Verification of the Taylor (minimum energy) state in a spheromak

G. W. Hart, A. Janos, D. D. Meyerhofer, and M. Yamada

Citation: Physics of Fluids (1958-1988) **29**, 1994 (1986); doi: 10.1063/1.865627
View online: http://dx.doi.org/10.1063/1.865627
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof1/29/6?ver=pdfcov
Published by the AIP Publishing

Verification of the Taylor (minimum energy) state in a spheromak

G. W. Hart,[a)] A. Janos, D. D. Meyerhofer, and M. Yamada
Princeton Plasma Physics Laboratory, Princeton, New Jersey 08544

(Received 4 December 1985; accepted 24 February 1986)

Experimental measurements of the equilibrium in the S-1 spheromak [M. Yamada, J. Sinnis, H. P. Furth, M. Okabayashi, G. Sheffield, T. H. Stix, and A. M. M. Todd, in *Proceedings of the US-Japan Symposium on Compact Toruses and Energetic Particle Injection* (Princeton Plasma Physics Laboratory, Princeton, NJ, 1979), p. 171] by use of magnetic probes inside the plasma show that the final magnetic equilibrium is one that has relaxed close to the Taylor (minimum-energy) state, even though the plasma is far from that state during formation. The comparison is made by calculating the two-dimensional μ profile of the plasma from the probe data, where μ is defined as $\mu_0 j_{\|}/B$. Measurements using a triple Langmuir probe proved evidence to support the conclusion that the pressure gradients in the relaxed state are confined to the edge region of the plasma.

I. INTRODUCTION

Of fundamental importance to understanding the physics of spheromaks is the concept of the Taylor minimum energy state. Taylor[1] showed that the minimum energy configuration for a given global magnetic helicity (defined as $\int dV\, \mathbf{A} \cdot \mathbf{B}$, where the integration is over the whole plasma volume) was the state where $\mathbf{j} = (\mu/\mu_0)\mathbf{B}$, where μ is a constant in space. Here μ_0 is the permittivity of free space, $\mathbf{A}$ is the vector potential, $\mathbf{B}$ is the magnetic field satisfying $\mathbf{B} = \nabla \times \mathbf{A}$, and $\mathbf{j}$ is the current density. It is expected that a plasma has a tendency to approach this state through some sort of energy dissipative process. The purpose of this paper is to show that the measured magnetic structure of the S-1 spheromak[2] plasma in its equilibrium state is close to the Taylor minimum energy state.

The reversed-field pinch and the spheromak are the two major divisions in the class of configurations based on the Taylor state. They differ mainly in that the toroidal field in the spheromak is generated by plasma currents rather than externally imposed by toroidal field coils, and therefore the spheromak need have no conductors or transformers that link the plasma torus.

The S-1 spheromak was designed to produce a spheromak configuration by the inductive generation of both poloidal and toroidal currents.[2] As these currents are injected into the plasma, the configuration relaxes to the one with the minimum magnetic energy allowed by the constraints on the system. The conjecture that the primary constraint is the conservation of the injected helicity leads to the Taylor state. It is important in gaining an understanding of the physics of spheromaks to see how close this condition is to being fulfilled in the many different ways that spheromaks can be formed.

Several authors have studied force-free states theoretically.[3–5] Experimentally, Turner *et al.*[6] examined the field profiles in their gun-generated spheromak and found that they matched those expected for the Taylor state. Conversely, Hart *et al.*[7] in their combination z-θ pinch-formed spheromak found by comparison of both field and $j_{\|}/B$ profiles that the inclusion of pressure gradient terms was important in their equilibrium. Recently, Knox *et al.*[8] have used the image currents in the flux conserver of their gun spheromak to characterize their force-free equilibrium and have found deviations from the Taylor state.

