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## Improving time reversal focusing through deconvolution: 20 questions

T. J. Ulrich\*, Brian Anderson, Pierre-Yves Le Bas, Cedric Payan, Johannes Douma and Roel Snieder

\*Corresponding author's address: Mail Stop D443, EES-17: GEOPHYSICS LANL, Los Alamos, New Mexico 87545, [tju@lanl.gov](mailto:tju@lanl.gov)

Time reversal is a proven technique for focusing wave energy in time and space. A known side effect of the process is the presence of temporal side lobes and spatial fringes, which can be undesirable depending upon the application. A deconvolution, also known as an inverse filter, technique has been developed to improve temporal compression of the focal signal. This presentation will explore the use of deconvolution in comparison to standard time reversal and provide insight into the process through visualizations of the wave-fields. Attention will also be given to the effect of the deconvolution technique upon nonlinear elasticity applications such as the time reversed nonlinear elasticity diagnostic (TREND).

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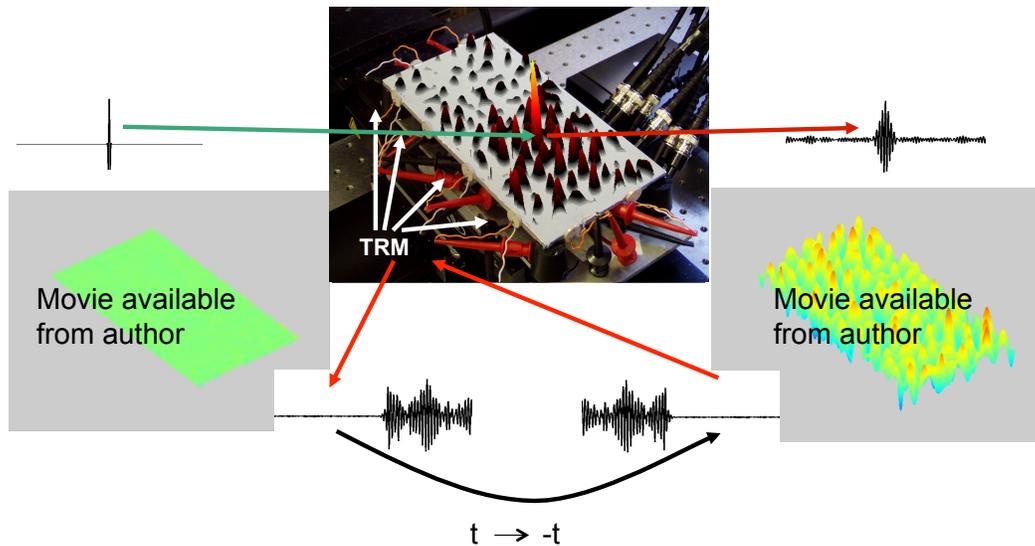
## Introduction

Activity in time reversal (TR) research has been focused on many activities and applications since it was first employed by Parvelescu and Clay in the 1960's.<sup>1</sup> One of the areas with broad implications to all TR based research is in the improvement of the TR focus process through signal processing. *Improvement* can be a subjective word, so here we will refer to *improving* the temporal and spatial compression abilities of TR; thus, *improving* TR focusing implies tighter compression in both time and space. Various methods have been developed by others to accomplish one or both of these goals (i.e., time and/or space improvements).<sup>2-15</sup> Generally this involves large arrays of sources and receivers and an elaborate methodology of processing. In the case of some of these methods, one of the focus attributes may be improved, e.g., temporal compression, at the expense of the other, e.g., space. The goal of this work was to develop a simple processing scheme, with little a priori information being required, for use with small time reversal mirrors (i.e., transducer arrays), perhaps even with a single channel. Additionally, the focus of this work is done in solid media using elastic waves, and testing the developed procedure in this complex medium. Finally, the application for which this is being developed is to better focus energy in solid materials for the purpose of detecting and imaging cracks and other nonlinear mechanical defects. Initial measurements on all above topics will be covered herein. This work will be presented as a series of questions and answers as indicated by the *20 questions* portion of the title.



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## What do you mean by “Time Reversal”?



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Q1: What do you mean by Time Reversal?

