Eigenvalue equalization applied to the active minimization of engine noise in a mock cabin

Jared K. Thomas[a]
Stephan P. Lovstedt[b]
Jonathan D. Blotter[c]
Scott D. Sommerfeldt[d]
Brigham Young University
435 Crabtree Building
Provo, Utah 84602

ABSTRACT

A number of active noise control applications involve the need to control multiple stationary frequencies, a single time-varying frequency, or some combination of the two. The most common control approach is typically based on some version of the filtered-x algorithm. For this algorithm, the convergence and tracking speed is dependent on the eigenvalues of the filtered-x autocorrelation matrix, with these eigenvalues being frequency dependent. To maintain stability, the system must be implemented based on the slowest converging frequency that will be encountered, which can lead to significant degradation in the overall performance of the control system. This paper will present an approach which largely overcomes this frequency dependent performance, maintains a relatively simple control implementation, and improves the overall performance of the control system. The control approach is called the eigenvalue equalization filtered-x (EE-FXLMS) algorithm and its effectiveness is demonstrated through an application to engine noise in a mock cabin. Experimental results show that the EE-FXLMS algorithm provides as much as 3.5 dB additional attenuation compared to the normal filtered-x algorithm.

1. INTRODUCTION

The most common control approach for the active noise control (ANC) of stationary or time-varying frequency noise is the filtered-x (FXLMS) algorithm[1,2]. Though the FXLMS algorithm has proven successful for many applications, one of its limitations is that it exhibits frequency dependent convergence and tracking behavior leading to a significant degradation in the overall performance of the control system.

Solutions to the frequency dependent problem have been proposed such as the higher harmonic filtered-x (HLMS) algorithm by Clark and Gibbs[3], and similar work by Lee *et al.*[4], the Filtered-x Gradient Adaptive Lattice (FxGAL) algorithm by Vicente and Masgrau[5], the work of Kuo *et al.*[6], and the modified FXLMS algorithm[7]. The drawback of most of these approaches is that they either increase the computational burden of the algorithm, increase the algorithm's complexity, or are only effective for specific applications.

This paper discusses a new approach which largely overcomes this frequency dependent problem, and improves the overall performance. The approach is simple to implement, can be

---

[a] Email address: jthomas@byu.edu
[b] Email address: s_lovstedt@yahoo.com
[c] Email address: jblotter@byu.edu
[d] Email address: scott_sommerfeldt@byu.edu

added to existing FXLMS algorithms with minor modifications, and does not increase the computational burden of the algorithm. The effectiveness of the approach will be demonstrated through an application to engine noise in a mock cabin.

2. BACKGROUND

For this research, a feedforward implementation of the FXLMS algorithm is used. The FXLMS algorithm involves adaptive signal processing to filter the reference signal in such a way that the measured residual noise is minimized. The measured residual is called the error signal and for this research it will be measured as an energy density (ED) quantity. The advantages of an ED based FXLMS algorithm[8] in enclosures[9,10] and for tractor engine noise[11], are well documented. Before introducing the new approach, a brief derivation of the FXLMS algorithm is given.

A. FXLMS

The goal of the FXLMS algorithm is to reduce the mean-squared error of the error signal at a location where the sound is to be minimized. Boucher, Elliot, and Nelson[12] provide a good reference for the derivation of the single channel FXLMS algorithm, which is shown in block diagram form in Figure 1. In the figure, and in all equations presented, the variable t is used as a discrete time index and the variable z is used as a discrete frequency domain index. Signals in

Figure 1. Block diagram of the FXLMS algorithm.