Earlier measurements[9] on S-1 indicated that there was a preferred state for the final equilibrium that was very insensitive to the initial currents injected into the plasma. Recently, it was found[10] that resistive tearing modes caused conversion of poloidal flux to toroidal flux, thus allowing the relaxation of the plasma to its preferred state. The amplitude of these magnetic fluctuations dropped to the noise level of $\leqslant 2\%$ after the formation phase of the discharge. These measurements are in accord with Taylor's hypothesis of a global relaxation to a minimum-energy state constrained by a single conserved quantity.

The clearest way to determine how close this relaxed plasma is to the Taylor state is to measure the μ profile, where μ is defined by $\mu_0 \mathbf{j} \cdot \mathbf{B}/B^2$. If μ is constant in space, then the plasma is in the Taylor state and μ is the eigenvalue of the equilibrium, $\nabla \times \mathbf{B} = \mu \mathbf{B}$. Solving the axisymmetric Grad–Shafranov equation, it can be shown that

$$\mu = \mu_0 \frac{\mathbf{j} \cdot \mathbf{B}}{B^2} = I'(\psi) + \mu_0 \frac{p'(\psi) I(\psi)}{B^2}, \qquad (1)$$

where $I(\psi) = RB_t$ is the poloidal current stream function, $p(\psi)$ is the pressure, and the prime indicates differentiation with respect to ψ. In order for μ to be a spatial constant, the second term must vanish. Since its numerator is a function of ψ only, and the B^2 in the denominator is not, in general, that term will not even be constant on a flux surface and, therefore, cannot be constant in the whole plasma volume as required by the Taylor state. Therefore, the constancy of μ requires that I be a linear function of ψ, so that $I'(\psi)$ is constant, and $p'(\psi) = 0$.

It is interesting to note that a low-temperature spheromak plasma can have a large β. In the results of Hart *et al.*[7] on the PS Spheromak, significant deviations from the force-free constant μ state resulting from finite plasma pressure effects were found in a low-temperature (5 eV) plasma.

[a)] Present address: Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84602.

II. EXPERIMENTAL RESULTS

We report on this measurement for a typical case in S-1.[11] Because the measurements were made with magnetic probes inserted directly into the plasma, we were limited to measuring a plasma with an electron temperature of less than about 30 eV. The electron density was in the range of 3–8×10^{13} cm^{-3}, the peak poloidal field was 1.8 kG, and $\langle\beta\rangle\sim5\%$.

The magnetic probe was supported by a boom along the symmetry axis of the machine and extended radially into the plasma. Full profiles of the magnetic field in both the radial and axial directions could be obtained by moving the probe in the axial direction on a discharge-to-discharge basis. The probe was also rotated toroidally to confirm the axisymmetry of the fields. The reproducibility required was possible because of the installation of figure-8 coils[11] to stabilize the $n=1$ shift mode during the initial part of the discharge. The data were integrated using active integrators, and then digitized and read by the computer system.

From these two-dimensional plots of the magnetic field structure, many of the quantities essential for characterizing the equilibrium can be calculated, including the poloidal flux ψ, the poloidal current RB_t, all three components of the current density j, the q profile, and the μ profile.

A set of contour plots of the poloidal flux showing the time history of a discharge from 0.2 to 0.5 msec is shown in Fig. 1. Formation is completed by 0.4 msec. After 0.5 msec the $n=1$ shift mode sets in and terminates the discharge. If the flux core were not present, the separatrix radius in the midplane at 0.5 msec would be about 1 m.

