LETTERS TO THE EDITOR

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Extraction of plate bending stiffness from coincidence angles of sound transmission measurements (L)

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The bending stiffness in a homogeneous, isotropic, thin plate is experimentally derived from measurements of coincidence angles extracted from supercritical sound transmission versus frequency measurements. A computer controlled turn table rotates a plate sample and a receiver array, placed in the near field of the plate. The array is used to track the transmitted sound through the plate, generated by a far-field stationary source, using beam forming. The array technique enables measurement of plates measuring only one wavelength in width. Two examples are used for proof of concept, including an aluminum plate in air and an alumina plate under water.
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I. INTRODUCTION

Bending stiffness, or flexural rigidity, is most commonly measured using static methods by measuring deflection in cantilever beam samples. These measurements typically require certain specific sample sizes in order to ensure elastic deformation and so that the moment of inertia can be predicted properly. Numerous static methods exist for these purposes. The bending stiffness or bending wave speed can also be extracted using guided waves, as described in the review article by Chimenti[1] and the recent work by Cornwell and Berthelot.[2]

An alternative method was introduced by Luukkala *et al.*[3] to measure the bending stiffness of copy paper using sound transmission. Their work set up a stationary sample under test and utilized a loudspeaker and a microphone that were mounted to a rotation stage. As they rotated the loudspeaker and microphone, the sound transmitted through the sample was measured. Increases in the sound transmission could be seen at the coincidence angles which correspond to the matching of the bending wave number in the sample with the component acoustic wave number parallel to the sample. Their work relied on the high directivity of their loudspeaker to prevent diffraction. Thus the sample under test had to be sufficiently large to reduce diffraction around the sample for the frequency range of interest. Their samples measured 50 cm square in size or approximately 150 wavelengths square in size for the nominal lowest frequency they used (100 kHz).

The purpose of this paper is to present a method to extract the bending stiffness of submerged or airborne plate samples using the array beam forming insertion loss measurement.[4,5] This method is used to measure sound transmission through the plate under test as a function of angle at various frequencies, similar to the work of Luukkala *et al.* For thin plates, above the critical frequency of the plate, there exist angles of maximal sound transmission mirrored about normal incidence, called coincidence angles. By extracting the coincidence angle as a function of frequency from measurement and fitting it to a theoretical expression, the bending stiffness can be calculated, assuming the mass density is determined. The advantage of the technique used here, developed by Anderson *et al.*[4] and Shaw and Anderson,[5] over that proposed by Luukkala *et al.*, is that a line array of sensors is used as a beam former to increase the sensitivity of the receiver towards the sound source and reduce the sensitivity to diffracted waves, thereby allowing testing of smaller plate samples. Here we test samples that are approximately 1 wavelength in the width dimension for the lowest frequency tested (25 kHz). This enables a greater flexibility in the size required for acoustical extraction of a sample's bending stiffness.

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Coincidence angles in sound transmission occur when the stiffness and mass terms governing the transmission cancel each other, representing a type of resonant transmission of sound whose amplitude is governed by the damping in the plate. The coincidence angle, θ_{CO}, is found at grazing angles ($\pm 90^\circ$ from normal incidence) at the critical frequency (the frequency at which the plate's anti-symmetric bending wave speed equals the sound speed in the fluid) and it decreases towards normal incidence as frequency increases according to the following Bernoulli–Euler plate theory equation:[6]

$$\sin^2\theta_{CO} = \frac{c^2}{\omega}\sqrt{\frac{m}{D}}, \quad (1)$$

where c is the fluid sound speed, ω is the angular frequency, m is the mass per unit area of the plate, and D is the bending stiffness which depends on Young's modulus and Poisson's ratio.

II. EXPERIMENTAL METHODS

Two sets of measurements of coincidence angles versus frequency are used as proof of concept for the proposed method of extracting the bending stiffness. One experiment utilizes a $1.02 \times 3.05 \times 24.38\,\text{cm}^3$ alumina plate that is submerged in water, and the other utilizes a $0.05 \times 3.8 \times 40.6\,\text{cm}^3$ aluminum plate in air.