the time domain are represented as lower case letters while capital letters are used in the frequency domain. Vectors in each domain are represented as bold letters.
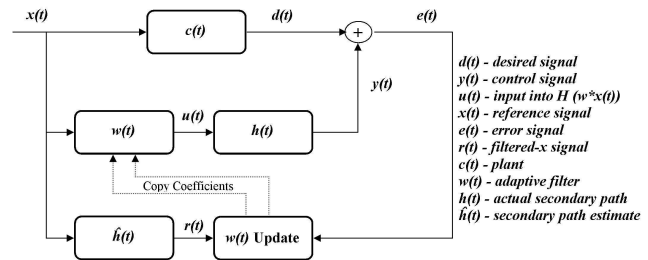
The mean-squared error is a quadratic function (a "bowl") with a unique global minimum. For each iteration of the algorithm, $\mathbf{W}(z)$, an adaptive finite impulse response (FIR) control filter, takes a step of size μ, the convergence coefficient, times the gradient in search of a single global minimum that represents the smallest attainable mean-squared error. The control filter update equation for $\mathbf{W}(z)$ can be expressed in vector notation as

$$\mathbf{w}(t+1) = \mathbf{w}(t) - \mu e(t)\mathbf{r}(t) \tag{1}$$

where $e(t)$ is the error signal and $\mathbf{r}(t)$ and $\mathbf{w}(t)$ are defined as

$$\mathbf{r}^T(t) = \left[r(t), r(t-1),...,r(t-I+1)\right], \text{ and } \mathbf{w}^T(t) = \left[w_0, w_1,...,w_{I-1}\right]. \tag{2}$$

The filtered-x signal, $\mathbf{r}(t)$, is the convolution of $\hat{\mathbf{h}}(t)$, the estimate of the secondary path transfer function, and $\mathbf{x}(t)$, the reference signal. The secondary path transfer function is an impulse response that includes the effects of A/D and D/A converters, reconstruction and anti-alias filters, amplifiers, loudspeakers, the acoustical transmission path, and the error sensor.

B. Secondary path transfer function

One difficulty in implementing the FXLMS algorithm is that the secondary path, represented as $\mathbf{H}(z)$ in Figure 1, is unknown. An estimate, $\hat{\mathbf{H}}(z)$, of the secondary path must be used. The estimate is obtained through a process called system identification (SysID).

The SysID process is performed either online (while ANC is running), or offline (before ANC is started). For the fastest convergence of the algorithm, an offline approach is used. The

offline SysID process consists of playing white noise through the control speaker(s) and measuring the output at the error sensor. The measured transfer function is a FIR filter, $\hat{\mathbf{h}}(t)$, that represents $\hat{\mathbf{H}}(z)$. The coefficients of $\hat{\mathbf{H}}(z)$ are stored and used to run control.

3. FXLMS LIMITATIONS

The inclusion of $\hat{\mathbf{H}}(z)$, while necessary for algorithm stability, degrades performance by slowing convergence. One reason for the decreased performance is the delay associated with $\hat{\mathbf{H}}(z)$. For many ANC applications, such as enclosures of less than a few meters, the delay is on the order of 10 ms and convergence is still rapid[13]. A more significant problem is that the inclusion of $\hat{\mathbf{H}}(z)$ causes frequency dependent convergence behavior. The frequency dependence can be better understood by looking at the eigenvalues of the autocorrelation matrix of the filtered-x signal.

The eigenvalues of the autocorrelation matrix of the filtered-x signal relate to the dynamics or time constants of the system modes. Typically, a large spread is observed in the eigenvalues, corresponding to fast and slow modes of convergence. The slowest modes limit the performance of the algorithm because it convergences the slowest at these modes. The fastest modes have the fastest convergence and the greatest reduction potential, but limit how large of a convergence parameter, μ, can be used[14]. For stability, μ is set based on the slowest converging mode (the maximum eigenvalue), leading to degraded performance. If μ is increased, the slower states will converge faster, but the faster states will drive the system unstable.