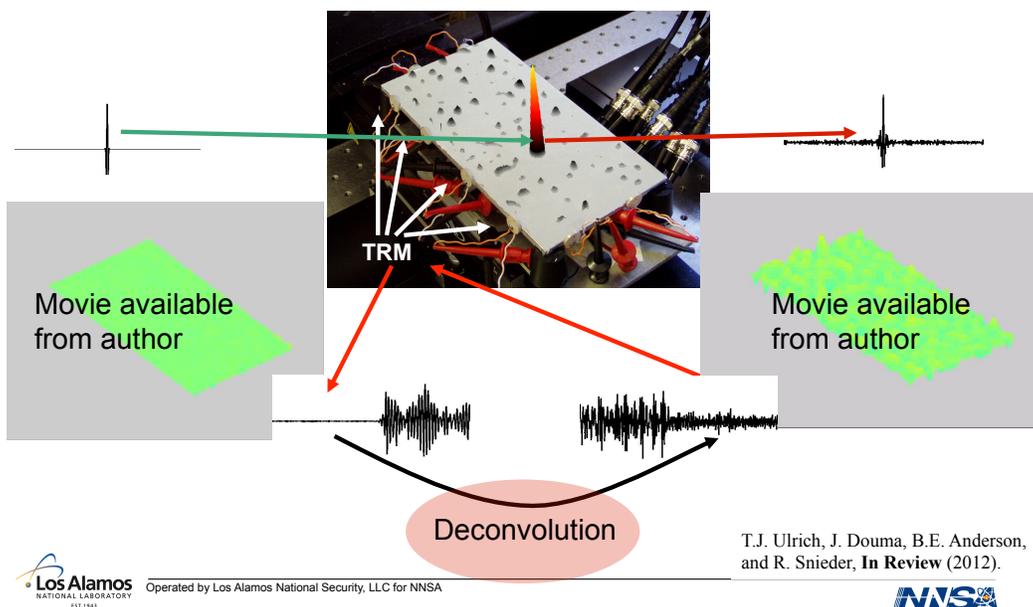
A1: Time Reversal (TR) is simply the use of reciprocity and time reversal invariance of the wave equation.<sup>16-17</sup> It is called Time Reversal because the process involves recording a response,  $R(t)$ , due to a source function,  $S(t)$ , and then broadcasting  $R(-t)$  for the purpose of reconstructing some version of  $S(t)$ , thus focusing  $S(t)$ . Usually this is explained as the source emitting at location A, received at location B, the  $R(-t)$  being broadcast at location B and the resulting focus (sometimes referred to as the spatial and temporal compression) occurring at location A. Quite often, however, the receiver at location B is not capable of being used as a source. In this case a method referred to as Reciprocal Time Reversal is employed where  $R(-t)$  is broadcast from location A and the focus occurs at location B. This form of TR is commonly used to focus energy, which is then used as a probe for material properties, such as in the case of the Time Reversal Elastic Nonlinearity Diagnostic (TREND).<sup>18</sup> It is this reciprocal version of TR, which is used exclusively in this study.

NOTE: TR is also known as matched filtering, phase conjugation and auto-focusing.



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## How is deconvolution different?



Q2: How is deconvolution different?

A2: In principle, and in application, it is nearly identical to TR. The difference lies in the processing between reception of  $R(t)$  and broadcast of  $R(-t)$ . To be more specific  $R(-t)$  is not used at all, rather a deconvolution operation is performed on the received signal  $R(t)$ , and it is this new signal,  $g(t)$ , which is broadcast from the original source location and used to focus the wave energy. We will cover more details of the deconvolution operation that is performed, but we will skip that momentarily to answer a few other questions.

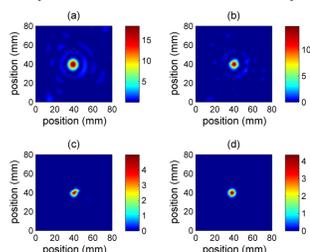
NOTE: Movies are available showing wave fields (space and time) for 1) the forward propagation, 2) the TR focusing and 3) the deconvolution focusing by contacting the author ([tju@lanl.gov](mailto:tju@lanl.gov)).



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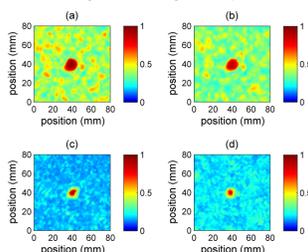
## What does it look like?

### Squared Wave Field Map



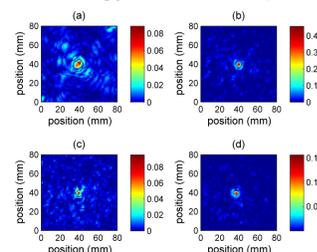
- a) Classical TR (pulse)
- b) Classical TR (chirp)
- c) Deconvolution (pulse)
- d) Deconvolution (chirp)

### Symmetry Map



Ulrich et al, Symmetry-based imaging condition in time reversed acoustics, *J. Applied Phys.*, **104** (6), (2008)

### Energy Current Map



Anderson et al, Energy current imaging method for time reversal in elastic media, *Appl. Phys. Lett.* **95**, 021907 (2009).

Deconvolution appears to have a tighter temporal and spatial focus for all imaging conditions.



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Q3: What does it look like?

A3: There are many ways to visualize the TR and deconvolution focusing processes. Here we have chosen to use three imaging conditions: 1) the instantaneous amplitude taken at the focal time for each point in space; 2) a spatial map of the symmetry imaging condition;<sup>19</sup> 3) a spatial map of the energy current imaging condition.<sup>20</sup> The first of these images is squared to provide a fair comparison of one imaging condition to the others.

Also, we have used various  $R(t)$ 's from different source functions. First is the direct  $R(t)$  corresponding to an impulse like source (a). The second is an impulse response  $R'(t)$ , calculated from a cross-correlation of a chirp source with the received signal from the chirp,

$$R'(t) = S(t) \otimes R(t) .$$

Both  $R(t)$  from (a) and  $R'(t)$  from (b) were used in the deconvolution process to produce the results for (c) and (d).

