a time-domain phenomenon, although its effect is often quantified in terms of its impact on the spectrum. Convective and temperature effects produce an amplitude-dependent sound speed that causes local distortion of the pressure waveform.[1] In the preshock region, nonlinear propagation alters the arrival times of acoustic pressure waveform values, but does not change the amplitude density itself.[2] Because of the shift in arrival times, metrics based on, or correlated with, the time derivative can be effective indicators of the nonlinear distortion present in a measured signal. Tracking of the time derivative with range allows for identification of cumulative nonlinear effects as shocks begin to form.

Nonlinear propagation of stationary, broadband noise signals may best be understood statistically, as different portions of the random waveform travel at different rates and shocks form at different distances. Amplitude, peak frequency, source extent and directionality, bandwidth, the Fourier spectral shape and phase, geometric spreading, atmospheric absorption, and other phenomena may all impact nonlinearity. The relative contributions of the various source characteristics and propagation phenomena make the characterization of nonlinearity in jet noise a challenging problem. There have been various studies examining nonlinear propagation effects in jet noise, beginning in the 1970's. Nonlinearity in full-scale jet noise was considered by Pernet and Payne,[3] who examined the anomalous lack of atmospheric absorption in jet noise flyover data. Blackstock[4] predicted nonlinear propagation from T-38 measurements, and Ffowcs Williams *et al*.[5] discussed nonlinear wave propagation in the context of jet crackle. Morfey and Howell[6] and Crighton and Bashforth[7] both attempted to develop spectrally-based nonlinear propagation models. Downing *et al*.[33] have identified probable nonlinear effects in legacy low bypass commercial engines. Far-field nonlinearity in modern tactical aircraft noise propagation has been modeled for the F-18,[8-11] F-22,[9,12,13] and F-35[14] and examined for a full-scale engine by Schlinker *et al.*[15] In laboratory-scale measurements, Gallagher and McLaughlin[16] and Gallagher[17] conducted early far-field nonlinear propagation studies, that were followed by Petitjean and McLaughlin[18] and Gee *et al.*[19] on heat-simulated Mach 1.5 jets, and Petitjean *et al*.[20] on an unheated Mach 1.92 jet.

It has been only relatively recently that researchers have begun to examine acoustical nonlinearities in near-field propagation. Analysis of near-field nonlinearity is complicated because assumptions like spherical spreading or propagation exclusively along a given observation radial may not be valid for some frequencies. Full-scale investigations have been carried out on F-35 and F-22 data[21,22] and reveal growth of cumulative nonlinear effects with distance, particularly in the aft direction. Shocks are not present at the source but are formed during the course of propagation. For laboratory-scale measurements, Gee *et al*. have reached a similar conclusion for a laboratory Mach 2.0 jet using three different analysis techniques – bicoherence analysis,[23] quadspectrum-based indicators,[24] and the skewness of the pressure time derivative (derivative skewness).[25] Mora *et al*. have used the derivative skewness for heated Mach 1.5 jets to show cumulative nonlinearities in the near field. Baars *et al*.[26,27] used the derivative skewness and another measure, the wave steepening factor (WSF) proposed by Gallagher,[16,17] to study the near and far-field acoustics of a Mach 3.0 unheated jet. They concluded, however, that cumulative nonlinear effects were not present in their data. Because of the conclusions of Baars *et al*., we now have greater uncertainty regarding the presence of nonlinear effects in laboratory jets.

This paper revisits the previous near-field nonlinearity analysis on F-35 AA-1 data, which employed the derivative skewness, and extends it to include a variation of the WSF and a more extensive discussion of the nonlinear behavior as a function of engine condition and angle. Quantitative comparisons to theoretical nonlinear acoustics studies by Muhlestein[28] and Muhlestein *et al.*[29] are made. The previous laboratory-scale studies that employed one or both of these indicators also are summarized. We find that these two metrics reveal different characteristics of the nonlinear propagation of jet noise.

II. Nonlinearity Metrics

A. Average Steepening Factor

The first nonlinearity metric we consider is the average steepening factor (ASF), which we define as the inverse of the WSF. The WSF was defined by Gallagher[16,17] as the absolute value of the ratio of the mean negative derivative in the waveform divided by the mean positive derivative, written as

$$\mathrm{WSF} = \left| \frac{E\left[\frac{\partial p}{\partial t}\Big|_-\right]}{E\left[\frac{\partial p}{\partial t}\Big|_+\right]} \right|. \tag{1}$$

Gallagher described its evolution for a nonlinearly propagating sine wave as beginning at 1 and then decreasing toward "approximately" zero for a pure N-wave. He applied it to different laboratory-scale supersonic jets, and some of the results are repeated in Table 1. The most significant change in WSF occurred for a Mach 2.5 jet excited at St=0.16 for which the OASPL at 30 D_j was 160 dB. He found WSF decreased from 0.73 to 0.48 between 30 and 80 D_j. Without the jet excitation though, the WSF increased between 30 and 80 D_j, indicating no nonlinear effects were observed. At a greater Reynolds number, there was a reduction in WSF for both the excited and non-excited jet, but the fact that WSF > 1 at 30 D_j for both cases is puzzling because it indicates that the negative slopes in the waveform at 30 D_j are, on average, greater than the positive slopes. We view this as unusual because the jet noise we have studied – both the pressure waveform and the derivative – is consistently either Gaussian or positively skewed (greater positive outliers) at both the laboratory and full scales and a WSF >1 would likely result in a negative skewness.

Table 1. Partial summary of Gallagher results.[17] *The WSF at 50 D_j for the Re=5e4 cases was 0.84 and 0.97 for the excited and natural jets. Gallagher attributes the subsequent increase in WSF between 50 and 80 D_j to wall reflections.

Jet Mach Number	Reynolds Number	Excitation Strouhal Number	OASPL at 30 D_j	WSF at 30 D_j	WSF at 80 D_j
2.5	8700	0.16	160	0.73	0.48
2.5	8700	--	154.5	0.80	0.93
2.5	50000	0.29	148.5	1.08	0.99*
2.5	50000	--	148	1.12	1.02*

Baars *et al.*[26] recently reintroduced the jet noise community to the WSF as part of their investigations of an unheated Mach 3.0 jet. They plotted a normalized WSF, which they described as $(1-\mathrm{WSF})/(1-\mathrm{WSF})_{\max} \times 10$. From visual inspection, their maximum WSF = 0.64 (which is $(1-\mathrm{WSF})_{\max} = 0.36$) aligned with the maximum OASPL direction (~45°) at a distance of ~115 D_j from an origin 20 D_j downstream. At a distance of 50 D_j, the normalized WSF was approximately 7.7, which corresponds to WSF = 0.72. At a radial distance of 35 D_j (the closest observation location given the nearest sideline distance of 25 D_j), WSF = 0.75. Thus, between 35 and 115 D_j along the peak radiation direction, there is a reduction in WSF of ~15%.

We consider the definition of WSF somewhat unfortunate, as one might naturally assume that a "wave steepening factor" would increase with additional waveform steepening, rather than decrease. For a pure N-wave, where the positive derivative goes to infinity, the steepening is infinite and therefore a more intuitive indicator of the steepening is the inverse of the WSF. Consequently, we have chosen to define the average steepening factor $\mathrm{ASF} = \mathrm{WSF}^{-1}$, such that it begins at 1 and then increases as the waveform steepens. Its name connotes not only the steepening indicator, but the fact that it is the ratio of two expectation values. For this definition, the Baars *et al.* result ranges from ASF = 1.33 to 1.56, which is more than a 17% increase over the propagation range.

The behavior of WSF was only loosely defined by Gallagher[16,17] as having a range of [1,0]. This leaves many important questions pertaining to its application unanswered. For example, how quickly should values change as shocks begin to form in the waveform? How does jet noise differ from a sine wave? What of finite sampling rates or a limited signal-to-noise ratio? These questions are essential, especially since Baars *et al.*[26] have contended that the WSF decrease seen in their data is inconsistent with cumulative nonlinear effects.