The autocorrelation matrix of the filtered-x signal is defined as

$$E[\mathbf{r}(t) * \mathbf{r}^T(t)] \qquad (3)$$

where $E[]$ denotes the expected value of the operand which is the filtered-x signal, $\mathbf{r}(t)$, multiplied by the filtered-x signal transposed, $\mathbf{r}^T(t)$. It has been shown that the algorithm will converge and remain stable if the chosen μ is less than $2/\lambda_{max}$[12], where λ_{max} is the maximum eigenvalue of the autocorrelation matrix in the range of frequencies targeted for control.

4. EIGENVALUE EQUALIZATION

A. Eigenvalue simulation

If the variance in the eigenvalues was minimized, the algorithm could converge at the same rate at each frequency. The autocorrelation matrix is directly dependent on the filtered-x signal, which is computed by filtering the reference signal with $\hat{\mathbf{H}}(z)$. The reference signal is often taken directly from the sound field and cannot be changed. Changes can be made to $\hat{\mathbf{H}}(z)$, but must be done carefully. Errors in its estimation contribute to lower convergence rates and instability. Estimation errors can be considered in two parts: amplitude estimation errors and phase estimation errors[15]. Phase estimation errors greater than +/- 90 degrees cause algorithm instability[12], but errors as high as 40 degrees have little effect on the performance[12]. Magnitude estimation errors can be compensated for by the choice of μ[16]. Ideally, changes would be made to the magnitude information of $\hat{\mathbf{H}}(z)$, while the phase information is preserved.

The idea to remove the variance in the eigenvalues by changing the magnitude coefficients of $\hat{\mathbf{H}}(z)$, while preserving the phase, will be referred to as the eigenvalue equalization filtered-x (EE-FXLMS) algorithm approach. The remainder of this paper will focus on one method of adjusting the magnitude coefficients that is simple to implement, and offers significant improvement in the overall sound reduction.

B. EE-FXLMS

The procedure for implementing the EE-FXLMS is to adjust the coefficients of $\hat{\mathbf{H}}(z)$ as follows:

1. Get time domain impulse response $\hat{\mathbf{h}}$(t) for each $\hat{\mathbf{H}}$(z) through an offline SysID process
2. Take the Fast Fourier Transform (FFT) to obtain $\hat{\mathbf{H}}$(z)
3. Divide each value in the FFT by its magnitude and multiple by the mean value of the FFT
4. Compute the inverse FFT to obtain a new $\hat{\mathbf{h}}$(t) and use the new modified $\hat{\mathbf{h}}$(t) in the FXLMS algorithm as normal

This procedure flattens the magnitude coefficients of $\hat{\mathbf{H}}$(z) while preserving the phase. It is an offline process done directly following SysID, and can be incorporated into any existing algorithm with only a few lines of code. As an offline process, it adds no computational burden to the algorithm while control is running. The results of the flattening process can be seen in Figure 2.
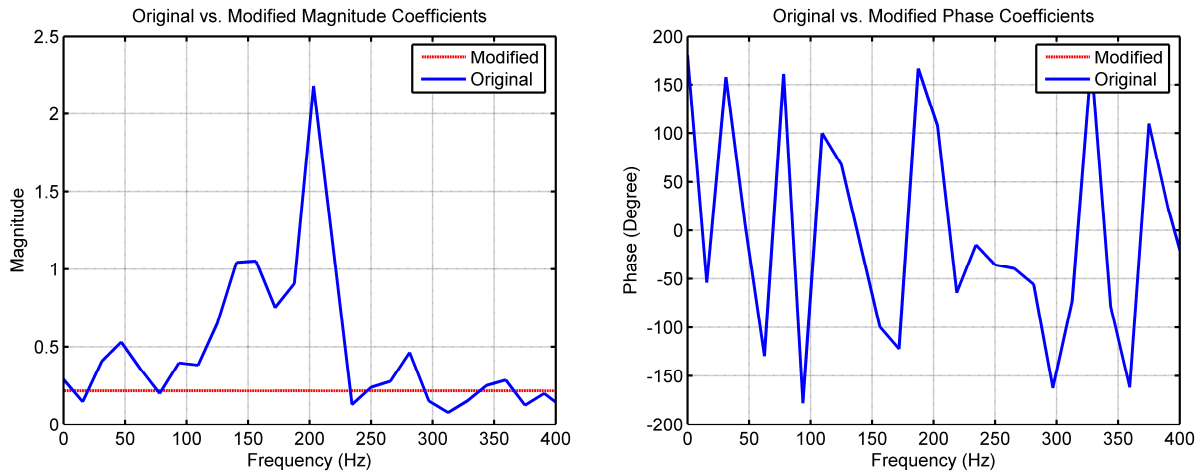