For all imaging conditions, the deconvolution focus is tighter than the TR focus.



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## What's going on?



Time Reversal

$$F_{TR}(t) = R(-t) \star R(t) \approx \delta(t)$$

$$F_{TR}(\omega) = R^*(\omega) \times R(\omega) \approx 1$$



Deconvolution

$$F_D(t) = g(t) \star R(t) \equiv \delta(t)$$

$$F_D(\omega) = g(\omega) \star R(\omega) \equiv 1$$

$$g(\omega) = \frac{1}{R(\omega)}$$

For more details search keyword "inverse filter" (esp. Tanter in TR context)



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Q4: What's going on?

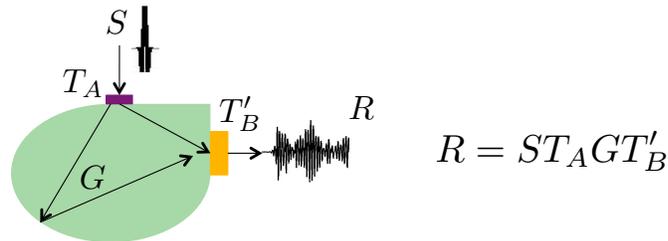
A4: Getting back to the details of the deconvolution we can compare the standard TR focusing process, which amounts to an auto-correlation of the original received signal, to the deconvolution focusing procedure. The deconvolution makes the assertion that a  $g(t)$  can be found, from the received signal, that will optimally focus. Transforming into the frequency domain (where we will remain for the rest of this paper, unless otherwise noted) we can see that the deconvolution of  $R(t)$  with a  $\delta(t)$  results in what is commonly referred to as an inverse filter.

Tanter et al have shown that an inverse filter improves temporal focusing when used with large arrays and processed in a more sophisticated fashion employing singular value decomposition (SVD) on large matrices of signals.<sup>2,6</sup> Here we explore the ability to do this more simply using only a single channel and a single operation where no SVD is necessary, and with the complex structure of elastic waves in a bounded solid medium.



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## What's Really Going On?



### Time Reversal Focusing

$$F_{TR} = (ST_AGT'_B)^* T_AGT'_B$$

$$= |T_A|^2 |T'_B|^2 S^*$$

 $\neq$ 

### Deconvolution Focusing

$$F_D = \frac{1}{ST_AGT'_B} T_AGT'_B$$

$$= \frac{1}{S}$$

Deconvolution compensates for the transducer response.



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Q5: What's really going on?

A5: To get a more physical handle on what is happening we need to consider a realistic system. In an elastic wave experiment we typically have a transducer acting as a source (at location A). A source function  $S$  is put into the transducer, which changes  $S$  by its own response ( $T_A$ ). The signal then scatters through the medium with Green function  $G$  and is recorded at location B by a receiver, which also changes the reception with its receiver response ( $T'_B$ ). Thus the received signal  $R$  is simply the product of all of these responses, assuming we operate in the frequency domain. Putting  $R^*$  (i.e., the complex conjugate of  $R$ , equivalent to a time reversal operation in the time domain) into the source transducer at A and following the same path through the system we can arrive at a mathematical representation of the TR focused signal. Similarly we can do the same for the deconvolution, or inverse filter,  $R^{-1}$ . Doing so reveals the differences between TR and deconvolution focusing, where it is apparent that the TR process is heavily colored by the source and receiver responses, while the deconvolution is not. Further the deconvolution recreates an inverted version of the source function, a fact overlooked in previous applications of the inverse filter for TR-type focusing due to the fact that sources are typically assumed (or designed) to be delta functions.



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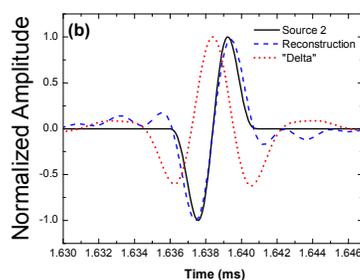
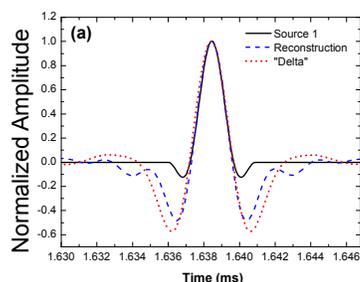
## But I wanted a delta function. How do I get it?

Recall:

$$F_D = \frac{1}{ST_A GT'_B} T_A GT'_B = \frac{1}{S}$$

So just multiply by  $S$

$$F_\delta = \frac{S}{ST_A GT'_B} T_A GT'_B = 1$$



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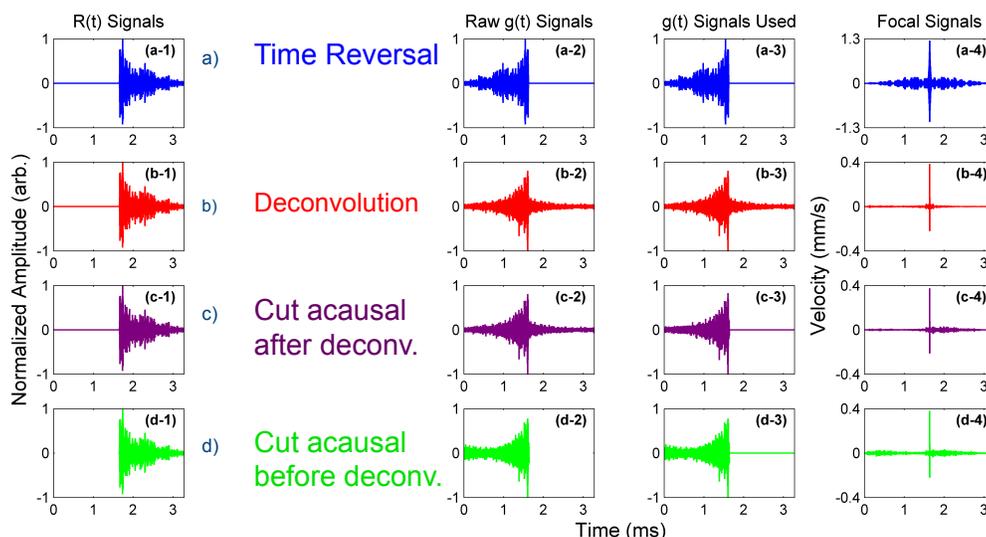
Q6: How do I focus a delta function?

A6: To truly focus a delta function is very difficult as one must have an infinite bandwidth signal to reconstruct a true  $\delta(t)$ . It is possible, however, to approximate a  $\delta(t)$  with whatever bandwidth is present. To do so, one must simply multiply (in the frequency domain) the original source function by the deconvolution, or inverse filter, result (i.e.,  $S(\omega)g(\omega)$ ). Following this through a reciprocal TR type experiment, as was done previously, you can see the  $F_\delta$  will approximate a  $\delta(t)$ . This can be done experimentally and compared to some other impulse-like source function. Here source function 1 was a 200kHz toneburst (single period) in a  $\sin^2$  envelope, while source function 2 is identical to source function 1 with the exception of a  $90^\circ$  phase shift. This means that each source function has the exact same bandwidth available, thus performing a deconvolution focusing experiment with and without the multiplication of the original source function (i.e., to approximate a  $\delta(t)$ ) should work equally well, or poorly, for either of these responses. When the experiments are performed as prescribed and we can see that deconvolved responses  $g_1(t)$  and  $g_2(t)$  each focus and are able to reconstruct their respective source functions quite well. Additionally, they each produce an identical approximation for a  $\delta(t)$ , as expected due to identical bandwidths being present in each signal.



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## So how do I do it?



Need acausal signal before deconvolution for cleanest focus.

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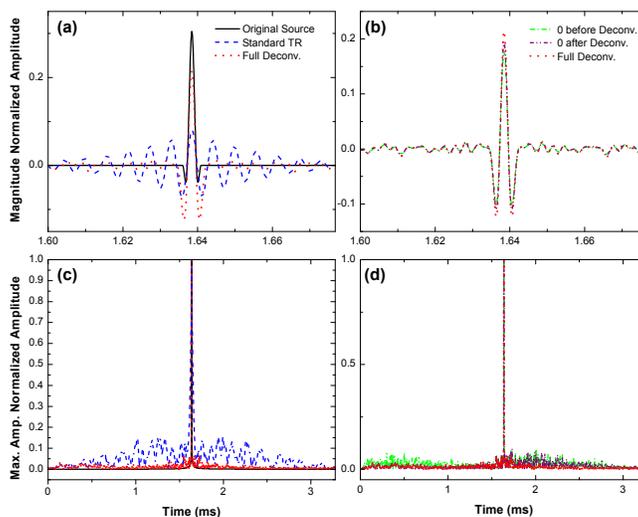
Q7: How do I really do this?

A7: In order to actually perform an experiment of this type there are a few details that can easily be missed, notably, the optimal signal to be used for the deconvolution. To illustrate this point we can look at four different cases a) classical TR, b) full deconvolution, c) zeroed deconvolution and d) partial deconvolution. Column 1 above shows the portion of the signal being recorded, note the *acausal* signal recorded in the first half of the signal (i.e., before the source function is broadcast). It should be noted that for all cases the source function was centered at  $t = 1.6384$  ms. For cases (a) – (c) the  $R(t)$  used for the processing is identical, for (d) the acausal portion is removed. Column 2 shows the raw  $g(t)$  signals immediately after processing, with case (a) being a simple TR process and all other being a deconvolution of the  $R(t)$  shown in column 1; note the change to the signal at early times for case (d) when compared to (b) and (c). The actual  $g(t)$  sent to the transducer is shown in column 3. Cases (a) and (b) are unchanged from column 2, while case (c) zeros out the acausal portion of the signal and (d) similarly sets that portion of the signal to zero in order to be of the same length as all other cases. Finally, the focal signals that result from the propagation of the  $g(t)$  signals used are shown in column 4. It is obvious that the full deconvolution is the cleanest and tightest in terms of temporal compression. It is also apparent that not using the acausal signal, i.e., in the manner of either cases (c) or (d), introduces additional noise and asymmetry to the focal signal, though both are still marked improvements over classical TR. One final detail of note is the fact that when  $g(t)$  is amplified to the same transducer input gain as the classical TR signal  $R(-t)$ , the resulting focal amplitude for all of the deconvolution methods is always less than classical TR amplitude, a fact that may make classical TR more desirable anytime maximizing focal amplitude is the primary use for focusing.



