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Optimal near-field error sensor locations for active control of fan noise using multipole expansions

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ABSTRACT

Recent developments in the active control of cooling fan noise have used error sensors to drive the pressure at locations in the near field to zero[1]. Theoretical mapping of near-field pressure during minimization of sound power reveals the location of pressure nulls that can be used to optimize the location of the error sensors. To this point, the locations of error sensors have been determined by modeling both the fan and the control loudspeakers as point monopoles. However, noise from an axial fan has multipole as well as monopole characteristics. The multipole characteristics of the fan can be obtained using a procedure based on the work of Martin and Roure[2]. Pressure values are obtained over a hemisphere in the far field of a primary source and the contributions from multipoles up to the second order, centered at the primary source, may be calculated using multipole expansions. The source characterization is then used in the aforementioned theoretical near-field calculation of pressure.

1. INTRODUCTION

The electrical components used in common appliances around the contemporary home and workplace generate heat and, therefore, require the use of a cooling system Fan noise characterization is a widely studied field of acoustics due to its applications in the cancellation of unwanted noise. Experts in fan noise have utilized various methods to characterize fan noise such as acoustical measurement[3–7], mathematical derivation[8–12], and numerical analysis[13–16] The central motivation for many of the referenced studies is to more effectively cancel the unwanted noise from cooling fans. Although passive techniques may be used to cancel some of the unwanted noise produced by cooling fans, the focus of this paper is optimization of the active cancellation of noise from an axial cooling fan. The fan used in this study is a 60 mm axial cooling fan.

Active cancellation of noise from any primary sound source requires the use of error sensors whose placement has an effect on the amount of global attenuation achievable by the active control system[1,17–21]. The central role of the active control system is to drive the pressure at the error sensor to zero[22]. As the acoustic pressure at the error sensor approaches zero, the radiated sound field from the actively controlled system is globally minimized. Gee and Sommerfeldt showed that the locations of the error sensors in the near-field of the primary

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source may be determined from a theoretical calculation of the near-field pressure that occurs as a result of the minimized sound power between the primary source and control actuators[1]. In this experiment, the theoretical near-field pressure null that results from the aforementioned sound power minimization is calculated assuming that the primary source radiates as a point monopole acoustic source. This model has limited accuracy if the primary sound source does not have point monopole characteristics.

The radiation of sound from axial cooling fans can be characterized as monopole, dipole, and quadrupole type noise[11]. The spectral content of the monopole noise is discrete in nature, correlating with the blade passage frequency of the fan as well as its harmonics, and is caused by volume displacements in the flow of the fan. The dipole noise from a fan is caused by both the steady and unsteady rotating forces and is the source of discrete and broadband noise in the fan spectrum. The quadrupole noise is broadband in nature and is caused by turbulent flow from the fan. Because fans have multipole characteristics, it is necessary to explore the contribution from each multipole (i.e. monopole, dipole, quadrupole) that may be used to characterize the sound radiation from any individual cooling fan. A more accurate model of the sound radiation from a cooling fan can then be used to possibly improve the prediction of error sensor placement in active noise control and increase global sound power reductions.

2. MULTIPOLE EXPANSION

A. Spherical Harmonic Coefficients

A multipole expansion may be used in characterizing the acoustic radiation from any arbitrary noise source. Because a spherical harmonic expansion is similar in both derivation and usage to a multipole expansion, it is necessary to discuss the differences between them[23]. The spherical harmonic expansion for a sound source is used to determine the spherical harmonic coefficients and is calculated from the complex pressure field measured on a sphere surrounding the noise source[2,24–25]. In the multipole expansion, the previously determined spherical harmonic coefficients are used to derive the complex source strength of individual multipoles that correspond to specific spherical harmonics (i.e. monopole, dipole, quadrupole). These multipoles are point sources whose phase and amplitude are determined from the spherical harmonic coefficients. For the dipoles and quadrupoles the point sources are separated by a distance, *d*. The maximum value of this distance is the radius of the smallest sphere containing the source of interest[25].

