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Study of the effects of photoelectron statistics on Thomson scattering data

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A computer code has been developed which simulates a Thomson scattering measurement, from the counting statistics of the input channels through the mathematical analysis of the data. The scattered and background signals in each of the wavelength channels are assumed to obey Poisson statistics, and the spectral data are fitted to a Gaussian curve using a nonlinear least-squares fitting algorithm. This method goes beyond the usual calculation of the signal-to-noise ratio for the hardware and gives a quantitative measure of the effect of the noise on the final measurement. This method is applicable to Thomson scattering measurements in which the signal-to-noise ratio is low due to either low signal or high background. Thomson scattering data from the S-1 spheromak have been compared to this simulation, and they have been found to be in good agreement. This code has proven to be useful in assessing the effects of counting statistics relative to shot-to-shot variability in producing the observed spread in the data. It was also useful for designing improvements for the S-1 Thomson scattering system, and this method would be applicable to any measurement affected by counting statistics.

INTRODUCTION

The effect of photoelectron statistics on a Thomson scattering measurement is difficult to quantify. The method usually used to evaluate a system is to calculate the signal-to-noise ratio of the total collected signal.[1] While this gives a good measure of the quality of the system, it does not give a quantitative measure of variations in the measured temperature that can be expected. This is a result of the fact that while the statistics of both a single channel and the total signal are well known, the interaction of multiple channels of data with the least-squares fitting used to evaluate the temperature and density is not easily calculated. This makes it difficult to assess whether the variations in the measurement of a plasma temperature are due to statistical fluctuations or to real shot-to-shot changes. This paper discusses a method of making this assessment.

I. EXPERIMENT

The S-1 spheromak Thomson scattering system is a single point system that can be moved to different points in the plasma on a shot-to-shot basis. The system has six wavelength channels spread over the range from 35 to 314 Å from the ruby laser line. The output from the photomultipliers on each of these channels is gated into a LeCroy 2249A current integrator for approximately 80 ns during the laser pulse. A second set of integrators is gated on for approximately 160 ns immediately after the laser pulse to obtain a background subtraction signal. After the signals are read and digitized, the result is then computer readable.[2]

Since the integrators do not actually count photons, but rather accumulate charge, it is necessary to calibrate the correspondence between the signal level on the integrators and the number of photoelectrons produced in the photomultipliers. We used two methods to obtain this number, both of which produced the same result. The first was to measure the height and width of single photoelectron pulses coming from the photomultipliers and use the known charge sensitivity of the integrators to calculate the number of photoelectrons per digitizer count. The second method used the fact that a repeated measurement of a photon signal will be distributed according to Poisson statistics. A constant signal from a lamp was put into the polychrometer, and the signal level from the photomultipliers was repeatedly sampled. The mean and standard deviation of those measurements was then calculated. From the properties of the Poisson distribution, the standard deviation of the measurement is proportional to the square root of the number of photoelectrons.[3] The constant of proportionality is the same as that relating the signal to the number of photoelectrons. Therefore, using the above data we can calculate that constant. If S is the signal level, σ is the standard deviation of the signal, C is the constant of proportionality between the signal and the number of photoelectrons, and P is the number of photoelectrons, then

$$S = CP \tag{1}$$

and

$$\sigma = C\sqrt{P}\,. \tag{2}$$

This gives the result that

$$C = \sigma^2/S\,. \tag{3}$$

The gain of the photomultipliers was adjusted until one count on the integrator approximately equaled one photoelectron on the photomultiplier.

We shall consider the case of S-1 operating with figure-8 coils.[4] At the operating parameters of 40 eV, electron density of 3×10^{13} cm^{-3}, and a laser energy of 13.5 J, the channel closest to the laser line ($\Delta\lambda = 46$ Å) has an average signal level of 50 photoelectrons, with a background light level of 63 photoelectrons during the laser pulse. The large background light level appears to be caused mostly by molecular hydrogen emission from the thick blanket of neutral gas surrounding the plasma.[2]

Because of the low signal level and the large background light, there was a large spread in the measured temperatures from this system, and it became essential to differentiate the scatter in the measurement caused by photoelectron statistics from that caused by the shot-to-shot variability of the experiment. For that purpose we developed the code described here.