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## How can we tell which is better?



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Q8: How can I tell which is better?

A8: "A picture is worth a thousand words," as they say. Here we have performed a standard magnitude (or vector style) normalization on each of the focal signals as well as the original source function (i.e., the one sent to the transducer). In (a) it is apparent that the  $F_D(t)$  signal is a much better reconstruction of  $S(t)$  than  $F_{TR}(t)$ , which is significantly wider and exhibits excessive ringing before and after the focal time (due to transducer response as we have shown earlier). The normalization also shows that the deconvolution focusing is more efficient as the relative normalized amplitude is much higher for  $F_D(t)$  than  $F_{TR}(t)$ , and even approaches that of  $S(t)$ . Panel (c) shows the same signals as (a) but for the entire duration of the signal. Panels (b) and (d) compare the 3 cases of deconvolution focusing discussed in the previous slide.



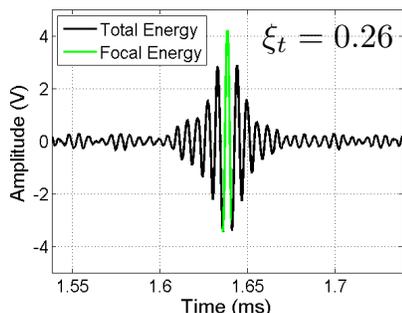
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## How do we quantify which is better?

Define an Energy ratio:  $\xi = \frac{E_{window}}{E_{total}}$

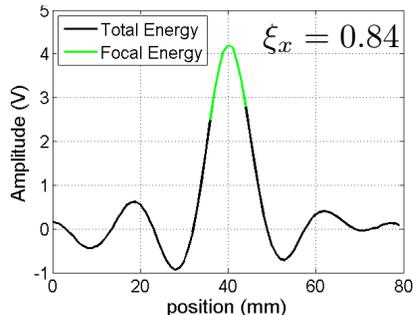
### Temporal Energy Ratio $\xi_t$

*window* defined by original source duration.



### Spatial Energy Ratio $\xi_x$

*window* defined by diffraction limit.



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Q9: How can we *quantify* which is better?

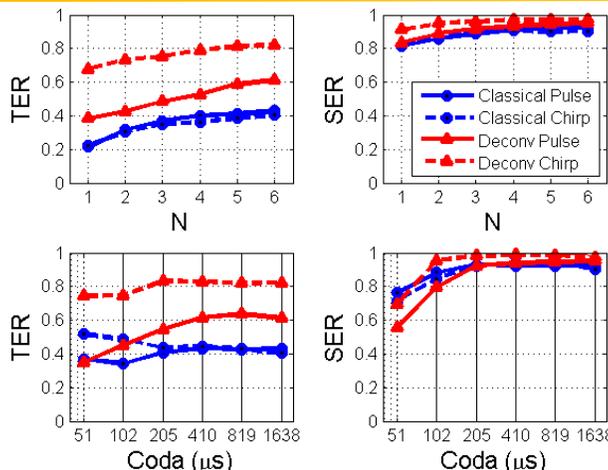
A9: To quantify which is better we define two energy ratios, one for the temporal compression and another for the spatial. These energy ratios ( $\xi_t$  or TER; and  $\xi_x$  or SER) essentially show the amount of energy that is within the original source region (defined by the source duration in time or by the diffraction limit in space) as opposed to the amount of energy that remains outside of that region (in time or space). For  $\xi_t$ , it is possible to measure for what amounts to infinite time (due to short durations of the signals), thus capturing the complete elastic wave energy of the measured component in the system at the focal point. For  $\xi_x$ , however, we cannot measure the wavefield everywhere in space (namely missing the inside of the sample) so we are limited to performing the calculation over a limited region. However, as long as this region is larger than the focal region, and remains constant for all focusing experiments, we can perform the calculation and compare each method to the others in a relative (rather than absolute) and quantitative manner. The values of both ratios should approach 1 for *perfect* focusing.



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## Okay, so what if ...

... you add more channels?



... or use more coda?

*Both TR and Deconvolution improve with increasing the number of channels (i.e., TRM elements) and the length of the coda (i.e., amount of scattering)*



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Q10: What if you add more channels?

Q11: What if you use more coda?

A10 & 11: In both cases classical TR and deconvolution focusing both improve, as expected with more channels and/or more coda (i.e., the length of scattered signal used). The most efficient is using the correlation calculated impulse response from a chirp used in the deconvolution process. This isn't a surprise as that technique benefits from the higher signal to noise measurement of recording a chirp response rather than the response from a short transient pulse. The deconvolution process results in a focus that is more spectrally uniform and thus we would expect it to produce a tighter focus temporally. However, as these results point out, the focus is tighter spatially as well for the deconvolution process.



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## Great, but how can I use this?