The spherical harmonic coefficients may be obtained by the Helmholtz equation for an acoustic pressure. The following equation is a representation of the Helmholtz equation in spherical coordinates,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\hat{p}(r,\theta,\varphi)}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\hat{p}(r,\theta,\varphi)}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\hat{p}(r,\theta,\varphi)}{\partial\varphi^2} \tag{1}$$

where r is the distance from the source to the edge of the measurement sphere, $\hat{p}$ is the complex acoustic pressure, θ is the zenith angle, and φ is the azimuth angle. The time-harmonic solution to this equation can be expressed as a summation in the following manner,

$$\hat{p}(r,\theta,\varphi)=\sum_{n=0}^{N}\hat{h}_n^{(1)}(kr)\sum_{m=0}^{n}\hat{A}_{nm}P_n^m(\cos\theta)\cos m\varphi+\sum_{n=1}^{N}\hat{h}_n^{(1)}(kr)\sum_{m=1}^{n}\hat{B}_{nm}P_n^m(\cos\theta)\sin m\varphi \tag{2}$$

where $\hat{h}_n^{(1)}(kr)$ is the spherical Hankel function for outgoing spherical waves, $P_n^m(\cos\theta)$ is the associated Legendre polynomial, and A_{nm} and B_{nm} are the complex spherical harmonic coefficients up to a specified order, N. In matrix form the solution becomes,

$$\overline{\mathbf{P}} = \mathbf{T}\overline{\mathbf{A}} \tag{3}$$

where $\overline{\mathbf{P}}$ is a vector of complex pressure values, $\mathbf{T}$ is a matrix of spherical harmonics and Hankel functions, and $\overline{\mathbf{A}}$ is a vector of spherical harmonic coefficients. If the complex pressure is measured over a hemisphere surrounding the primary sound source the spherical harmonic coefficients may be calculated using a least squares approximation[2,26] of the inverse of the **T** matrix

$$\overline{\mathbf{A}} = \left(\mathbf{T}\mathbf{T}^H\right)^{-1}\mathbf{T}^H\overline{\mathbf{P}} \tag{4}$$

where $\mathbf{T}^H$ is the Hermitian transpose of the **T** matrix. The values calculated for the spherical harmonic coefficients indicate the relative contribution of each individual multipole. An example of the typical values calculated for spherical harmonic coefficients is given in Table 2. The multipoles calculated for a second order (N = 2) approximation is found in Table 1.

Coefficient	Multipole
A_{00}	*Monopole*
A_{01}	*Dipole (centered on the z-axis)*
A_{11}	*Dipole (centered on the x-axis)*
B_{11}	*Dipole (centered on the y-axis)*
A_{02}	*Pseudo-longitudinal Quadrupole (z-axis)*
A_{12}	*Lateral Quadrupole (xz plane)*
B_{12}	*Lateral Quadrupole (yz plane)*
A_{22}	*Axial Lateral Quadrupole (xy axis)*
B_{22}	*Lateral Quadrupole (xy plane)*

Table 1: The specific type of multipole corresponding to each spherical harmonic coefficient up to the second order (*N*=2).

B. Calculation of Multipole Source Strengths

The source strength of each contributing multipole in the expansion is derived from the analytical expression for that specific multipole[27–28]. As an example, the analytical expression for the z-axis dipole can be expressed in two ways,

$$\hat{p}(r,\theta,\varphi) = \frac{\rho_0 c k^3}{2\pi}\hat{q}_{dz} d\cos\theta\left[-e^{-jkr}\left(\frac{1}{kr} - \frac{j}{(kr)^2}\right)\right], \qquad \text{(5),(6)}$$
$$\hat{p}(r,\theta,\varphi) = \hat{A}_{01}Y_0^1(\cos\theta)h_1^{(1)}(kr)$$

where k is the acoustic wave number, c is the speed of sound, ρ_0 is the ambient density of the medium, d is the separation distance of the point sources, and $\hat{q}_{dz}$ is the source strength of the two point sources. The two equivalent expressions are solved for the source strength of the dipole sources to yield

$$\hat{q}_{dz} = \frac{2\pi}{\rho_0 c k^3 d} \hat{A}_{01} \tag{7}$$

The source strengths of all the other multipoles in the expansion are derived in a similar manner. The pressure field anywhere outside the minimum sphere containing the sound source may be reconstructed as a superposition of multipole pressures. A map of the near-field pressure may be obtained using the source strengths and point source configurations of each individual multipole.