Muhlestein *et al.*[28,29] have investigated the theoretical behavior of the WSF (and ASF) from a theoretical nonlinear acoustics perspective and included finite sampling and signal-to-noise considerations in their investigations. One result of particular interest is the behavior of the ASF for the Earnshaw solution to the problem of lossless nonlinear propagation of a planar sinusoid. Muhlestein *et al.*[29] found it to be

$$\mathrm{ASF} = \frac{\pi + 2\sigma}{\pi - 2\sigma}, \qquad \sigma < \frac{\pi}{2}, \tag{2}$$

where σ is the propagation distance relative to the shock formation distance. For the lossless case, the shock constitutes the entire rise portion of the waveform for $\sigma \geq \pi/2$ (the N-wave case referred to by Gallagher), resulting in an infinite ASF (zero WSF). Regardless of the nature of the propagation, Eq. (2) provides insight into quantifying the nonlinear regime based on the ASF value. For example, in an initially sinusoidal waveform that has newly formed shocks consistent with those present at $\sigma = 1$, ASF = 4.5 from Eq. (2). How ASF changes for different types of waveforms, is more tenuous at present, but Muhlestein *et al*.[29] show trends for numerical noise propagation that are consistent with the initially sinusoidal behavior.

B. Derivative Skewness

The derivative skewness, or the skewness of the pressure waveform time derivative, is written for a zero-mean derivative process as

$$\mathrm{Sk}\left\{\frac{\partial p}{\partial t}\right\} = \frac{\mathrm{E}\left[\left(\frac{\partial p}{\partial t}\right)^3\right]}{\mathrm{E}\left[\left(\frac{\partial p}{\partial t}\right)^2\right]^{3/2}}. \tag{3}$$

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After its introduction by McInerny as a useful measure for high-amplitude aeroacoustic noise analysis,[30] the derivative skewness has been more extensively applied to jet and rocket noise to examine data for nonlinear and shock content.[9,13,19- 21,30-,33,,25,26] However, only recently have efforts been made to quantify its behavior relative to shock formation. Shepherd *et al*.[34] considered the change in waveform and derivative statistics for $\sigma < 1$ and found that a rapid, exponential increase in derivative skewness accompanied the formation of shocks. Muhlestein *et al*.[35] confirmed the behavior with a plane-wave tube experiment for both sinusoids and noise. The change in derivative skewness has been analytically treated by Muhlestein[28] and Reichman *et al*.[36] For $\sigma \ll 1$, $\mathrm{Sk}\{\partial p/\partial t\} \approx 3\sigma/\sqrt{2}$ and for $\sigma \to 1$, $\mathrm{Sk}\{\partial p/\partial t\} \approx (1-\sigma^2)^{-3/4}$. The derivative skewness has been used by Gee *et al.*[21, 22] to quantify the shocks present in the near field of high-performance tactical aircraft. They indicated that because of the significant growth of derivative skewness, jet crackle could be expected to evolve as a propagation phenomenon.[22]

Laboratory-scale studies have also shown growth of derivative skewness as a function of propagation range, particularly along the maximum radiation direction. Gee *et al*.[25] examined unheated Mach 2.0 (ideally expanded) and Mach 1.8 (overexpanded) laboratory-scale data from a 3.49 cm diameter nozzle and found increasing derivative skewness out to the maximum measurement distance of 75 D_j. For both the overexpanded and ideally expanded jets, the derivative skewness was maximum along the peak OASPL direction. Although the maximum $\mathrm{Sk}\{\partial p/\partial t\}$ values were only $\mathrm{Sk}\{\partial p/\partial t\} \approx 3$, these results upheld the general conclusions of the previous study involving F-35 data.[21] Mora *et al.*[31] used a 1.38 cm convergent-divergent conical nozzle in overexpanded, underexpanded, and on-design conditions to examine the effect of expansion and temperature on the change in higher-order statistics. They showed increases in derivative skewness and kurtosis along the maximum radiation angle using a dense near-field array out to a sideline distance of 18 D_j and a separate far-field arc at 81 D_j. For the near-field array, the maximum $\mathrm{Sk}\{\partial p/\partial t\}$ values were on the order of 1, whereas at the far-field arc, they were approximately 1.5, showing further increases between the near and far fields.

Baars *et al*.[26] also plotted a spatial map of $\mathrm{Sk}\{\partial p/\partial t\}$ for their 1 cm exit diameter, Mach 3.0 jet experiment. The maximum derivative skewness $\mathrm{Sk}\{\partial p/\partial t\} = 1.96$ occurred at about 120 D_j before decreasing, but at a shallower angle (~40°) than the WSF. At 35 D_j along 45° (sideline offset distance of 25 D_j), the value is about 61% of the maximum, or $\mathrm{Sk}\{\partial p/\partial t\} = 1.20$. Note that these values for the Mach 3.0 unheated jet are relatively consistent with the Mora *et al.* [31] 600 K jet that had a similar convective Mach number of approximately 1.3. For example, the Baars *et al*. map shows a derivative skewness of approximately 1.55 at 81 D_j along the maximum radiation direction. Note that the remainder of the Mora *et al*. measurements, which showed growth from zero skewness to approximately unity at 18 D_j, occur inside the first observation location of Baars *et al*. Thus, it is possible that for the Mach 3.0 jet, the most significant growth of $\mathrm{Sk}\{\partial p/\partial t\}$ occurred *before* the first observation location. The greater derivative skewness values seen in the Gee *et al*.[25] unheated supersonic data relative to the other laboratory studies could be caused by differences in jet conditions, jet scale, experimental setup, and/or sampling rate. Further investigations are required to elucidate these differences, but examination of ASF and $\mathrm{Sk}\{\partial p/\partial t\}$ at full-scale help provide baselines for laboratory-scale experiments.

III. F-35 AA-1 Measurement

The F-35 AA-1 static run-up measurements were conducted 18 October, 2008 at Edwards Air Force Base (EAFB), CA. The measurements were made jointly by the Air Force Research Laboratory, Blue Ridge Research and Consulting, LLC and Brigham Young University. Photographs of the tied-down aircraft are displayed in Figure 1.

Figure 1. Tied-down F-35AA aircraft, along with tripods of the near-field microphone array.

Measurements[37,14,25] were made using 6.35 mm Type 1 free-field and pressure microphones located at a height of 1.5 m (5 ft). The pressure microphones were oriented skyward, for nominally grazing incidence. The free-field microphones were pointed toward the plume, aimed at a microphone array reference position (MARP) approximately 6.7 m aft of the aircraft. This MARP, which is about 7-8 nozzle diameters downstream of the engine exit plane (the same scaled distance used for a previous F-22A experiment in 2004),[12] was set as the origin for defining observation angles. The microphone array represents the most spatially extensive measurements of a military jet aircraft to date. During the test, the average wind speed was less than 1 kt and the ambient pressure was virtually constant at 0.914 kPa. Temperature and relative humidity varied from 7 – 16 °C and 21-27%, respectively.

Data acquisition for the array described in this paper (the "near-field" array involving all the microphones closer than 76 m)[37] was carried out using a National Instruments® 8353 RAID server connected to a PXI chassis containing PXI-4462 cards. Analog input ranges for each channel were adjusted (in 10 dB increments) for low and high-power settings, based on the sensitivity of each microphone, in order to maximize the dynamic range of each of the 24-bit cards. The system sampling frequency was varied between 96 and 204.8 kHz. The lower sampling rate was required because of slower hard drive write speeds for the early-morning tests while the data acquisition system was cold and during afterburner, where system vibration was greater. The data acquisition system was located forward of the aircraft and to the sideline (about 70°) at an approximate distance of 35 m. Data sampled at 96 kHz are described in this paper.

As examples of waveform characteristic shapes and derivatives, data from the near-shear layer and 38 m microphones along 120° are displayed in Figure 2 – Figure 5. Figure 2 and Figure 3 are from 50% ETR, where there is nearly zero pressure skewness and only slight positive derivative skewness at the closest microphone, which is at a radial distance of 4.6 m from the angle origin. At 38 m, the derivative skewness has unmistakably increased, with large, infrequently occurring values indicating waveform steepening. Figure 4 and Figure 5 are from 130% ETR, which reveal both significant pressure and derivative skewness. Visual inspection of the waveforms at 4.6 m shows positively skewed waveforms with relatively large derivative values, but without the "N-wave" shape associated with typical acoustic shocks. The scenario has changed dramatically by 38 m, however. There are multiple shocks present in the waveform, which are easily identified in the time derivative of the waveform. The derivative skewness increases from 3.1 to 9.7 through the course of propagation away from the jet. Further spatial analyses as a function of engine condition and inclusion of ASF in the analysis are shown in the next section.