Figure 2. Original and modified magnitude and phase coefficients of $\hat{\mathbf{H}}$(z)

Figure 2 shows the original and modified $\hat{\mathbf{H}}$(z) magnitude coefficients and shows that the phase information of $\hat{\mathbf{H}}$(z) has been preserved. In Figure 2, the two lines representing the original and modified phase of $\hat{\mathbf{H}}$(z) are directly on top of each other.

In practice, it is too computationally demanding to obtain a real-time estimate of the autocorrelation matrix. An offline estimate of the autocorrelation matrix is made by taking an actual $\hat{\mathbf{H}}$(z) model from a mock cabin and importing it into a numerical computer package. If a single frequency reference signal is used, λ_{max} can be computed for that frequency. If the simulation is repeated over a range of frequencies, λ_{max} for each frequency can be found. Figure 3 shows an offline simulation using both the original $\hat{\mathbf{H}}$(z) and the new modified $\hat{\mathbf{H}}$(z), and tonal inputs from 0-400 Hz.
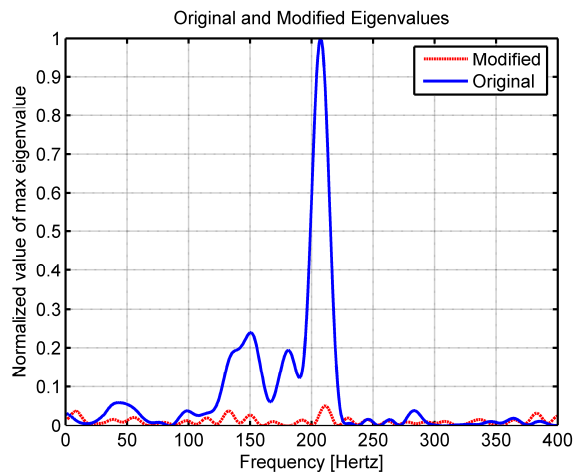

Figure 3. Original and modified eigenvalues.

In Figure 3, the eigenvalues in both the original, and modified case, have been normalized by the largest of the original eigenvalues. The modified eigenvalues are more uniform ("flat") over

all frequencies. While not the optimum, the improved modified eigenvalues make a noticeable improvement in the algorithm's performance.

5. EXPERIMENTAL RESULTS

The performance advantages of the EE-FXLMS control approach were verified for the case of a single frequency noise, single time-varying frequency noise, and recorded tractor engine noise.