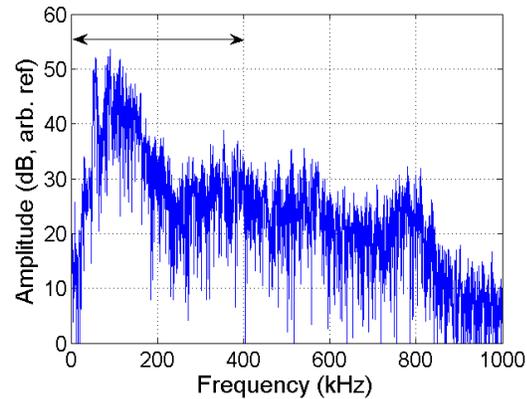
Signal transmission/communications

Recall:

$$F_{\delta} = \frac{S}{ST_A GT'_B} T_A GT'_B = 1$$

Again, just multiply

$$F_X = \frac{XS}{ST_A GT'_B} T_A GT'_B = X$$



Anderson, Ulrich and Le Bas,  
*manuscript in preparation* (2012).

Example:  $X(t)$        $F_{TR}(-t)$        $F_X(t)$



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Q12: How can we use this?

A12: There are many ways one might find to use deconvolution or TR focusing. Here we extend to one example (communications), while later we will address another (nondestructive evaluation). The experiment performed for this application was to communicate an audio signal through a  $10 \times 10 \times 10 \text{ cm}^3$  silica glass block using resampled audio in the ultrasonic range (bandwidth shown). The results for focusing this communication from one point on the structure to another are best demonstrated through audio, thus three sound files are available (contact the author at [tju@lanl.gov](mailto:tju@lanl.gov)) for the example given above, 1) original source function, 2) TR focus and 3) deconvolution focus. From these the communication is intelligible (once down sampled back to audible range) using deconvolution focusing, while it is not when using classical TR.



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## But what about nonlinearity?

### Doped Glass Block

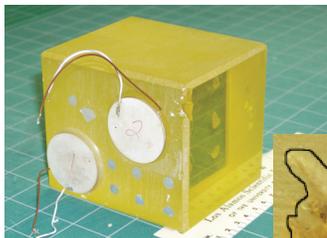
~ 10 x 9 x 9 cm

Density = 6300 kg/m<sup>3</sup>

Q = 1500

c<sub>L</sub> = 4500 m/s

c<sub>S</sub> = 2300 m/s



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Compare in 2 sample types

(localized and volumetric nonlinearity)

Use SSM Scalerandi scaling subtraction method

### Mortar Blocks

~ 10 x 10 x 5 cm

Density = 2200 kg/m<sup>3</sup>

Q = 150

c<sub>L</sub> = 3500 m/s

c<sub>S</sub> = 1800 m/s



Q13: What about nonlinearity? [i.e., nonlinear elasticity<sup>21</sup>]

A13: From the fact that the classical TR focusing produces higher amplitude focal signals, given a fixed amplifier gain, it is not unreasonable to expect that TR will be superior for nonlinear elasticity measurements due to the fact that the nonlinear response is strain (i.e., amplitude) dependent. To explore the effectiveness of the deconvolution vs. TR focusing for nonlinear elasticity measurements, e.g., TREND for NDE, we will present results from samples with localized damage (i.e., surface crack in a glass block) and distributed damage (i.e., microcrack networks in thermally damaged mortar). The scaling subtraction method (SSM) developed by Scalerandi et al<sup>22</sup> will be used to quantify the nonlinear elastic response. One significant difference in the rest of the measurements presented here compared to the earlier measurements given in this paper is that from here on we utilize a narrower band of frequencies for our source function. The main reason for doing this is to avoid using a particular source frequency whose 2<sup>nd</sup> harmonic would also be present in the source function. Three samples will be used for four measurements:

1. Doped glass block (DGB): one measurement on the crack and one a few centimeters away on an undamaged region.
2. M400 – Mortar sample thermally damaged at 400° C for 3 hours.
3. M20 – Mortar sample kept at 20° C (i.e., room temperature) to avoid thermal damage. Properties for the samples can be found above.

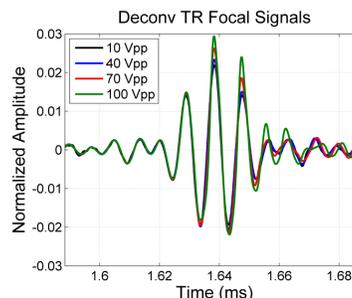
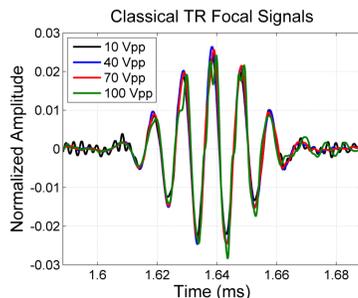


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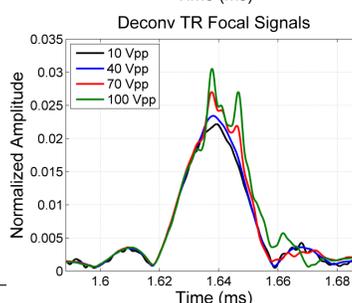
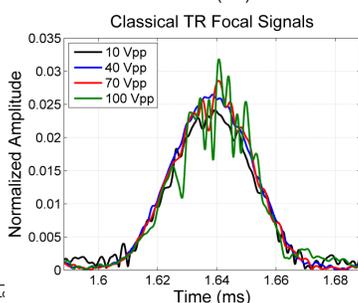
## How do TR and deconvolution compare?