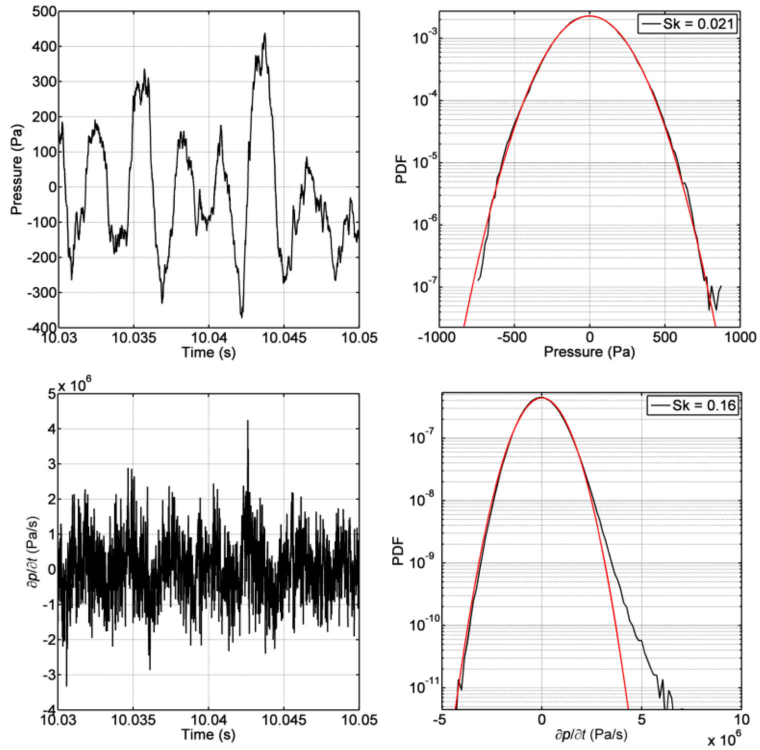

Figure 2. Waveform and derivative segments, along with the 30 s PDFs, for the closest microphone (4.6 m) along the 120° radial at 50% ETR. Gaussian PDFs are also shown and the pressure and derivative skewness are given in the respective legends.

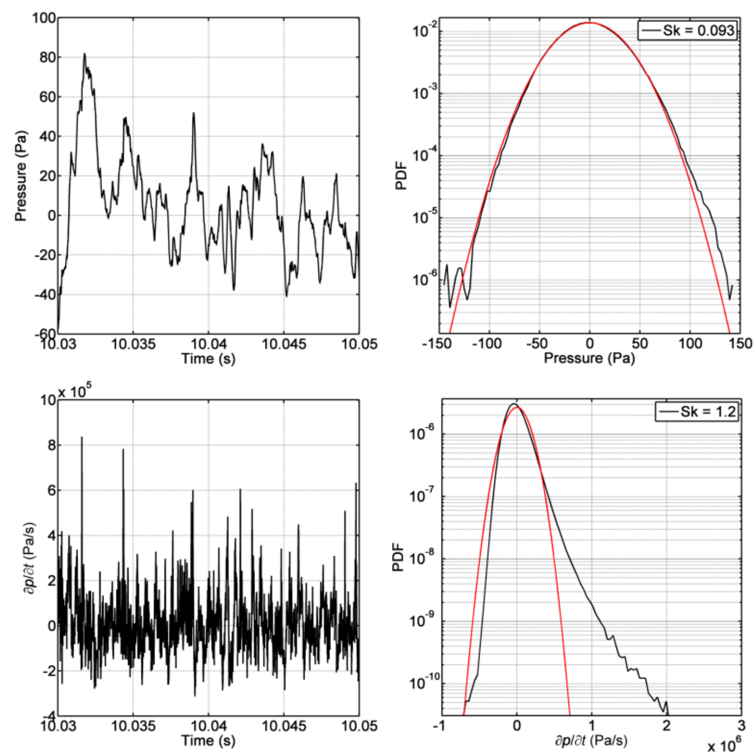

Figure 3. Waveform and derivative segments, along with the 30 s PDFs, for the 38-m microphone along the 120° radial at 50% ETR.

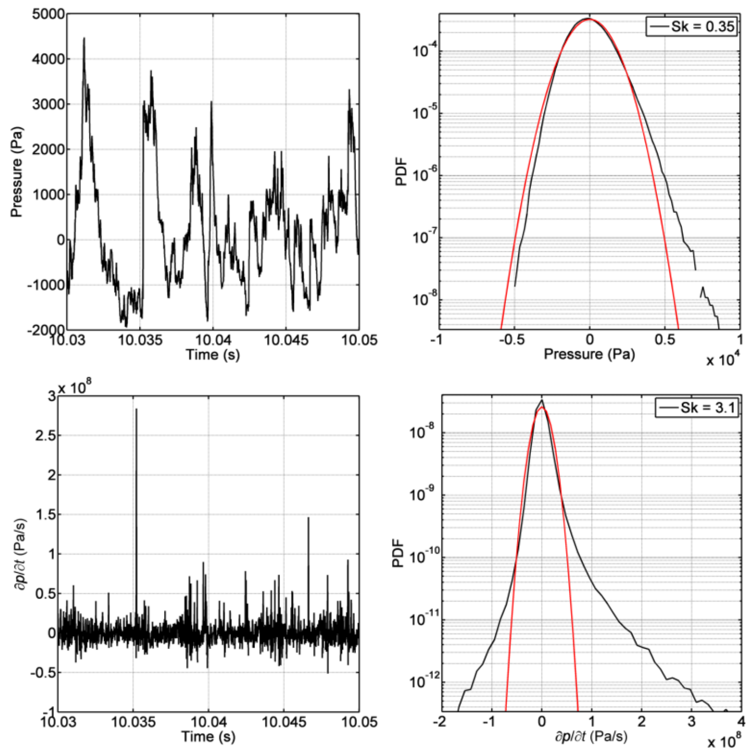

Figure 4. Waveform and derivative segments, along with the 30 s PDFs, for the closest microphone (4.6 m) along the 120° radial at 130% ETR.

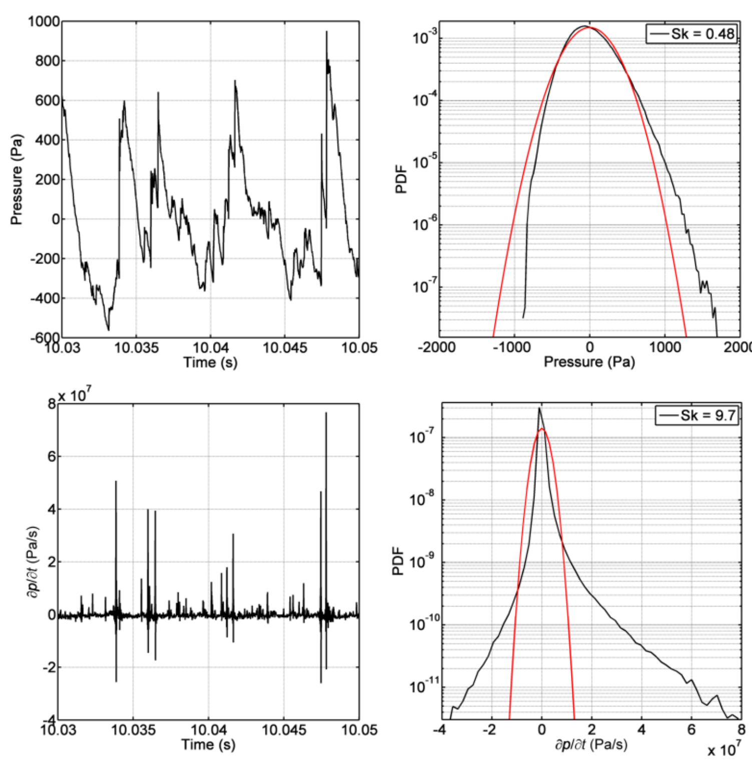

Figure 5. Waveform and derivative segments, along with the 30 s PDFs, for the 38-m microphone along the 120° radial at 130% ETR.

