Finite-Pressure-Gradient Influences on Ideal Spheromak Equilibrium

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Spatially resolved measurements of the magnetic field of a spheromak have been analyzed and compared with expectations for the ratio of $j_{\parallel}/B$ from the pressure-gradient-free Taylor model and a model with pressure due to Morikawa. Better agreement is found with the model containing finite pressure.

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The spheromak configuration has been the object of theoretical and experimental investigation in the last few years because of possible technological simplicities in fusion reactor design as well as for scientific interest.[1-6] It combines the internally closed magnetic surfaces of toroidal devices with the lack of plasma-linking coils as in the mirror configuration. In addition to these practical features the study of the spheromak may lead to a better understanding of plasma physics in part because the spheromak is a nearly minimum-magnetic-energy configuration. Much of the recent theoretical work on spheromaks has been in the context of force-free configurations. Taylor[7] has argued that the minimum-magnetic-energy state allowed by given magnetic helicity (defined as $K=\int dv\,\vec{A}\cdot\vec{B}$, where $\vec{A}$ is the magnetic vector potential for the magnetic field $\vec{B}$ and the integral is over the entire closed confinement region) is a force-free state with the ratio of current density to magnetic field a spatial constant. This Taylor state has been used for many of the theoretical studies of spheromak equilibria and stability.[8] From an experimental viewpoint, however, a force-free state is not very interesting because it provides no plasma confinement.

The purpose of this paper is to show how data on the current distribution in the PS experiment deviate from the force-free Taylor state and in addition how the data appear to be consistent with at least one model containing pressure gradients.

The PS experiment forms a spheromak by a combination of z and θ discharges. In the experiment a bias field in the z direction is created by a capacitor-bank discharge, followed by a z discharge originating from a ring of electrodes. The direction of the applied external field is then reversed, making a z-directed component of field on the outer edge which is opposite to the trapped initial bias field. Field line reconnection occurs at the ends trapping both toroidal and poloidal flux in a spheromak configuration. This experiment and formation scheme are fully described in another paper.[5] In this paper we discuss analysis of magnetic-probe data from a case which, in Ref. 5, was referred to as the slow formation scheme and which uses both a metallic liner and figure-8 coils for $n=1$ stabilization. For the results reported here typical parameters are $|B| \leq 6$ kG, $n \cong (1\text{–}5)\times 10^{15}$ cm^{-3}, $T_e \cong 5$ eV, and the separatrix radius in the midplane is about 10–11 cm. Since the Alfvén transit time across the minor radius is less than $\frac{1}{2}\,\mu$sec there is good reason to expect that after about 5 μsec the configuration should be in mechanical equilibrium. The plasma appears to pass through successive magnetohydrodynamic (MHD) equilibrium states in a resistive time scale. In addition, no acceleration is observed at this time or later. On this basis we compare results with MHD equilibrium models.

In Fig. 1 we plot surfaces of constant poloidal flux obtained from magnetic-probe data. Data sufficient to make flux plots were only taken in half the experimental chamber. Symmetry with respect to the $z=0$ plane was confirmed by sampling positions for $z<0$. As can be seen in the figure the configuration at this time (25 μsec into the discharge) is oblate with a separatrix half-length to radius ratio of 0.6, which is about optimum for stability against the $n=1$ tilt-shift mode. The question we are interested in is how close is this equilibrium to the Taylor state?

A useful measure of how close a configuration is to the Taylor state is the ratio of the parallel current density to the magnitude of the field, $j_{\parallel}/B$. In the Taylor state $\vec{j}$ is parallel to $\vec{B}$ and the ratio of their magnitudes is constant in space and equal to k/μ_0, where k is the eigenvalue of the equilibrium, i.e., $\nabla\times\vec{B}=k\vec{B}$. A plot of $j_{\parallel}/B$ in the $z=0$ midplane at 25 μsec is shown in Fig. 2. It can be clearly seen that $j_{\parallel}/B$ is not a spatial constant.