A. Experimental setup

The experiments were conducted inside a mock tractor cabin with nominal dimensions of 1.0 m x 1.5 m x 1.1 m. The cabin has a steel frame, 0.01 m thick plywood sides, and a 0.003 m thick Plexiglass® front panel. A speaker placed under a chair served as the sound source and three loudspeakers were setup in a two channel control configuration. The control signal for each channel was routed through a crossover circuit sending the low-frequency content (below 90 Hz) to a subwoofer on the cabin floor, and the high-frequency content (above 90 Hz) to one of two smaller satellite speakers mounted in the cabin's top corners. A 2-D error sensor consisting of four equally spaced microphones around a small disk was placed on the ceiling near where the operator's head would be. A photo of the cabin, error sensor, and speakers is seen in Figure 4.
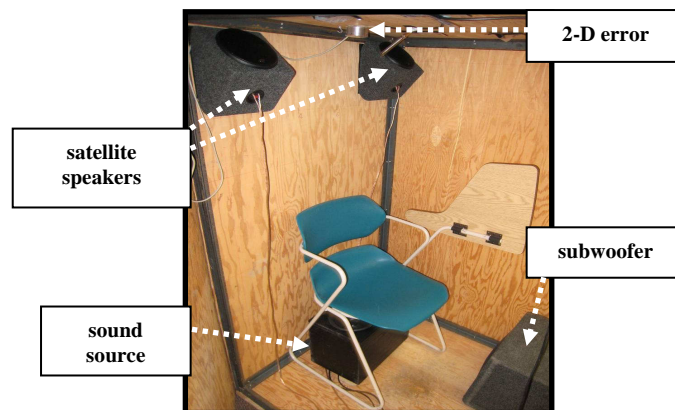

Figure 4. Photo of inside of mock cabin.

The algorithms were implemented on a Texas Instruments TMS320C6713 DSP, capable of 1,350 million floating point operations per second. Both adaptive control filters consisted of 32 taps, and all secondary path transfer functions were modeled with 128 taps. All input channels were simultaneously sampled at 2 kHz, and all signals had 16 bits of resolution. Fourth-order Butterworth lowpass filters (400 Hz cutoff) provided anti-aliasing and reconstruction of input and output signals, respectively. Performance was monitored using ten precision microphones arranged in two horizontal planes, located (0.15 m) and (0.5 m) from the cabin ceiling.

B. Single frequency disturbance

A function generator was used to generate single sinusoids at 50 Hz, 80 Hz, 113 Hz, 125 Hz, 154 Hz, 171 Hz, and 195 Hz; frequencies where the response of the cabin is large and where the normal FXLMS control is expected to have poor performance. The convergence coefficient μ was determined experimentally by finding the largest stable μ and then scaling it back by a factor of ten to ensure stability. The measured performance was the amount of attenuation in dB and the convergence time. The convergence time was a measure of how long it took the error signal,

from the time that control was enabled to reach 1/e of its initial value, where e is the base of the natural logarithm. Each measurement was performed three times and the results are shown in Table 1. In Table 1, EE refers to EE-FLXMS control, and normal refers to FXLMS control.

Table 1. Results of single frequency experimentation

Code Type	Freq. (Hz)	μ	Attenuation (dB)		Convergence Time (sec)	
			Avg.	Stdev.	Avg.	Stdev.
Normal	50	1.E-08	37.85	0.34	0.26	0.02
EE	50	4.E-08	36.98	0.41	0.21	0.04
Normal	80	1.E-07	20.35	0.06	0.21	0.05
EE	80	2.E-07	21.77	0.02	0.30	0.02
Normal	113	1.E-08	13.69	0.01	0.44	0.04
EE	113	5.E-08	15.63	0.01	0.44	0.02
Normal	125	6.E-08	22.43	0.02	0.14	0.01
EE	125	7.E-07	23.68	0.01	0.11	0.01
Normal	154	3.E-08	1.91	0.01	5.00	0.01
EE	154	1.E-07	5.26	0.03	0.21	0.00
Normal	171	2.E-07	3.94	0.02	2.01	2.59
EE	171	9.E-07	6.61	0.01	0.36	0.16
Normal	195	9.E-08	16.43	0.29	0.49	0.12
EE	195	3.E-07	15.32	0.08	0.36	0.04
Total Average		Normal	**16.66**	0.08	**1.22**	0.40
		EE	**17.89**	0.11	**0.27**	0.04

The results in Table 1 show that on average, the EE-FXLMS converged about a second faster and had about 1 dB more attenuation. The convergence speed was more uniform for EE-FXLMS with a standard deviation less than 0.05 seconds compared to 0.4 seconds or greater for normal FXLMS. Table 1 also shows that the greatest decreases in convergence time occurred at 154 Hz and 171 Hz, with a difference of several seconds being seen at these frequencies. 154 Hz and 171 Hz correspond to the two largest resonant modes of the cabin below 200 Hz.