IV. Results and Analysis

The primary purpose of this paper is to apply the ASF to the F-35 AA-1 data, compare its behavior to the characteristics of $\mathrm{Sk}\{\partial p/\partial t\}$, and to extract insight regarding the radiated field as a function of engine condition. Connections to prior model-scale studies are also considered. Figure 6 – Figure 11 contain spatial maps of OASPL, $\mathrm{Sk}\{p(t)\}$, $\mathrm{Sk}\{\partial p/\partial t\}$, and ASF for six engine conditions of the F-35 AA-1 from 25% to 150% ETR (full afterburner). Microphone locations are shown and a cubic interpolation is used between data locations. The nature of the cubic interpolation is not physical, as it does not assume propagation along a radial, but rather triangulates between data points. Idle is not shown because of its similarity to 25% ETR. Note that Gee *et al.*[38] previously discussed different percentile-based crest factors present as a function of engine condition, and $\mathrm{Sk}\{p(t)\}$ and $\mathrm{Sk}\{\partial p/\partial t\}$ have been shown for 50% ETR and/or 100% ETR in conjunction with discussions on crackle and near-field shock formation. [21,22] For 25% ETR, where the inlet noise dominates the jet noise and thus can be considered a "low-amplitude" benchmark, the skewness, derivative skewness, and ASF are all essentially negligible. Note that although the derivative skewness and ASF are different quantities with different color scales, their spatial behavior is similar for this case. As engine power increases, the maximum pressure skewness, derivative skewness, and ASF all increase. The pressure skewness, $\mathrm{Sk}\{p(t)\}$, which has been shown by Gee *et al.*[22] to not correlate well with nonlinearity and shock formation, does not have distinct propagation trends. It has been included here because of the previous studies that have included its calculation and its relevance in describing correlation between the directionality in growth of the different parameters.

In comparing the characteristics of $\mathrm{Sk}\{\partial p/\partial t\}$ and ASF over the range of engine conditions, we first note that both measures grow with range, particularly in the aft direction. As mentioned previously, the nonphysical nature of the interpolation possibly creates nulls in the more sparsely populated regions, but the trends are still clear. Given that $\mathrm{Sk}\{\partial p/\partial t\}$ growth has been previously shown to be correlated with increased nonlinearity, these results are firm validation of the potential utility of ASF (or WSF) as a nonlinearity measure. This validation is required given that Baars *et al.*[26] concluded they did not see cumulative nonlinear distortion with their examination of WSF and the original Gallagher study[17] produced mixed results depending on jet excitation and Reynolds number. Regarding engine condition, at high powers, the increase in maximum ASF for high powers is very small. For example, between 75% and 130% ETR, the maximum derivative skewness decreases slightly but the angular range above a threshold of significant derivative skewness (e.g., $\mathrm{Sk}\{\partial p/\partial t\} > 4$) increases by more than 20°. Furthermore, the minimum derivative skewness is appreciably larger at all angles for 130% ETR. The same trend is true for ASF. Although the maximum ASF increase is not as large, both the minimum ASF and the angular range above a threshold of 1.4 increase with engine condition.

The spatial dependence of the derivative skewness and ASF, although similar, are not the same. The maximum ASF grows more slowly with engine condition – at 25% it exhibits essentially Gaussian statistics and ASF = 1. On the other hand, at military power, which represents a >40 dB increase in OASPL, the maximum average steepening only approaches ASF ≈ 2. The derivative skewness on the other hand, increases two to three orders of magnitude. This points to a difference in sensitivity that should be accounted for when ASF used as a nonlinearity indicator. Additionally, note that the ASF is generally broader around its maximum growth direction for the higher engine power. This width appears to be caused by the linear dependence of ASF on the derivative values, whereas $\mathrm{Sk}\{\partial p/\partial t\}$ depends on their cube. Thus, the derivative skewness is more sensitive to the formation and strengthening of shocks, which correspond to large values of the derivative.

To more quantitatively evaluate nonlinearity changes as a function of range and engine condition, Table 2 summarizes the growth of the two nonlinearity measures in the aft jet noise radiation direction. The table contains the minimum ASF and derivative skewness from the two microphones closest to the shear layer and nozzle exit and the maximum value found along the 38-m arc. Also included is the percentage increase for each engine condition and nonlinearity measure. Note the monotonic growth in ASF at each set of microphone locations as a function of engine condition. Considering the increase between the near-shear-layer and the 38 m microphones, the ASF growth is negligible for the low-amplitude benchmark of 25% ETR, is moderate for 50% ETR, with a 22.1% increase, and then ranges from a 59.8% to 75.3% increase for the higher power conditions. For $\mathrm{Sk}\{\partial p/\partial t\}$, the dependence on the cube of the derivative values results in much greater percentage increases. With increasing engine power, there is a monotonic increase of $\mathrm{Sk}\{\partial p/\partial t\}$ in the near-shear-layer data. At 38-m, however, 75% ETR has the second largest value, exceeded only by maximum afterburner (150% ETR). The reason for the apparent decrease in derivative skewness growth for 100% ETR is not clear at the present. It may relate to a physical phenomenon, but also may simply reveal a sensitivity of $\mathrm{Sk}\{\partial p/\partial t\}$ values as shocks begin to form and large positive outliers are emphasized. In any case, the 75 –150% ETR data at 38 m represent significant increases from near-shear-layer behavior, with values that are significantly greater than those seen in the laboratory.

Table 2. Summary of ASF and $\mathrm{Sk}\{\partial p/\partial t\}$ growth as a function of engine condition from the minimum at the two microphones closest to the shear layer to the maximum at 38 m in the maximum jet noise direction between 90 and 150°.

ETR (%)	ASF			$\mathrm{Sk}\{\partial p/\partial t\}$		
	Near shear-layer Min	*38 m, 90-150° Max*	*Percentage Increase*	*Near shear-layer Min*	*38 m, 90-150° Max*	*Percentage Increase*
25	1.00	1.02	1.97%	2.80e-03	0.070	2380%
50	1.03	1.26	22.1%	0.161	1.62	909%
75	1.11	1.77	59.8%	0.709	10.2	1340%
100	1.13	1.99	75.3%	1.36	8.71	540%
130	1.21	2.02	67.1%	3.00	9.65	221%
150	1.25	2.04	63.2%	3.34	11.0	228%

Thus far, the primary consideration has been the growth of the two nonlinearity measures with distance. An additional difference between derivative skewness and ASF is the directionality of the maximum growth direction. The results of Baars *et al.*[26] for their Mach 3.0 jet show that the maximum ASF occurs at an angle approximately 5° greater (relative to the jet exhaust centerline) than the maximum derivative skewness. Although the F-35 AA-1 measurement density is not as finely resolved as one would like, the ASF growth appears to also be centered 5-10° farther toward the sideline than $\mathrm{Sk}\{\partial p/\partial t\}$. The 130% ETR result shows that the derivative skewness follows the OASPL direction, whereas there is a closer correlation between ASF and the pressure skewness. Some insights into the difference between the ASF and the $\mathrm{Sk}\{\partial p/\partial t\}$ may be gained by considering this correlation further.

For a heated supersonic jet, the maximum directivity is associated with Mach wave radiation produced by relatively coherent turbulent structures within the jet. These quasi-planar wavefronts lend themselves to waveform steepening and shock formation as has been observed in computational studies, Schlieren photography, and evidence from the acoustic data. It is this Mach-wave based shock formation that is emphasized by $\mathrm{Sk}\{\partial p/\partial t\}$, with its emphasis of the positive derivative outliers. One may then ask, why then does the pressure skewness peak upstream of the OASPL? In examining shock formation for ideally versus overexpanded model-scale jets, Gee *et al.*[21] filtered a skewed waveform to show that the waveform skewness is caused by relatively low-energy, high-frequency components that generally have a maximum directivity upstream than that of the OASPL. So, if $\mathrm{Sk}\{\partial p/\partial t\}$ emphasizes shocks, and ASF is correlated with the directionality of high-frequency energy, what nonlinearity does ASF tend to emphasize?

We believe that ASF, because of its linear dependence on the pressure time derivative is correlated with the mean distortion in the waveform, hence our choice of moniker, "average" steepening factor. Although relatively few acoustic shocks can lead to a large increase in $\mathrm{Sk}\{\partial p/\partial t\}$, their impact on ASF will be far less. On the other hand, the naturally occurring large derivatives associated with high-frequency components in the waveform will tend to be emphasized by ASF because of their greater rate of occurrence. If we consider that the shock formation distance is inversely proportional to frequency, these high-frequency components that exist through the waveform – not just as part of Mach wavefronts - will tend to undergo their own nonlinear propagation. With its linear dependence on the derivatives, the ASF tracks the mean distortion of the smaller-amplitude, high-frequency components as well as the high-amplitude Mach waves. Gee *et al*. (2008) showed that the far-field propagation from the F-22 Raptor is appreciably nonlinear at 90°, even at 90% power. This angle is not believed to be dominated by Mach wave radiation, but rather fine-scale radiation[39] with a higher peak frequency. Given the fact that it does not weight the shock-specific content, it is reasonable, therefore, that the ASF would peak in a direction upstream of the derivative skewness.