Possible causes of the deviation of $j_{\parallel}/B$ from a constant value can be seen by examining the

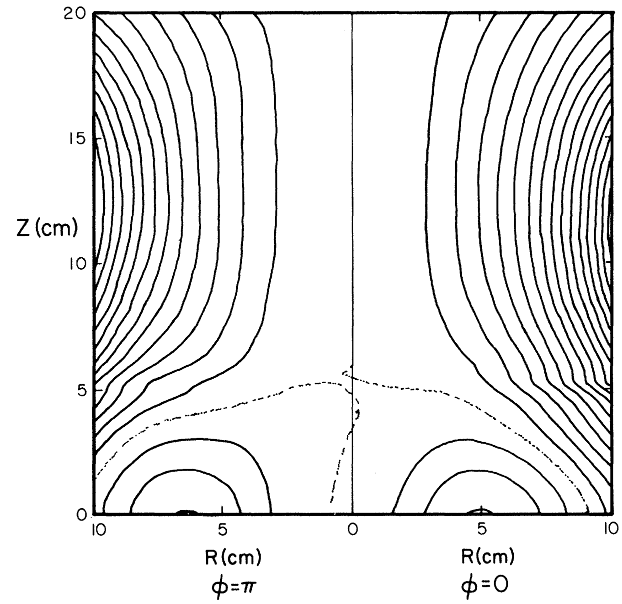

FIG. 1. Surfaces of constant poloidal flux at 25 μsec in one half of the cylindrical (R, φ, z) plane where $\varphi = 0, \pi$. Surface contours are separated by $2\pi \times 5$ kG cm^2. The dashed line is the $\psi = 0$ separatrix.

axisymmetric equilibrium expression for $j_{\parallel}/B$ using the poloidal current stream function $I(\psi) = rB_{\varphi}$, and the pressure $p(\psi)$, where ψ is the poloidal flux function:

$$\frac{j_{\parallel}}{B} = \frac{I'(\psi)}{\mu_0} + \frac{p'(\psi)I(\psi)}{B^2}, \tag{1}$$

(the prime indicates differentiation with respect to ψ). The Taylor state corresponds to $p'(\psi) = 0$ and $I(\psi) = k\psi$. There are two possible causes of the variation of $j_{\parallel}/B$. One is $p'(\psi) = 0$ with I a nonlinear function of ψ. In this case $j_{\parallel}/B$ is a function of ψ and thus would be an extremum on the magnetic axis. The second possible cause of variation is that $p'(\psi) \neq 0$. In this case $j_{\parallel}/B$ would not be a function of ψ alone because of the B^2 in the denominator of Eq. (1), and would tend to peak outside the magnetic axis. We have determined $I(\psi)$ from experimental data along the $z = 0$ midplane; the results are shown in Fig. 3. It is apparent that deviations from linearity of $I(\psi)$ will not explain the double-humped behavior of Fig. 2.

A model for static MHD equilibrium with a linear $I(\psi)$ and the mathematically simplest nonzero pressure gradient, $p = (P/\mu_0)\psi$, where P is a constant, was investigated a number of years ago by Morikawa.[9] He found a solution to the Grad-Shafranov equation with spherical boundary condi-

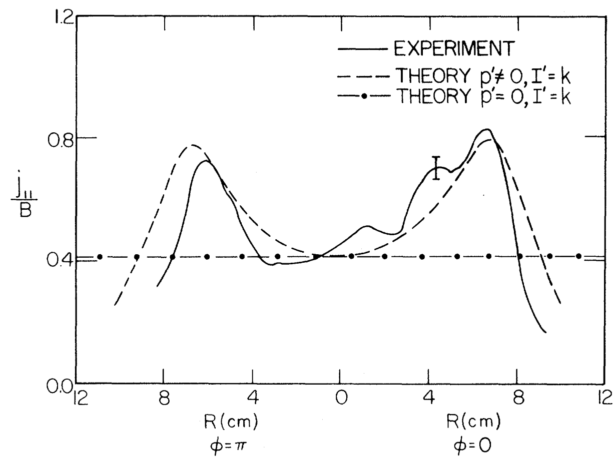

FIG. 2. A plot of theoretically and experimentally determined values of $j_{\parallel}/B$ along the $z = 0$ line in the $\varphi = 0, \pi$ plane at 25 μsec. The error bar is an indication of the error associated with the shot-to-shot reproducibility of the field measurements.

tions. In the more general case with oblate or prolate boundaries, the flux function in spherical coordinates (ρ, θ, φ) can be written as the sum of a force-free part and a pressure-dependent part,

$$\psi = \sum_n c_n \frac{k\rho j_n(k\rho)}{k^2} P_n'(\cos\theta)\sin^2\theta - \frac{P\rho^2\sin^2\theta}{k^2}, \tag{2}$$

where $j_n(k\rho)$ is a spherical Bessel function and P_n is a Legendre polynomial. For our oblate case the boundary conditions can be matched with $c_3/c_1 = -0.4$ and all other coefficients zero. Using this solution to the equilibrium equation we compute $j_{\parallel}/B$ along the midplane. In Fig. 2 we plot the calculated $j_{\parallel}/B$ for the P and k parameters which best fit the data (see Table I).[10]

For the more general situation where $p \sim \psi^{\nu}$ we see that

$$j_{\parallel}/B = (k/\mu_0)(1 + \nu\beta/2). \tag{3}$$

The peaks are a measure of the maximum value of the local β $[\equiv 2\mu_0 p(\psi)/B^2]$ while the value on

TABLE I. Experimentally obtained values of the parameters k and P used in the linear ψ model.