3. MEASUREMENT AND DATA PROCESSING

The complex pressure is obtained over a hemisphere surrounding the cooling fan using an angular microphone array with a radius of approximately 1.5 meters and spanning 180° in θ (90° in each direction from the zenith). The array consists of thirteen ½" ICP microphones and is rotated in ten degree φ (azimuth) increments for a total of 234 measurements on the hemisphere surrounding the fan. In each measurement the complex pressure is calculated as shown,

$$\hat{y} = \left|x_{ref}\right| H_{xy}(j\omega) \tag{8}$$

where the expression in x is the square root of the power spectrum for the central and stationary microphone on the angular array and the expression in H is the transfer function between the central microphone and another individual microphone on the array.

The measured complex pressure is imported into MATLAB and the spherical harmonic coefficients are calculated using the least squares approximation (4). The source strength of each individual point source is calculated following the procedure in (5)–(7). These source strengths are then used to reconstruct the pressure field at the same 234 positions originally used in the measurement. The reconstructed pressure field is compared to the original measurement by calculating the mean error at each point on the hemisphere. A benchmark case was measured and processed using two loudspeakers 4¼" apart and wired 180° out of phase. The center of

4. EXPERIMENTAL RESULTS

A benchmark case was measured and processed using two 1" loudspeakers 4¼" apart and wired 180° out of phase. The center of the dipole loudspeaker arrangement was placed in the geographic center of the measurement hemisphere. The multipole reconstruction was accurate within a mean value of 0.5 dB. The results are shown in Fig. 1.

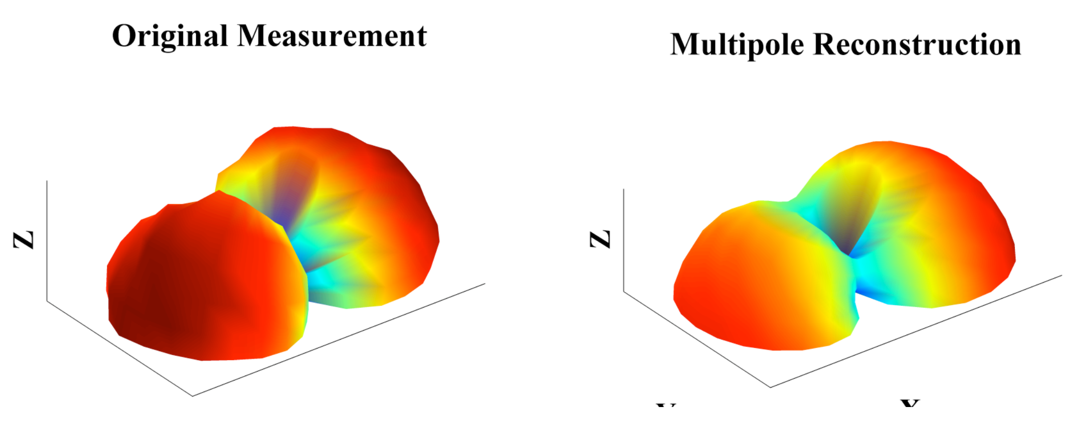

Figure 1: The dipole that was measured was located along the cartesian x-axis. The multipole reconstruction is plotted on the same dB color scale as the original pressure measurement.

A 60 mm cooling fan, installed in a rectangular box, was placed at the geographical center of the hemispherical measurement array. The complex pressure was measured and processed as discussed in Sec. 3 with a mean error of 0.6 dB. The spherical harmonic coefficients calculated indicate that the fan in the box may be characterized by a monopole and a z-axis dipole (Table 2). The monopole and z-axis dipole (A_{00} and A_{01}) contribute the most to the acoustic radiation of the fan. Each of the other multipoles radiated at 13 to 20 dB less than the monopole and z-axis dipole. A comparison of the results for the cooling fan is found in Fig. 2.