The argument that these two measures provide different viewpoints into the nonlinear propagation of jet noise is strengthened by using the evolution and the solutions to the Earnshaw solution for an initial sinusoid to describe the propagation relative to a shock formation distance. If we use 75% ETR as an example, the maximum derivative skewness is $\mathrm{Sk}\{\partial p/\partial t\}$=10.2. On the other hand, the maximum ASF value is ASF = 1.77. Because the results of Shepherd *et al.*[34] show that a derivative skewness in excess of 5 indicates $\sigma > 0.9$, we use the analytical approximation of Muhlestein[28] and Reichman *et al.*[36] as $\sigma \to 1$. This results in $\sigma \approx \sqrt{1 - \mathrm{Sk}\{\partial p/\partial t\}^{-4/3}} \approx 0.98$. On the other hand, if we consider σ based on ASF [see Eq. (2)], we obtain $\sigma = \pi(\mathrm{ASF} - 1)/2(\mathrm{ASF} + 1) \approx 0.44$.

The derivative skewness indicates the waveform has shocks present, which it does,[21] but the ASF suggests that we do not have significant shocks present. How can these two views be reconciled?

Recall that the quantitative description for ASF and $\mathrm{Sk}\{\partial p/\partial t\}$ in Sec. 2 is for an initial planar, sinusoid evolving into a shock-containing waveform. Each period consists of a single shock, with a pressure amplitude variation evolving from a bimodal distribution into a uniform distribution as the wave progresses from a sinusoid into a sawtooth.[34] On the other hand, Gaussian or near-Gaussian noise of equal standard deviation has larger amplitude values present in the waveform, but also has a greater number of lower-amplitude values. Figure 2 – Figure 5 help show this behavior. As these waveforms evolve nonlinearly, a large fraction of the waveform steepens more slowly than the equivalent sinusoid and other portions that steepen more quickly. Because of the cubing operation, the derivative skewness is likely to be dominated by the quickly-forming shocks, whereas the ASF is more heavily influenced by the relatively large number of lower amplitude portions. Thus, this analysis strengthens the argument that the two measures are sensitive to different aspects of the nonlinear propagation.

Note that the relative difference in sensitivity to the derivative portions of the shocks results in different sampling requirements. An analysis begun by Gee *et al.*[22] of the sampling requirements for derivative skewness has been extended and pursued in more rigorous fashion by Muhlestein,[28] Muhlestein *et al.*[29], and Reichman *et al.*[36] Because the derivative skewness emphasizes the positive derivatives at the shocks, undersampling the shocks can result in an artificial suppression of its growth with range as shocks form and strengthen. On the other hand, the ASF, with its reduced sensitivity to the relatively infrequent shocks, has reduced sampling requirements. With further investigation into the behavior for jet-like broadband noise signals, sampling considerations could help explain the greater differences between laboratory-scale and full-scale results for $\mathrm{Sk}\{\partial p/\partial t\}$ than for ASF.

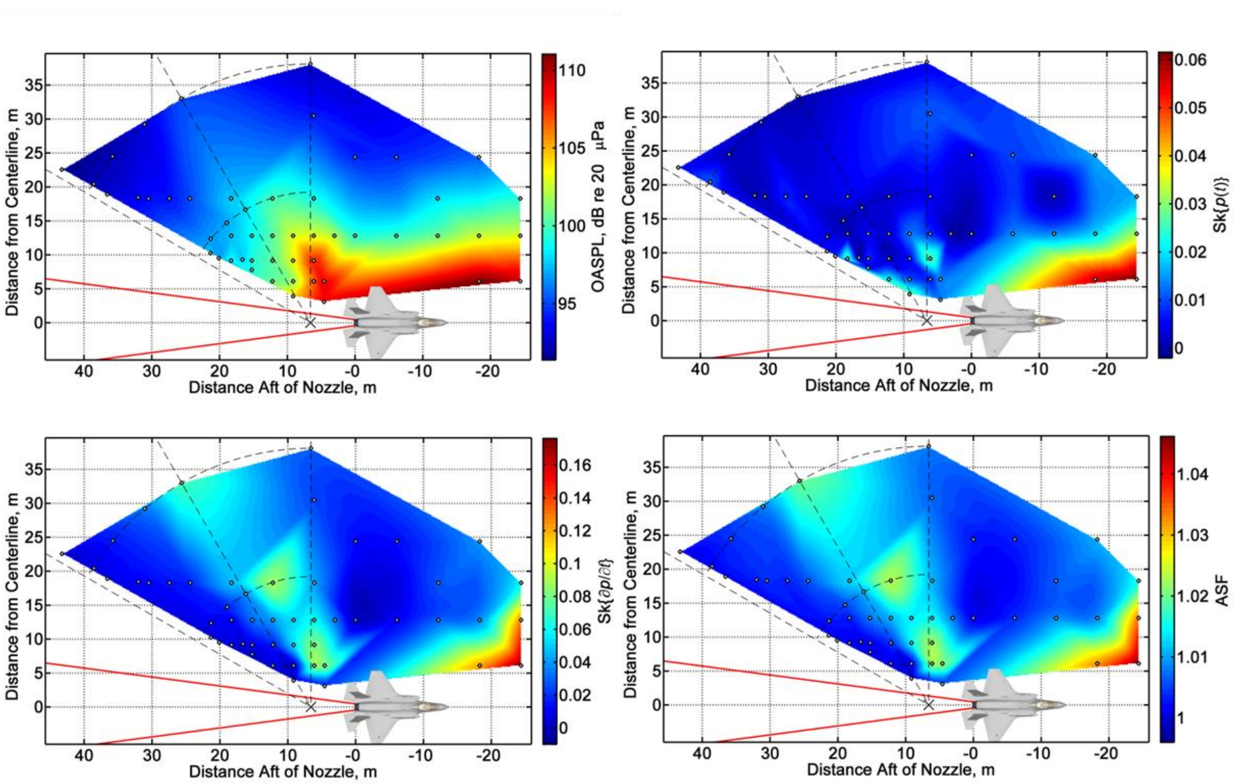

Figure 6. Clockwise from top left: Overall sound pressure level (OASPL), pressure skewness, $\mathbf{Sk}\{p(t)\}$, derivative skewness, $\mathbf{Sk}\{\partial p/\partial t\}$, and average steepening factor (ASF) for the F-35 AA-1 at 25% ETR.