For the submerged alumina plate, eight piezoelectric Tonpilz transducers were mounted to the plate with a polyurethane layer of 1.52 mm thickness between the transducers and the plate. The transducers were housed in an air-filled array module, which isolated the array from diffraction paths because of the impedance mismatch. A polyurethane acoustic window between the transducers and the water was used to isolate the array components from the water and for impedance matching to the water. The module was rotated with a computer-controlled system, and a source was placed approximately 6.1 m from the array module. Additional details on the experimental setup for the submerged alumina plate may be found in closely related studies published in Refs. 4, 7, and 8. Photos of the setup may be found in Figs. 1 and 2 of Ref. 7.

For the airborne aluminum plate, eight 6.35-mm (1/4-in.) G.R.A.S. microphones were mounted in a line array 5 mm behind the plate. The plate was taped to a foam baffle that helped to block sound energy from diffracting around the plate. The foam, plate, and microphones were attached to a computer-controlled turntable located in an ultrasonic anechoic chamber.[9] Three ultrasonic sources (one for each of three frequency bands) were placed approximately 90 cm from the plate. Additional details on the experimental setup for the airborne aluminum plate may be found in a closely related study published in Refs. 5 and 10. Photos of the setup may be found in Fig. 1 of Ref. 5.

In each experiment, the source emitted a narrowband signal, and a complex transfer function between the source signal and each of the eight receiving transducers was obtained. This procedure was repeated at each angular rotation increment between $\pm 90^\circ$. For each particular rotation increment, the transfer functions were then steered using beam forming in the direction of the source transducer, thus ensuring that the receiving array was most sensitive to sound incident from the desired direction (and less sensitive to waves diffracting around the plate and radiation from reflected waves traveling in the opposite direction). The experiment was conducted with and without the plate in place and thus an insertion loss, or transmission loss, could be determined by comparing the steered array sensitivity values with and without the plate. References 4 and 5 further explain the beam forming technique used.

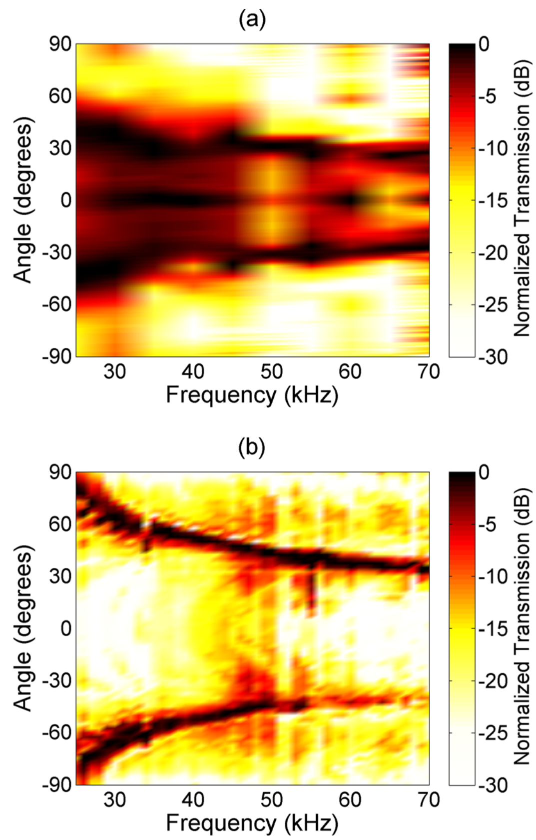

FIG. 1. (Color online) Normalized transmission versus rotation angle and frequency (a) for the submerged alumina plate and (b) for the airborne aluminum plate. The transmission versus angle measurement at each frequency is normalized to its peak value.

Normalized transmission results for each experiment are shown in Fig. 1 versus rotation angle and frequency. The data at each frequency are normalized to their respective peak values. One can note the coincidence angle ridges in each set of transmission data that vary from approximately $\pm 41^\circ$ at 25 kHz to $\pm 27^\circ$ at 70 kHz for the submerged alumina plate, and from approximately $\pm 90^\circ$ at 25 kHz to $\pm 38^\circ$ at 70 kHz for the airborne aluminum plate. Coincidence angles were extracted at each frequency by averaging the angles corresponding to the maximum sound transmission on either side of normal incidence.