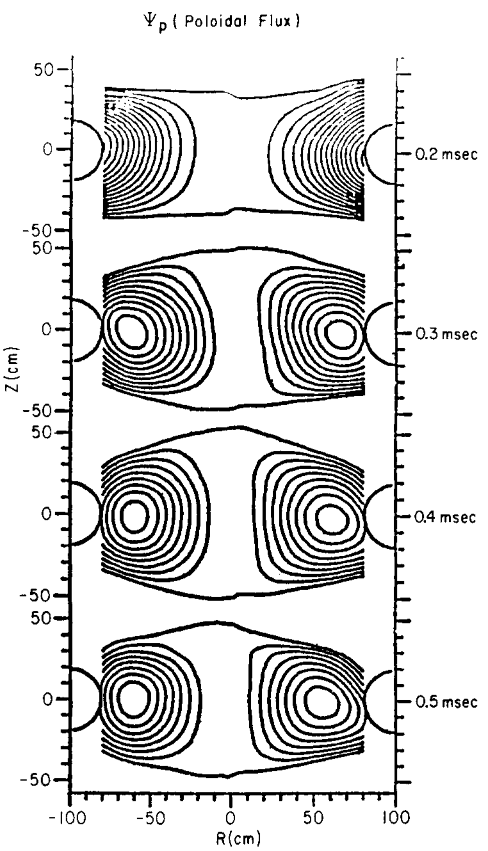

FIG. 1. Contour plot of poloidal flux in S-1 showing the time evolution of this discharge. The contour increments are 10 mWb. After 0.5 msec, the discharge terminates in an $n=1$ shift instability.

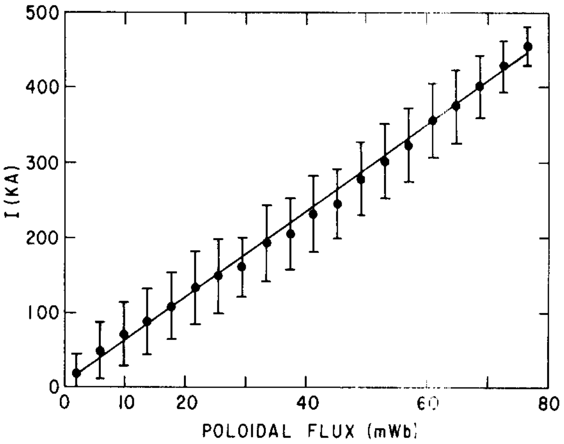

FIG. 2. Experimentally determined measurements of $I(\psi)=RB_t$ vs ψ. The slope of this curve is expected to be equal to μ in the Taylor state.

As a first check on whether the plasma is in a Taylor state at 0.5 msec, we examine a plot of I vs ψ as shown in Fig. 2, where $\psi=0$ represents the symmetry axis of the configuration. As can be seen, the fit is indeed linear. This is not true during formation; at 0.2 msec, the slope of the $I(\psi)$ curve is 2.5 times greater near the magnetic axis than near the separatrix. This indicates that the plasma is far from the Taylor state initially.

In a Taylor state, the slope of the I vs ψ curve is equal to the eigenvalue of the equilibrium. For a given plasma shape, the eigenvalue is inversely proportional to the midplane separatrix radius. The value of 5.5 for the eigenvalue, taken from the I vs ψ plot, combined with the length to radius ratio of 0.53, determined from the flux plots shown in Fig. 1, implies a separatrix radius of 1.0 m, based on the Taylor theory. This is in good agreement with that from the flux plot.

In order to calculate the μ profile, the current density must be calculated. The magnetic field data are first interpolated using a polynomial fit. This facilitates taking the curl of the fields to calculate the current density. The calculation assumes axisymmetry. Using this result to calculate the μ profile, we get the results shown in Fig. 3. Because of the

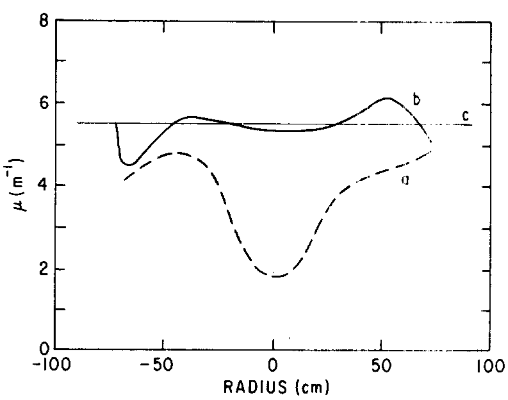

FIG. 3. A plot of $\mu=\mu_0\mathbf{j}\cdot\mathbf{B}/B^2$ across the midplane of the S-1 device. Curve a is at 0.2 msec, which is during the formation phase. Curve b is at 0.5 msec, which is after the equilibrium is established, and curve c is the expected value of curve b, based on the slope in Fig. 2.