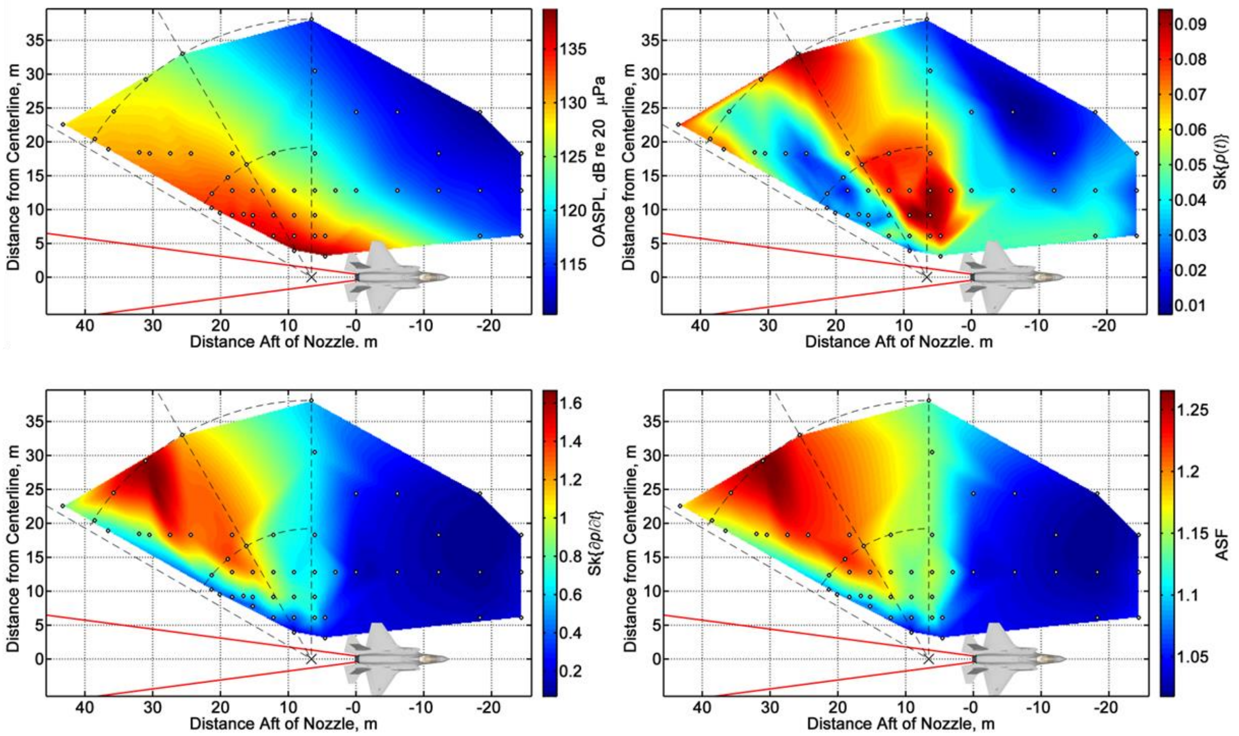

Figure 7. Same as Figure 6 but for 50% ETR.

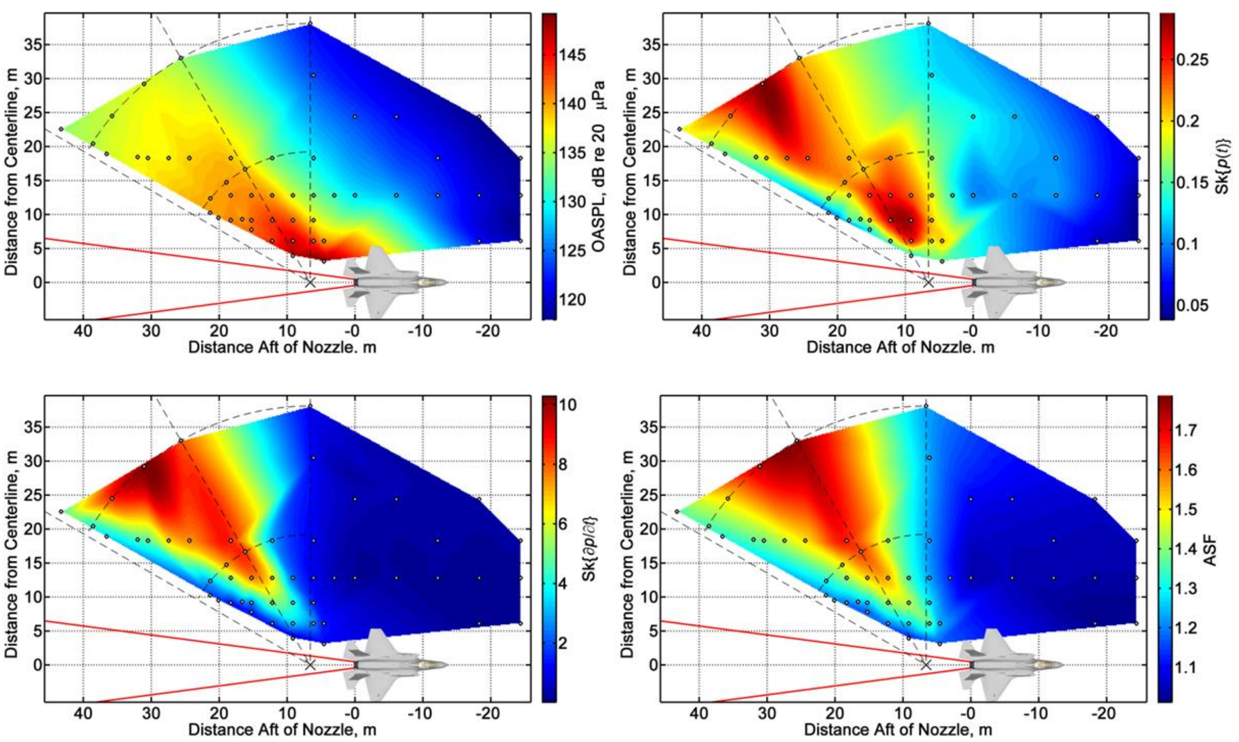

Figure 8. Same as Figure 6 but for 75% ETR.

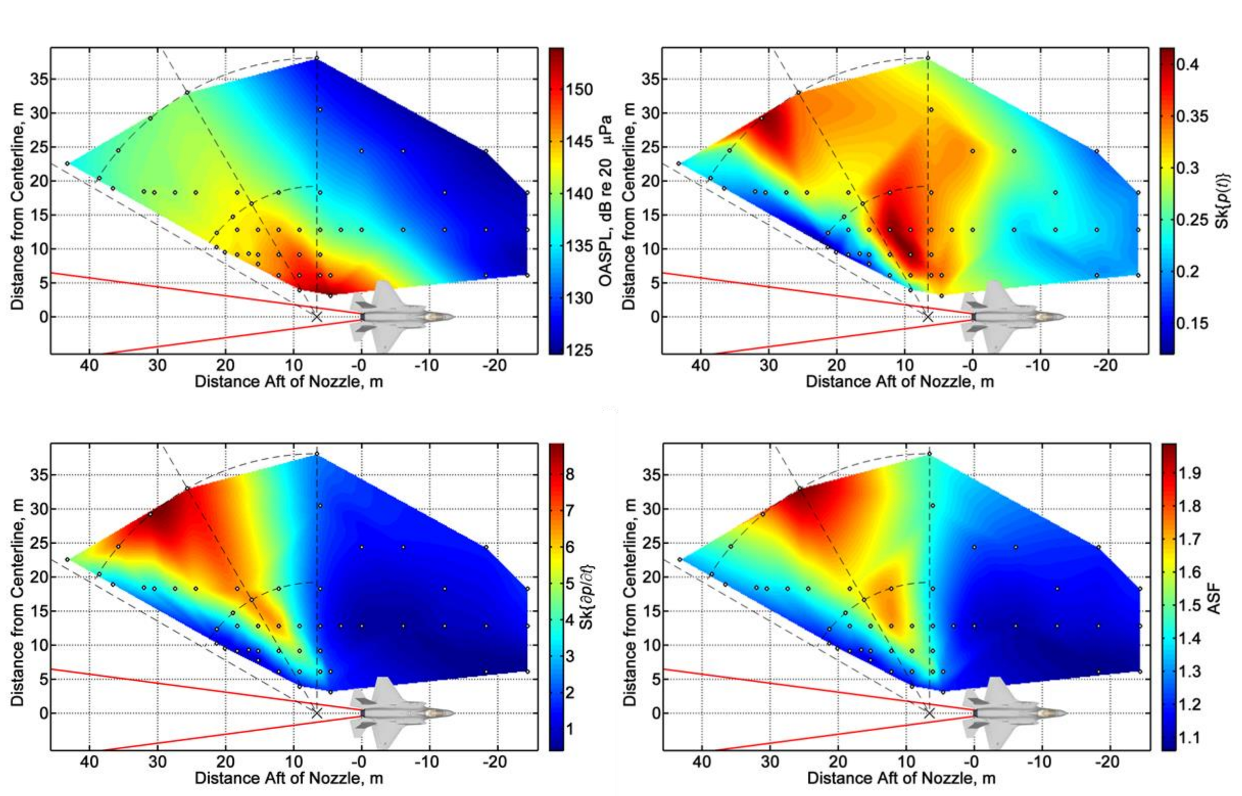

Figure 9. Same as Figure 6 but for 100% ETR.