II. COMPUTER SIMULATION

The code simulates the effect of the interaction of the Poisson statistics of the photoelectron counting process in each signal channel with the nonlinear fitting routine used to calculate the temperature and density from the data channels. If a given channel is expected to have a given number of photoelectron counts, μ, then the Poisson statistics of the measurement say that the probability of measuring a number n of photoelectrons is given by the expression

$$P(n,\mu) = \mu^n e^{-\mu}/n! \, . \tag{4}$$

The value which goes into the fitting routine is a simulated signal plus background (as with the first gate in the real experiment), from which a background signal (simulating the second gate) will be subtracted.

A set plasma temperature is put into the code, along with the expected mean signal and background levels for the central channel. For a given plasma temperature, specifying the signal in the central channel is equivalent to specifying the plasma density and optical throughput. The code calculates the expected signal for each channel from a Gaussian distribution of the signal versus wavelength, using the given temperature to calculate the width and the central signal for the height. The value for the signal plus background in a given channel is randomly chosen from a Poisson distribution with a mean equal to the expected signal plus the background level. The background for that channel is also chosen from a Poisson distribution, using twice the input background level as the mean (because in the experiment the background gate is twice as wide as the signal gate) and dividing the resultant number by two before subtraction. For example, if the signal expected in a channel is 15 photoelectrons and the background level is 50 photoelectrons, the signal plus background would be chosen from a Poisson distribution with a mean of 65. The background would be chosen from a distribution with a mean of 100, with the result divided by two before subtraction from the first value.

The above process is carried out for each signal channel, making proper allowance for its relative spectral width, and the results are put into a least-squares fitting routine.[5] The routine fits the signals to a function of the form

$$y_i = Ae^{-Bx_i^2} , \tag{5}$$

where y_i is the simulated signal and $x_i = \Delta\lambda_i$. At the temperatures we were measuring, the relativistic corrections to the above expression were insignificant. It is important to use a nonlinear least-squares fitting routine of this sort to prevent the noise in the outer channels from skewing the measurement.[6]

III. RESULTS AND DISCUSSION

The results of this comparison are shown in Fig. 1. The jagged curve shows an experimental histogram of 121 shots taken on S-1 with a plasma current of 270 kA and a fill pressure of 3 mTorr. This set of data was chosen for our analysis because the plasma was reproducible and was collected over a short period of time to minimize changing conditions. The smooth curve is the result of this code for a temperature of 40 eV, with the same mean signal and background levels as in the experimental data (50 signal and 63 background photoelectrons). The temperature distribution from the experimental data is in good agreement with the results of the simulation. This implies that in this case the variation in the measurements can be totally explained by the statistics of the measurement, without any contributions from shot-to-shot variations in the actual temperature.

There are many other uses for this code besides the one mentioned above. Using the code we were able to simulate the response of the system to different conditions such as changes in density and temperature. The most important function of the code has been to evaluate the various possible design changes, including spectral channel positions, photomultiplier quantum efficiency, and gate width and to choose the ones that would give the best improvement in performance without having to actually build the systems. This approach is not limited to Thomson scattering analysis, but could be applied to any system affected by counting statistics.

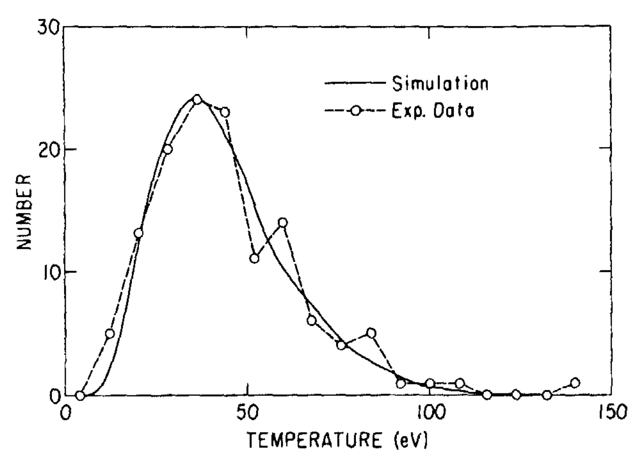

FIG. 1. Comparison between a histogram of 121 shots of experimental data from the S-1 spheromak and a histogram of the results of a run of the code with one million samples at a temperature of 40 eV, a signal level of 50 photoelectrons, and a background level of 63 photoelectrons. The simulation is scaled down to match the scale of the data.

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