Coefficient	Value
$\|A_{00}\|$	*0.0857*
$\|A_{01}\|$	*0.0730*
$\|A_{11}\|$	*0.0202*
$\|B_{11}\|$	*0.0143*
$\|A_{02}\|$	*0.0248*
$\|A_{12}\|$	*0.0142*
$\|B_{12}\|$	*0.0096*
$\|A_{22}\|$	*0.0051*
$\|B_{22}\|$	*0.0013*

Table 2: The absolute value of the spherical harmonic coefficient calculated from the complex pressure in a hemisphere surround the axial cooling fan.

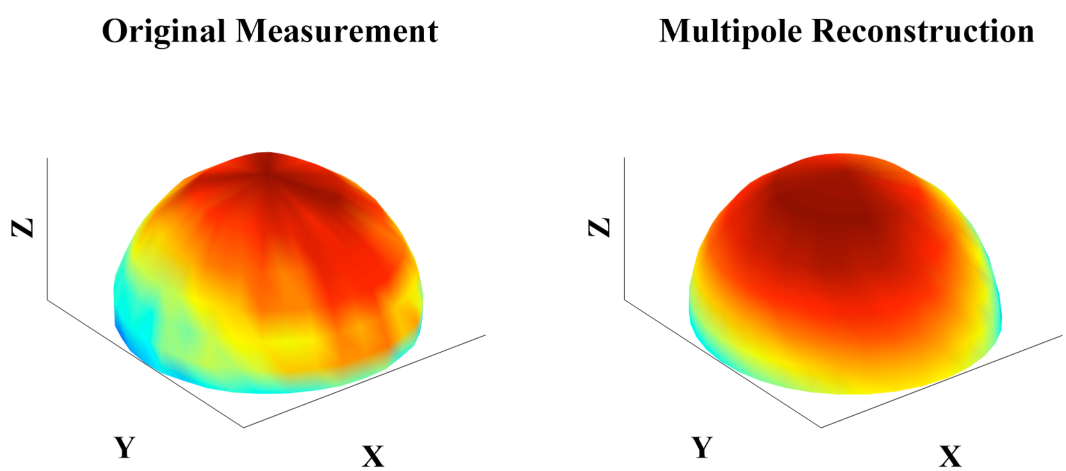

Figure 2: The measurement and reconstruction of the cooling fan at its blade passage frequency, 653 Hz. The multipole reconstruction is plotted on the same dB color scale as the original pressure measurement.

The monopole and z-axis dipole point sources were used to map the pressure in a plane located ¼" above plane containing the cooling fan. The map was calculated as the near-field pressure due to minimized sound power of the primary source(s) using four symmetrically located control actuators in the actual locations of previous control systems used for the active cancellation of fan noise. The pressure map was calculated both for the fan modeled as a single point monopole source as well as the monopole/dipole source found from the multipole expansion. The near-field pressure nulls differ between fan models, indicating the need to relocate the error sensors. As shown in Fig. 3 and Fig. 4 the pressure nulls between the control actuators become less defined and protrude further from the center.