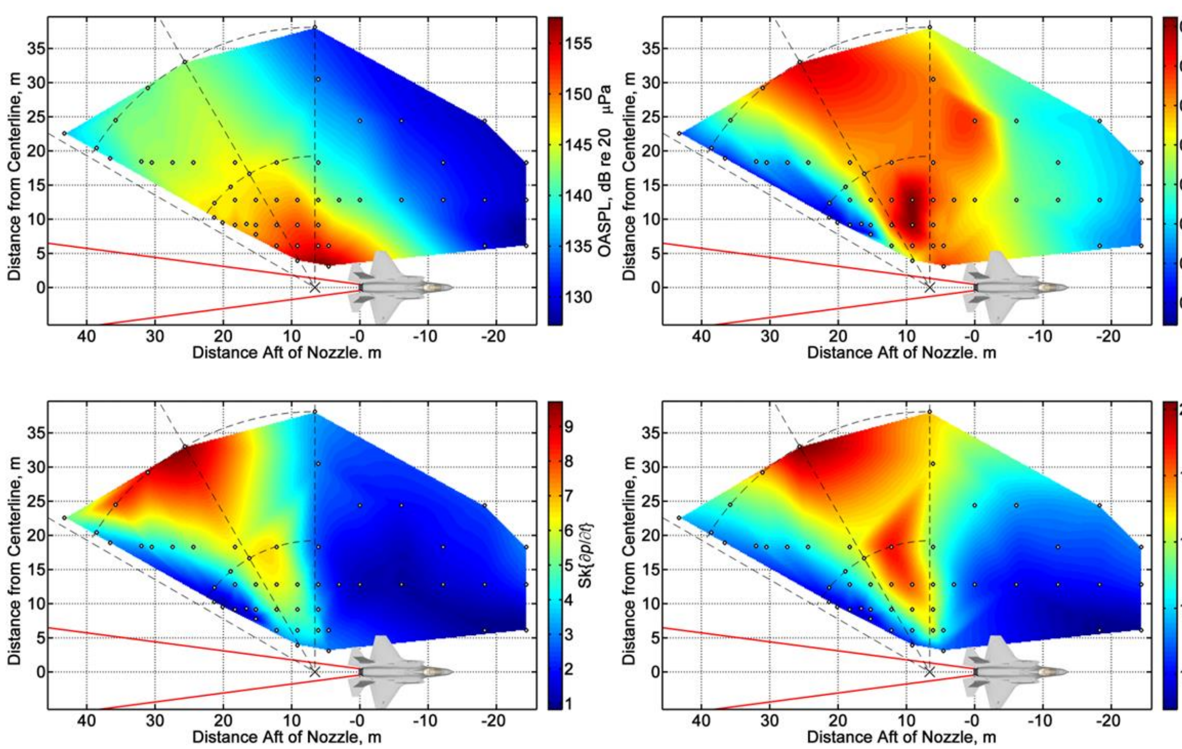

Figure 10. Same as Figure 6 but for 130% ETR.

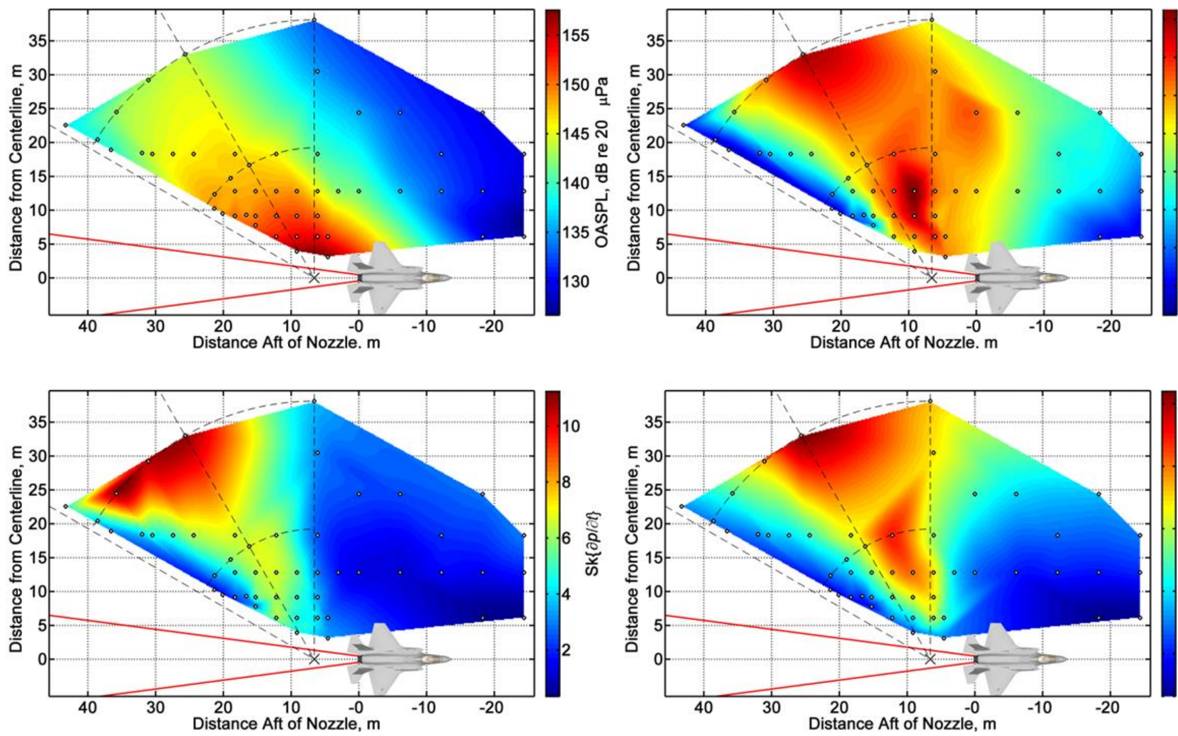

Figure 11. Same as Figure 6 but for 150% ETR.

V. Concluding Discussion

This analysis of F-35 AA-1 data has presented additional insights into the near-field nonlinear propagation of jet noise. However, the emphasis has been on examining the behavior of the average steepening factor (ASF) relative to the derivative skewness. The ASF has been defined as the inverse of the original wave steepening factor proposed by Gallagher[17] in order to have a more intuitive measure that increases with waveform steepening and shock formation, rather than towards zero. From the results, it appears that the ASF is not a very sensitive indicator of actual shock formation because of its linear dependence on the pressure time derivative. Instead, it more closely correlates with the overall steepening in the waveform because of the relative infrequency of the actual shocks. On the other hand, the derivative skewness, with its cubic dependence on the derivative, greatly emphasizes the high derivative values present at the shocks. However, this inherently requires a greater sampling rate in order to resolve the shock rise times. These nuances may lead to differences in conclusion regarding nonlinear propagation in the bandwidth and range-limited environments in which laboratory-scale measurements are made. However, for full-scale, high-performance jet aircraft there is no question that there is nonlinear propagation over a broad range of angles and jet conditions. This suggests that additional investigations into the scalability of laboratory measurements for supersonic jet noise generation and propagation are needed.

Acknowledgments

The authors acknowledge Dr. Alan T. Wall of the Air Force Research Laboratory for his contributions to the measurements and analyses described in this paper. The support of the F-35 Lightning II Joint Program Office is acknowledged. (Distribution A - Approved for Public Release; Distribution is Unlimited; JSF14-592.) The authors gratefully acknowledge funding from the Air Force Research Laboratory through the SBIR program and support through a Cooperative Research and Development Agreement (CRDA) between Blue Ridge Research and Consulting, Brigham Young University, and the U.S. Air Force. Some of the analysis has been performed under a Jet Noise Reduction program sponsored by the U.S. Office of Naval Research, with Joseph Doychak as program manager.