Method	k	P
Profile fit	0.45 ± 0.04	0.062 ± 0 01
$j_{\parallel}/B$	0.43 ± 0.08	0.059 ± 0.01
$2W_t/K$	0.43 ± 0.04	
I vs ψ	0.41 ± 0.06	

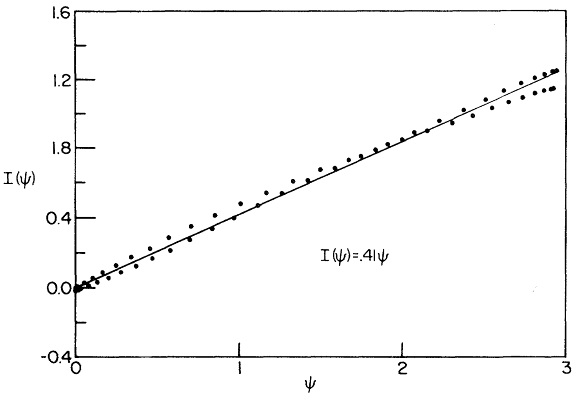

FIG. 3. Experimentally obtained dependence of $I(\psi) = rB_\varphi$ along the $z = 0$ line in the $\varphi = 0, \pi$ plane.

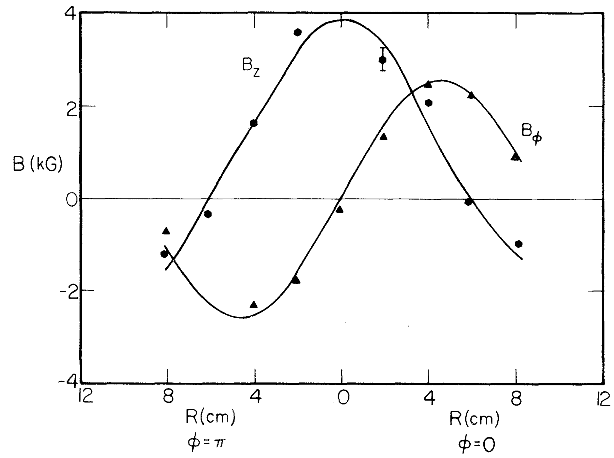

FIG. 4. Comparison of theoretically and experimentally derived profiles of the poloidal and toroidal field components in the $z = 0$ plane. The solid lines are from the model with the linear pressure dependence and the points are from the experiment. The error bracket indicates shot-to-shot reproducibility.

the symmetry axis ($\beta = 0$) gives k. The volume-averaged beta can also be expressed in a simple form involving measurable field quantities as

$$\langle\beta\rangle = 2\nu^{-1}(1 - 2W_t/W), \quad (4)$$

where $W_t = \int B_\varphi^2 dv/2\mu_0$, W is the total magnetic energy, and $\langle\beta\rangle = \int p\, dv/W$. For $\nu = 1$ the measured W_t and W give $\langle\beta\rangle = 0.4$ with an uncertainty of (15–20)%, which is consistent with the interferometrically measured line-averaged density of 2×10^{15} cm^{-3} and $\langle T_e + T_i\rangle = 10$ eV. More detailed measurements showing local values of beta were not performed. It should be understood that during the short time scale of this experiment pressure-driven instabilities may not have time to develop and hence this value of beta may not be realizable in a long-time confinement experiment.

A further comparison with the model can be made by comparing measured and calculated field profiles. In Fig. 4 we show a least-mean-squares fit of the model to the data.[10] The resulting values of P and k along with their standard deviation are shown in Table I. Finally, another measure of the value of the parameter k can be obtained from the ratio of the toroidal field energy to helicity inside the separatrix, $k = 4\mu_0 W_t/K$. This equation is independent of the pressure and follows from the fact that $K = 2\int A_\varphi B_\varphi dv$. We also show in Table I the value of k obtained from the measured W_t and K. The uncertainty in this case is based on the shot-to-shot variation in the field measurements.

We have found a simple technique for observing departures from zero-beta behavior in a spheromak by determining the parallel component of current, which in our spheromak is much larger than the perpendicular component. We have used this technique and compared our measurements with a model for the equilibrium. The good agreement suggests that the linear ψ pressure model is a reasonable substitute for the Taylor model for investigating pressure effects. It should be noted that other models for $p(\psi)$ might also be appropriate even though they have not been discussed here.

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