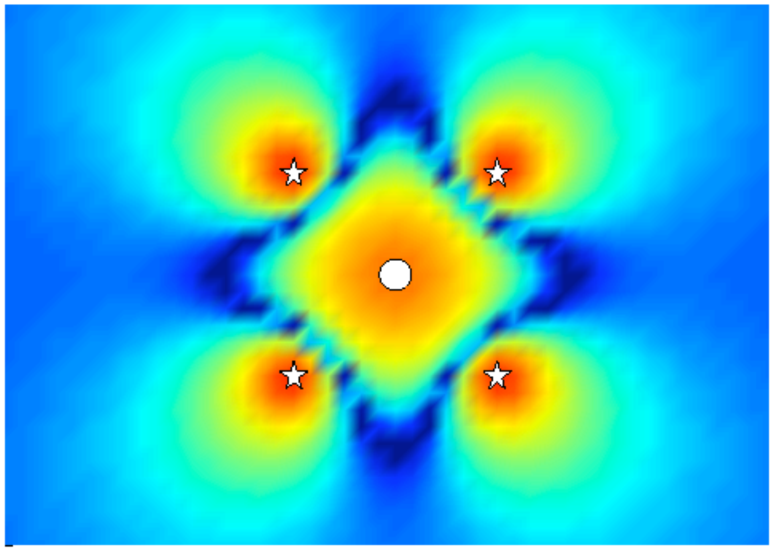

Figure 3: A theoretical prediction of the pressure nulls resulting from minimized sound power of the primary source. The map is plotted as the pressure referenced to the primary pressure field with the stars indicating the locations of the control actuators and the circle indicating the location of the primary source.

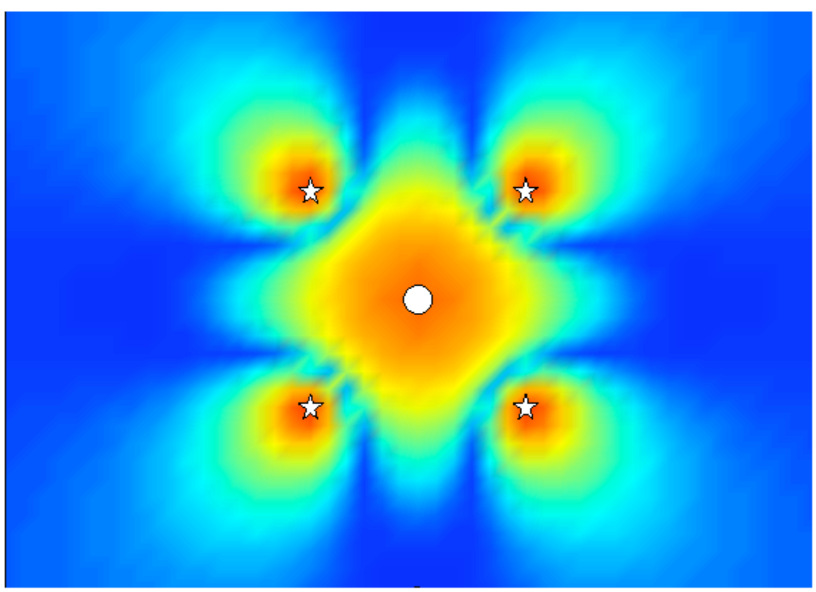

Figure 4: A theoretical prediction of the pressure nulls resulting from minimized sound power of the primary sources. The map is plotted in the same manner as in Figure 3. The z-axis dipole is not visible because the pressure is plotted in the xy plane only 0.25 inches above the plane of the monopole source and control actuators.

5. CONCLUSION

A multipole expansion may be used to more accurately model the noise from an axial cooling fan. The multipole expansion model of acoustic noise may be used for any type of noise source. For example, it can be difficult to find the optimum locations of near-field error sensors for many different types of fans in different installations. When the multipole expansion model of fan noise is used, such error sensor locations may be more accurately represented in the near-field pressure map. The accuracy of the multipole expansion may be calculated using the error between the reconstructed multipole pressure field and the original measurement. The mean error in both the benchmark case and the axial cooling fan was much less than one dB.

The noise from the axial fan can be characterized with a monopole and a dipole that is oriented in the flow of the fan. This dipole may be primarily due to the positive and negative pressure fluctuations as the air flows through the fan enclosure. In the near-field pressure map the presence of the dipole in the z-axis greatly altered the locations of the pressure nulls used in locating error sensors. In future work, the error sensors will be placed in the locations determined in the multipole expansion model of the fan and the amount of global attenuation achieved will be compared to the attenuation achieved when the error sensors are located according to a point source model of the fan.

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Spherical Harmonics equations