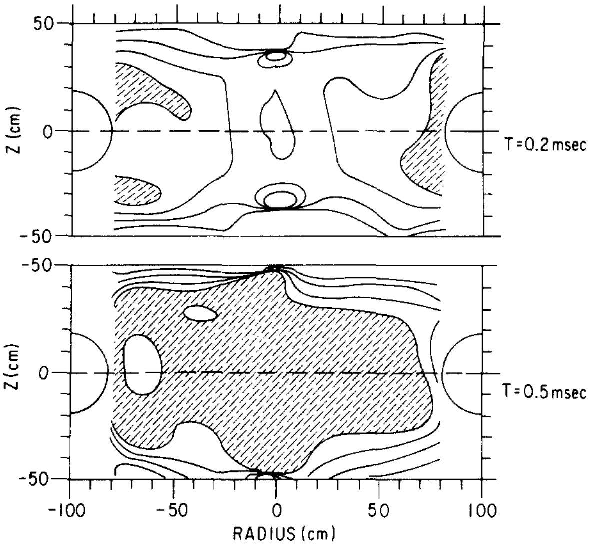

FIG. 4. A contour plot of μ both during formation and after. The shaded areas are those that are within $\pm 15\%$ of the nominal value of 5.5. The dashed line across the midplane shows where the curves of Fig. 3 were taken.

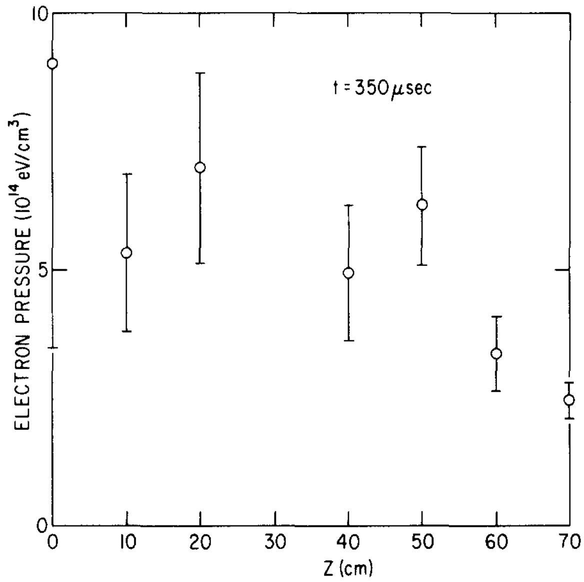

FIG. 5. Experimentally measured pressure profile, made by using a triple Langmuir probe. The plasma separatrix is located near $z = 50$ cm.

limitations of the numerical differentiation, the calculation of μ is only valid to $\pm 18\%$.

At 0.2 msec, shown as curve a in Fig. 3, there is a deficit of current density near the symmetry axis because the driving electric fields are strongest near the flux core. This is also reflected in the previously mentioned deviations of the $I(\psi)$ curve from linearity at this time. But by 0.5 msec into the discharge, the current has diffused into the center to produce a flat μ profile (curve b). The straight line (curve c) indicates the value of the slope of the I vs ψ curve at this time. The variation of curve b from the expected value is within the error bars of the measurement.

Figure 4 is a contour plot of μ across the plasma cross section at the same two times as in Fig. 3. At early times, the μ profile peaks near the flux core and has a current hole near the symmetry axis. At 0.5 msec, the profile is basically flat throughout the plasma volume, with the gradients all occurring near the separatrix. The shaded areas are those that are within 15% of the nominal value of 5.5 m^{-1}.