DGB

Waveforms



Envelopes



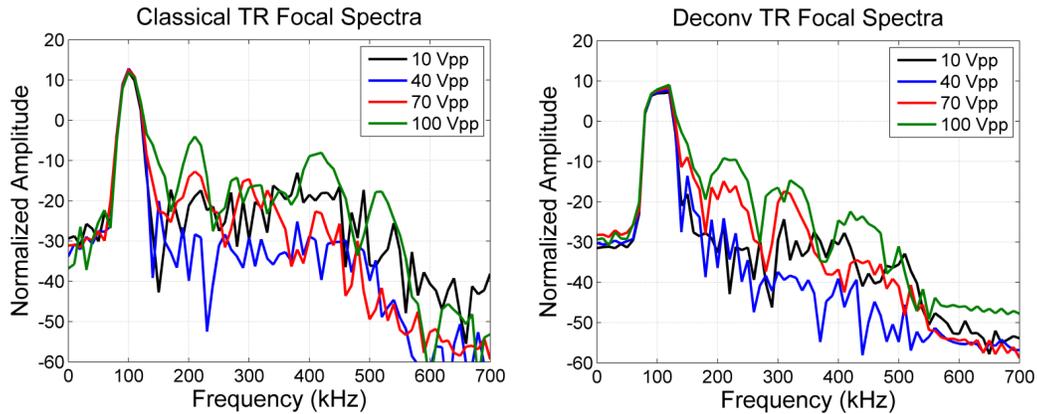
Q14: How do TR and deconvolution compare? [in a nonlinear elasticity context]

A14: The SSM requires multiple focal signals, each at increased amplitude steps. These focal signals are shown above, each normalized by their input voltage max amplitudes. These signals were measured on the crack. Distortion at the highest amplitudes is seen in both the TR and deconvolution focal signals and their respective amplitudes. Surprisingly, the distortion in the deconvolution signals is more easily discernable with the eye. Note also that the amplitude for the deconvolution focal signals is comparable to those for the classical TR focal signals. Apparently when a narrower frequency bandwidth is used for the source function, the deconvolution can produce nearly the same focal amplitudes as found in classical TR.



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## What do the spectra look like?



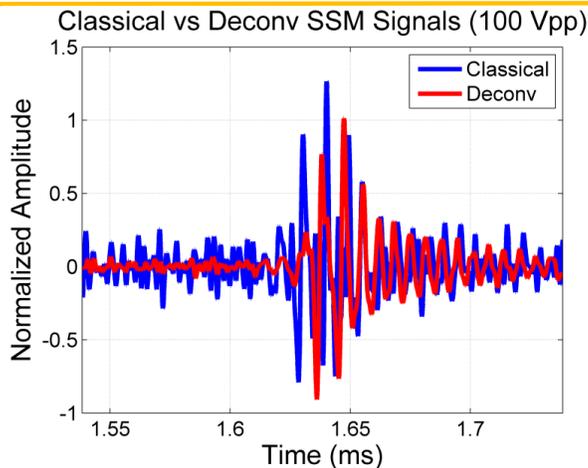
Q15: What do the spectra look like?

A15: Taking the focal signals shown previously and examining their spectral content, the presence of harmonics from the nominal 100kHz fundamental is clearly visible, again in both types of focal signals.



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## What does the SSM residual look like?



Sharper onset and longer ring-down seen with deconvolution focus



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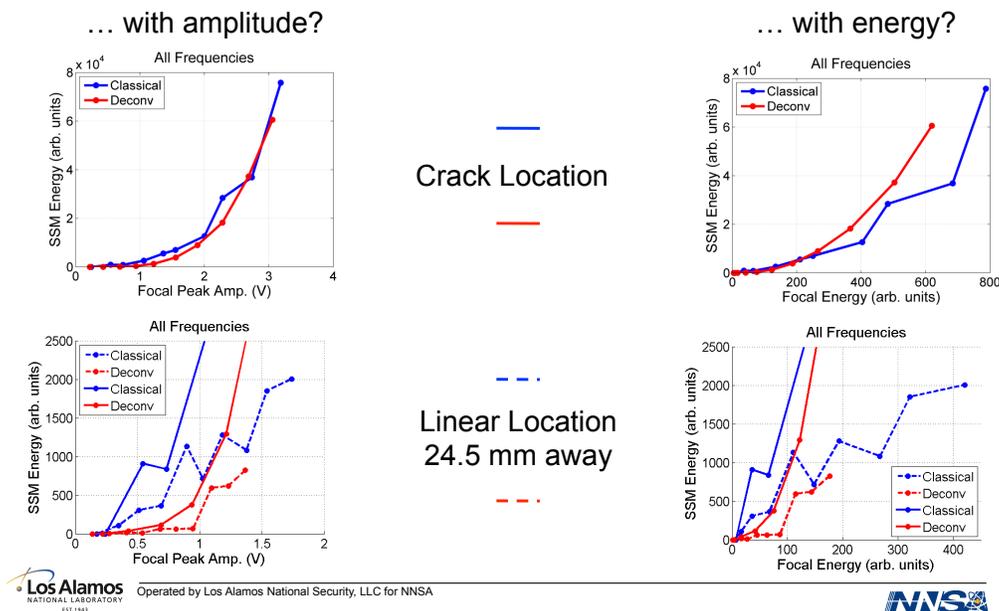
Q16: What does the SSM residual look like?