C. Single time-varying frequency
Several swept sine test signals with different sweeping rates were created. Each test signal consisted of a swept sine from 50–200 Hz and the rates ranged from 2 Hz/sec to 256 Hz/sec. The time-averaged sound pressure level over the entire duration of the test signal was measured with and without control running. The convergence coefficient μ was determined experimentally by finding the largest stable value then scaling it back by a factor of ten to ensure stability. The μ for EE-FXLMS control was 1e-7 and the μ for standard FXLMS control was 1e-8. The attenuation in dB is reported at the error sensor, at a microphone located at the operator's ear, as an average of the ten microphones placed in the two horizontal planes above and below the operator's head, as an average of the eight microphones closest to the operator's head, and as an average of the six microphones closet to the operator's head. Each measurement was repeated three times. The actual attenuation is not reported, but the difference in dB between EE-FXLMS and FXLMS is shown in Table 2. The actual attenuation (not shown) at any microphone location was as high as 9 dB for the slower sweeps and as low as 1 dB for the faster sweeps. A positive number indicates EE-FXLMS performed better.

Table 2. Results of time-varying frequency experimentation

Sweep Rate	Ear Mic Avg. Difference (dB)	Error Mic Avg. Difference (dB)	10 Mic Avg. Difference (dB)	8 Mic Avg. Difference (dB)	5 Mic Avg. Difference (dB)
2 Hz	2.86	3.56	2.26	2.97	3.24
4 Hz	1.33	2.16	0.77	1.40	1.64
8 Hz	0.72	1.45	0.35	0.80	0.98
16 Hz	0.64	1.06	0.44	0.72	0.83
32 Hz	0.14	0.33	0.08	0.19	0.24
64 Hz	0.15	0.22	0.19	0.20	0.21
128 Hz	-0.07	0.06	-0.08	-0.04	-0.01
256 Hz	-0.10	-0.01	-0.08	-0.06	-0.04

Total Averages						
	Ear Mic Avg.	Error Mic Avg.	10 Mic Avg.	8 Mic Avg.	5 Mic Avg.	Total Avg.
Average	0.71	1.10	0.49	0.77	0.89	0.79

The data show that on average over all of the data, EE-FXLMS performs 1.0 dB better than normal FXLMS at the error sensor and about 0.8 dB globally. The data also show that the slower the sweep rate the more advantage EE-FXLMS has. For the 2 Hz sweep rate, EE-FXLMS control provides 2.3-3.5 dB more reduction. At the fastest sweep rates, the differences were almost negligible. At the faster sweep rates, such as 128 Hz/sec, the algorithm has 0.0078 seconds (1/128 Hz/sec = 0.0078 sec/Hz) to convergence at each frequency. At a single frequency, EE-FXLMS control had the fastest convergence times at about 0.10 seconds; however, 0.10 seconds is still too slow to make a performance difference at the faster sweep rates.

D. Tractor engine noise

The performance advantages of the EE-FXLMS algorithm were tested on recordings obtained from a CAT wheel-loader tractor for different operating conditions. As part of the recordings, the engine tachometer signal was recorded to use as the reference signal. The recordings were played through the source speaker, and measurements were taken in the same manner as the single time-varying frequency measurements for both EE-FXLMS and normal FXLMS control. The results are shown in Table 3 for slow, medium, and fast sweep rates of the engine rpm. A positive number indicates that EE-FXLMS performed better.