If the pressure gradient were nonzero in the plasma, the effect on μ can be estimated. Hart *et al.*[7] show that if $I(\psi) = k\psi$ and $p(\psi) = P\psi^{\nu}$ then

$$\mu = k(1 + \nu\beta/2), \tag{2}$$

where the local $\beta = 2\mu_0 p(\psi)/B^2$. If the pressure was linear with ψ, in a plasma with $\langle\beta\rangle \sim 5\%$, the peak local β would be roughly 20% so that the deviation of μ from a constant would be 10%, within the error bars of the measurement.

The measurement of the electron pressure profile shows that the pressure gradient is near zero in the plasma interior. This supports the conclusion that μ is a constant in the plasma and that the plasma is in a Taylor state. A triple Langmuir probe[12] is used to measure the temperature and density as a function of time and space in a set of discharges similar to those discussed above. The discharges for this measurement differ in that these discharges are slightly less oblate than the case shown in Fig. 1. As shown in Fig. 5, the triple probe shows the pressure to be flat along z (parallel to the symmetry axis of the device at $R = 45$ cm) within the error bars of the measurement, dropping off near the separatrix, which is at $z = 50$ cm.

III. CONCLUSION

By examination of the two-dimensional field structure of the S-1 spheromak, we have determined that the equilibrium established in these low-temperature discharges is well described by the Taylor state model, with the deviations from the force-free state occurring near the edge of the plasma. Additional supporting evidence for this has been obtained by measuring the pressure profile directly. The experimental data for the S-1 spheromak equilibrium describe a state that is very similar to the quiescent phase of RFP discharges. In higher temperature plasmas, transport effects will be a mechanism tending to drive the plasma away from the Taylor state.

ACKNOWLEDGMENTS

This work supported by U.S. Department of Energy Contract No. DE-AC02-76–CHO-3073.

[1]J. B. Taylor, Phys. Rev. Lett. 33, 1139 (1974).
[2]M. Yamada, J. Sinnis, H. P. Furth, M. Okabayashi, G. Sheffield, T. H. Stix, and A. M. M. Todd, in *Proceedings of the US-Japan Symposium on Compact Toruses and Energetic Particle Injection* (Princeton Plasma Physics Laboratory, Princeton, NJ, 1979), p. 171.
[3]T. H. Jensen and M. S. Chu, Phys. Fluids 27, 2881 (1984).
[4]S. Riyopoulos, A. Bondeson, and D. Montgomery, Phys. Fluids 25, 107 (1982).
[5]P. M. Bellan, Phys. Rev. Lett. 54, 1381 (1985).
[6]W. C. Turner, G. C. Goldenbaum, E. H. A. Granneman, J. H. Hammer, C. W. Hartman, D. S. Prono, and J. Taska, Phys. Fluids 26, 1965 (1983).
[7]G. W. Hart, C. Chin-Fatt, A. W. DeSilva, G. C. Goldenbaum, R. Hess, and R. S. Shaw, Phys. Rev. Lett. 51, 1558 (1983).
[8]S. O. Knox, C. W. Barnes, G. J. Marklin, T. R. Jarboe, I. Henins, H. W. Hoida, and B. L. Wright, Phys. Rev. Lett. 56, 842 (1986).

[9]A. Janos, in *Proceedings of the Sixth U.S. Symposium on Compact Toroid Research and the Fifth U.S.-Japan Joint Symposium on Compact Toroid Research* (Princeton Plasma Physics Laboratory, Princeton, NJ, 1984), p. 97; submitted to Phys. Fluids.
[10]A. Janos, G. W. Hart, C. H. Nam, and M. Yamada, Phys. Fluids **28**, 3667 (1985).
[11]M. Yamada, R. Ellis, Jr., H. P. Furth, G. Hart, A. Janos, S. Jardin, F. Levinton, D. Meyerhofer, M. Mimura, C. H. Nam, S. Paul, A. Sperduti, S. von Goeler, F. Wysocki, and P. Young, in *Proceedings of the Tenth International Conference of Plasma Physics and Controlled Nuclear Fusion Research* (IAEA, Vienna, 1985), Vol. II, p. 535.
[12]S. L. Chew and T. Sekiguchi, J. Appl. Phys. **36**, 2363 (1965).