References

[1] Blackstock, M. F. Hamilton, and A. D. Pierce, "Progressive waves in lossless and lossy fluids," in Nonlinear Acoustics, edited by M. F. Hamilton and D. T. Blackstock (Academic, San Diego, CA, 1998).
[2] Webster, D. W. and Blackstock, D. T., "Amplitude density of a finite amplitude wave," *Journal of the Acoustical Society of America*, Vol. 65, No. 4 , 1979, pp. 1053–1054.
[3] Pernet, D. F. and Payne, R. C., "Non-linear propagation of signals in airs," *Journal of Sound and Vibration*, Vol. 17, No. 3, 1971, pp. 383-396.
[4] D. T. Blackstock, "Nonlinear propagation of jet noise," in Proceedings of the Third Interagency Symposium on University Research in Transportation Noise, University of Utah, Salt Lake City, UT (1975), pp. 389–397.
[5] Ffowcs-Williams, J. E., J. Simson and V. J. Virchis, "'Crackle': An annoying component of jet noise," *Journal of Fluid Mechanics*, Vol. 71, No. 2., 1975, pp. 251-271.
[6] Morfey, C. L. and Howell, G. P., "Nonlinear Propagation of Aircraft Noise in the Atmosphere," *AIAA Journal*, Vol. 19, No. 8, 1981, pp. 986-992.
[7] Crighton, D.G. and Huerre, P., "Shear-Layer Pressure Fluctuations and Superdirective Acoustic Sources," *Journal of Fluid Mechanics*, Vol. 220, 1990, pp. 355-368.
[8] Gee, K. L., Gabrielson, T. B., Atchley, A. A. and Sparrow, V. W., "Preliminary analysis of nonlinearity in military jet aircraft noise propagation," *AIAA Journal*, Vol. 43, No. 6, 2005, pp. 1398-1401.
[9] Gee, K. L., Sparrow, V. W.,James, M. M., Downing, J. M., Hobbs, C. M., Gabrielson, T. B., and Atchley, A. A.,"Measurement and prediction of noise propagation from a high-power jet aircraft," *AIAA Journal*, Vol. 45, No. 12, 2007, pp. 3003-3006.
[10] Brouwer, H. H., "Numerical simulation of nonlinear jet noise propagation," AIAA Paper 2005-3088, May 2005.
[11] Saxena, S., Morris, P. J., and Viswanathan, K., "Algorithm for the Nonlinear Propagation of Broadband Jet Noise", *AIAA Journal*, Vol. 47, No. 1, 2009, pp. 186-194.
[12] Gee, K. L., Sparrow, V. W., James, M. M., Downing, J. M., Hobbs, C. M., Gabrielson, T. B., and Atchley, A. A., "The Role of Nonlinear Effects in the Propagation of Noise from High-Power Jet Aircraft," *Journal of the Acoustical Society of America*, Vol. 123, No. 6, 2008, pp. 4082-4093.
[13] McInerny, S. A., Gee, K. L. Dowing, J. M., and James, M. M., "Acoustical nonlinearities in aircraft flyover data," AIAA Paper 2007-3654, May 2007.
[14] Gee, K. L., Downing, J. M., James, M. M., McKinley, R. L., McKinley, R. C., Neilsen, T. B., and Wall, A. T., "Nonlinear Evolution of Noise from a Military Jet Aircraft During Ground Run-up," AIAA Paper 2012-2258, June 2012.
[15] Schlinker, R. H., Liljenberg, S. A., Polak, D. R., Post, K. A., Chipman, C. T., and Stern, A. M., "Supersonic Jet Noise Source Characteristics & Propagation: Engine and Model Scale," AIAA Paper 2007-3623, May 2007.
[16] Gallagher, J. A. and McLaughlin, D. K., "Experiments on the Non-linear Characteristics of Noise Propagation from Low and Moderate Reynolds Number Supersonic Jets," AIAA Paper 81-2041, Oct. 1981.
[17] Gallagher, J., "The effect of non-linear propagation in jet noise," AIAA Paper 82-0416, January 1982.
[18] Petitjean, B. P. and McLaughlin, D. K. "Experiments on the Nonlinear Propagation of Noise from Supersonic Jets," AIAA Paper 2003-3127, May 2003.
[19] Gee, K. L. , Petitjean, B. P., McLaughlin, D. K. and Sparrow, V. W., "Nonlinear Propagation of Noise Radiated from Supersonic Jets," *Proceedings of Noise-Con 04*, edited by C. B. Burroughs and G. C. Maling, Jr., Noise Control Foundation, Poughkeepsie, New York, 2004, pp. 725-733.
[20] Petitjean, B. P., Viswanathan, K., and McLaughlin, D. K., "Acoustic pressure waveforms measured in high speed jet noise experiencing nonlinear propagation," *International Journal of Aeroacoustics*, Vol. 5, No. 2, 2006, pp. 193-215.
[21] Gee, K. L., Neilsen, T. B., Downing, J. M., James, M. M., McKinley, R. L., McKinley, R. C., and Wall, A. T., "Near-field Shock Formation in Noise Propagation from a High-power Jet Aircraft," *Journal of the Acoustical Society of America*, Vol. 133, No. 2, 2013, EL88 – EL93.
[22] Gee, K. L., Neilsen, T. B., Muhlestein, M. B., Wall, A. T., Downing, J. M., James, M. M., and McKinley, R. L., "On the Evolution of Crackle in Jet Noise from High-Performance Engines," AIAA Paper 2013-2190, May 2013.
[23] Gee, K. L., Atchley, A. A., Falco, L. E., Shepherd, M. R., Ukeiley, L. S.,Jansen, B. J., and Seiner, J. M., "Bicoherence analysis of model-scale jet noise," *Journal of Acoustical Society of America*, Vol. 128, No. 5, 2010, EL211-EL216.
[24] K. L. Gee, A. A. Atchley, L. E. Falco, and M. R. Shepherd, "Nonlinearity analysis of model-scale jet noise," 19th International Symposium on Nonlinear Acoustics, Tokyo, Japan, May 2012, AIP Conf. Proc. **1474**, 307-310 (2012).
[25] Gee, K. L., Neilsen, T. B., and Atchley, A. A., "Skewness and shock formation in laboratory-scale supersonic jet data," *Journal of the Acoustical Society of America*, Vol. 133, No. 6, 2013, EL 491 – 497.
[26] Baars, W. J., Tinney, C. E., and Wochner, M. S., "Nonlinear Noise Propagation from a Fully Expanded Mach 3 Jet," AIAA Paper 2012-1177 January 2012.
[27] Baars, W. J. and Tinney, C. E., "Shock structures in the acoustic field of a Mach 3 jet with crackle," *Journal of Sound and Vibration*, Vol. 333, No. 12, June 2014, pp. 2539-2553.
[28] Muhlestein, M. B. "Analyses of nonlinearity measures in high-amplitude sound propagation," M.S. Thesis, Dept. of Physics and Astronomy, Brigham Young University, Provo, UT, July 2013.
[29] Muhlestein, M. B., Gee, K. L., Neilsen, T. B., and Thomas, D. C., "Evolution of an average steepening factor for nonlinearly propagating waves," submitted to *Journal of Acoustical Society of America* (2014).
[30] McInerny, S. A., "Launch vehicle acoustics Part 2: Statistics of the time domain data," *Journal of Aircraft*, Vol. 33, No. 3, May-June 1996, pp. 518-523.
[31] Mora, P.,Heeb, N., Kastner, J., Gutmark, E., and Kailasanath, K., ""Effect of Scale on the Far-Field Pressure Skewness and Kurtosis of Heated Supersonic Jets," AIAA Paper 2013-616, January 2013.
[32] Gee, K. L., Kenny, R. J., Neilsen, T. B., Jerome, T. W, Hobbs, C. M., and James, M. M., "Spectral and statistical analysis of noise from reusable solid rocket motors," *Proceedings of Meetings on Acoustics*, Vol.18, 2013, 040002.
[33] Downing, J. M., Gee, K. L., McInerny, S. A., Neilsen, T. B., and James, M. M. "Do recent findings on jet noise answer aspects of the Schultz curve?," *Proceedings of Meetings on Acoustics*, Vol. 19, 2013, 040022.
[34] Shepherd, M. R., Gee, K. L., and Hanford, A. D., "Evolution of statistics for a nonlinearly propagating sinusoid," *Journal of Acoustical Society of America*, Vol. 130, No. 1, 2011, EL8-EL13.

[35] Muhlestein, M. B. and Gee, K. L., "Experimental investigation of a characteristic shock formation distance in finite-amplitude noise propagation," *Proceedings of Meetings on Acoustics*, Vol. 12, 2011, 045002.
[36] Reichman, B. O., Muhlestein, M. B., Gee, K. L., Neilsen, T. B., and Thomas, D. C., "Evolution of the pressure derivative skewness for nonlinearly propagating waves," submitted to *Journal of Acoustical Society of America* (2014).
[37] McKinley, R., McKinley, R., Gee, K. L., Pilon, T., Mobley, F., Gillespie, M., and Downing, J. M., "Measurement of Near-field and Far-field Noise from Full Scale High Performance Jet Engines," *Proceedings of ASME Turbo Expo 2010*. June 2010, Paper No. GT2010-22531.
[38] Gee, K. L., Neilsen, T. B., James, M. M., "On the crest factor of noise in full-scale supersonic jet engine measurements," *Proceedings of Meetings on Acoustics*, Vol. 20, 2013, 045003.
[39] Neilsen, T. B., Gee, K. L., Wall, A. T. and James, M. M. "Similarity spectra analysis of high-performance jet aircraft noise," *Journal of the Acoustical Society of America,* Vol. 133, No. 4, 2013, pp. 2116 – 2125.