III. RESULTS

The extracted coincidence angles versus frequency are shown in Figs. 2 (a) and 2(b) for the submerged alumina and airborne aluminum plates, respectively. A least squares optimization algorithm was performed to determine an optimal

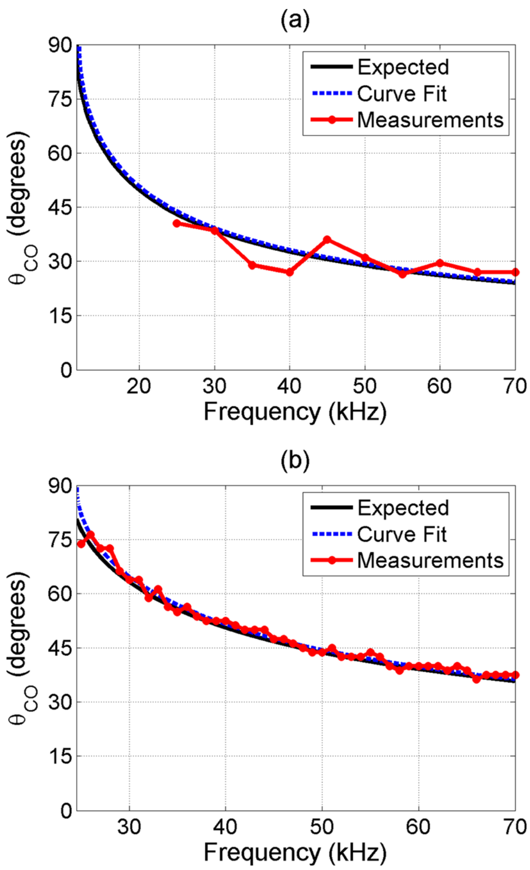

FIG. 2. (Color online) Coincidence angle as a function of frequency (a) for the submerged alumina plate and (b) for the airborne aluminum plate. The solid line represents the expected theoretical values when the manufacturer's specified material property values are used. The dashed line represents the optimal curve fit to the measured data. The solid line with circles represents the measured data with the circles appearing at the measurement frequencies.

bending stiffness value by fitting Eq. (1) to the data obtained for each experiment. The optimal fits to the data for each case are depicted in Fig. 2. The expected values, obtained by evaluating Eq. (1) with the manufacturer's published values are also depicted in Fig. 2. Note that the lowest frequency depicted in each subplot of Fig. 2 represents the critical frequency for the plate under test (at this frequency the plate's dispersive bending wave speed equals that of the speed of sound in the fluid).

In the case of the submerged alumina plate, the bending stiffness determined from using the measured value of $E = 391$ GPa, the manufacturer's published value of $\sigma = 0.22$, and a plate thickness of 10.2 mm is $D = 36\,000\,\mathrm{kg\,m^2/s^2}$ (units of momentum). The fitted value to the measured coincidence angle data is $D = 38\,000\,\mathrm{kg\,m^2/s^2}$. This represents a 5% difference.

In the case of the airborne aluminum plate, the bending stiffness determined from using the manufacturer's published values for aluminum alloy 2024 of $E = 73.1$ GPa and $\sigma = 0.33$ with a plate thickness of 0.5 mm, is $D = 0.855\,\mathrm{kg\,m^2/s^2}$ (units of momentum). The value from fitting the measured data is $D = 0.810\,\mathrm{kg\,m^2/s^2}$, representing a 5% difference. As assumed here, if the density is known and the Poisson's ratio is known, then the Young's modulus may be extracted, or if the Young's modulus is known then the Poisson's ratio may be extracted.

The discrepancy between the expected value of the bending stiffness (the calculated value based on material properties) and the extracted value from the curve fitting may be due to errors in assumed values for the material properties, a low density of frequencies for transmission measurements in the case of the submerged alumina plate, a course angular resolution in the case of the airborne aluminum plate (2.5° increments), or a breakdown of the Bernoulli–Euler beam theory. Further improvement could be made by averaging values through several measurement iterations.

IV. CONCLUSIONS

This paper has shown that one can extract the bending stiffness of a plate under test by measuring the coincidence angles of sound transmission as a function of frequency with 5% accuracy. Two experiments were presented, one using a submerged alumina plate in water, and the other with an airborne aluminum plate. This method provides a noncontact method of the measurement of bending stiffness of smaller plate samples (with a width of one wavelength) than previously reported studies (with a width of 150 wavelengths). However, it also requires an array of sensors for increased directional sensitivity as well as an anechoic measurement environment, or a large enough measurement facility to window out reflected sound from non-anechoic walls.

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