A16: Performing the scaling and then subtraction operations required for extracting the nonlinear response from the focal signals we see a clear SSM residual signal. Again, surprisingly, the distortion in the deconvolution (seen as the SSM residual) is more easily identified. For the classical TR focal signals, the nonlinearity rises out of the background only during the focal time and then disappears back into the background once the focal time window has passed. For the deconvolution SSM residual the background level of nonlinearity is much smaller prior to the focus and rises sharply during the focal window. This nonlinear signal then remains (i.e., rings down) beyond the focal time window. Certainly the physics (i.e., generation of the nonlinear response) is not different for the two different excitations (i.e., focal procedures), rather the improved temporal compression (i.e., minimization of temporal sidelobes in the focal signal) make it possible to make this stark observation (i.e., lower background signal before and after the focal window).



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## How does it evolve ...



Q17: How does it evolve with amplitude?

Q18: How does it evolve with energy?

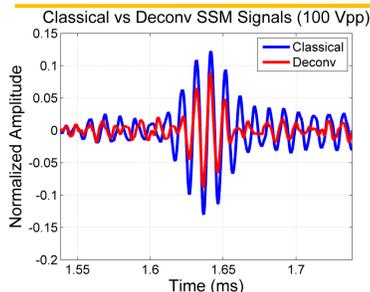
A17 & 18: Both methods show approximately equivalent responses as a function of amplitude. Given the same physics being responsible for the response in the two cases, this is expected. Also expected is the ability to achieve somewhat higher amplitudes when using classical TR, and thus producing larger nonlinear responses. However, the lack of temporal sidelobes means that a nonlinear response is not produced until the moment of focus, thus the ability to more accurately detect it and its onset time, is enhanced. This allows for a cleaner measurement. It should be noted that the sidelobes are inherent in the classical TR procedure and extremely reproducible, thus the apparent scatter in the nonlinear response seen as a function of amplitude is not a reproducibility issue, but rather is due to amplitude of sidelobes producing a nonlinear response outside of the expected window, and thus complicating the SSM response.

When looking at the energy dependence of the nonlinearity (with respect to the energy in the focal signal), the nonlinear response proves to be more efficiently generated from deconvolution focusing than with the classical TR type.



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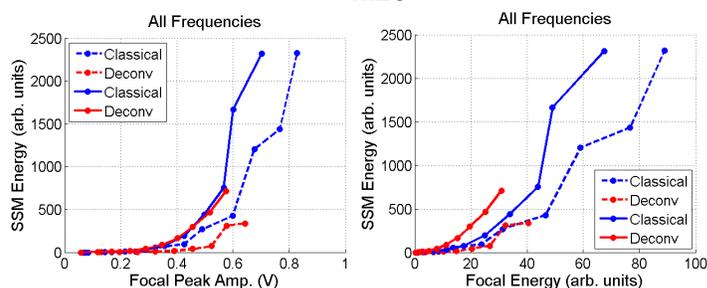
## What about for the mortar samples?



Sample: M400

Larger amplitudes from TR focusing produce larger NL response.

Both TR and deconvolution show higher degree of nonlinearity for the M400 (damaged) sample. Deconvolution is cleaner (less scatter) and more efficient with respect to the energy of the focus.



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Q19: What about for the mortar samples?

A19: Similar results can be seen when comparing the two focusing techniques in damaged and undamaged mortar. Once again, TR produces larger amplitudes and thus larger nonlinear responses, while the deconvolution produces the clear, more easily measured signals. As expected the nonlinearity is larger in the thermally damaged mortar (M400) than the undamaged mortar (M20). Other details in the waveforms from the mortar and glass block samples show features that may be used to distinguish localized damage (i.e., single cracks, delaminations, etc.) from volumetric damage (networks of micro-cracks). This is currently being investigated as an area of further research, so no speculation is presented at this time.

Q20: Do you have any references for this stuff?

A20: Currently in review is a paper on the details of deconvolution focusing, enhanced spatial and temporal compression and source reconstruction. Further manuscripts are in preparation and/or awaiting submittal on various related topics, e.g., nonlinear response from deconvolution focusing, application to communications through structures, a parametric study of quality of deconvolution focusing, etc. Other information about deconvolution, inverse filtering and a variety of other topics mentioned here are available from the open literature. Many publications from our research team on this and related subjects can be found on our website:

<http://www.ees.lanl.gov/ees11/geophysics/timerev/timerev.shtml>

A selection of other useful references of others can be found below.

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