Table 3. Results of tractor engine noise

Sweep Rate	Ear Mic Avg. Difference (dB)	Error Mic Avg. Difference (dB)	10 Mic Avg. Difference (dB)	8 Mic Avg. Difference (dB)	5 Mic Avg. Difference (dB)
Slow ramp	0.98	0.98	0.86	0.95	0.97
Medium Ramp	1.23	1.35	1.07	1.21	1.25
Fast Ramp	0.40	0.35	0.38	0.40	0.41

Total Averages						
	Ear Mic Avg.	Error Mic Avg.	10 Mic Avg.	8 Mic Avg.	5 Mic Avg.	Total Avg.
Average	0.87	0.89	0.77	0.85	0.87	0.85

Similar performance advantages for the EE-FXLMS were seen with the tractor recording simulations. On average EE-FXLMS performed 0.9 dB better at the error sensor and globally 0.85 dB better than the normal case.

6. CONCLUSIONS

A new eigenvalue equalization approach has been demonstrated for the case of engine noise in a mock cabin. It has been shown that adjustments to the magnitude coefficients of $\hat{\mathbf{H}}(z)$, while preserving the phase, leads to faster convergence times and increased attenuation. Flattening the magnitude coefficients leads to an average of 13 % additional attenuation and as much as 50 % additional attenuation for some cases. In terms of dB for the specific cases tested, this was seen as on average 1 dB additional attenuation and as high as 3.5 dB additional attenuation.

The strength of the EE-FXLMS approach is its simplicity. It can be incorporated into any FXLMS algorithm with only a few lines of code and because it is an offline process, it does not increase the computational burden of the algorithm.

Flattening the magnitude coefficients is but one of many possible methods for adjusting the magnitude coefficients to improve the performance of FXLMS based algorithms. Future work will focus on an optimization approach to finding the values of the magnitude coefficients that lead to the best performance.

7. REFERENCES

[1] D.R. Morgan, "An analysis of multiple correlation cancellation loops with a filter in the auxiliary path," IEEE Trans. on **ASSP 28**, 454-467 (1980).

[2] J.C. Burgess, "Active adaptive sound control in a duct: a computer simulation," J. Acoust. Soc. Am. **70**, 715-726 (1981).

[3] R. L. Clark and G. P. Gibbs, "A novel approach to feedforward higher-harmonic control," J. Acoust. Soc. Am. **96**, 926-936 (1994).

[4] S. M. Lee, H. J. Lee, C. H. Yoo, D. H. Youn, and II W. Cha, "An active noise control algorithm for controlling multiple sinusoids," J. Acoust. Soc. Am. **104**, 248-254 (1998).

[5] L. Vicente and E. Masgrau, "Performance comparison of two fast algorithms for active control," Proc. of ACTIVE 99, pp. 1089-1100 (1999).

[6] S. M. Kuo, X. Kong, S. Chen, and W. Hao, "Analysis and design of narrowband active noise control systems," Proc. of IEEE Int. Conf. on Acoustics, Speech, and Signal Processing **6**, 557-3560, (1998).

[7] M. Rupp and A. H. Sayed, "Modified FXLMS algorithms with improved convergence performance," IEEE Proc. of ASILOMAR-29 (1995).

[8] S. D. Sommerfeldt and P. J. Nashif, "An adaptive filtered-x algorithm for energy-based active control," J. Acoust. Soc. Am. **96**, 300-306 (1994).

[9] Y. C. Park and S. D. Sommerfeldt, "Global attenuation of broadband noise fields using energy density control," J. Acoust. Soc. Am. **101**, 350-359 (1997).

[10] J. W. Parkins, S. D. Sommerfeldt, and J. Tichy, "Narrowband and broadband active noise control in an enclosure using acoustic energy density," J. Acoust. Soc. Am. **108**, 192-203 (2000).

[11] B. Faber and S. D. Sommerfeldt, "Global control in a mock tractor cabin using energy density," Proc. ACTIVE '04 (Sept), edited by Randolph H. Cabell, and George C. Maling, Jr. (1994), a04_039(10).

[12] C. C. Boucher, S. J. Elliot, and P. A. Nelson, "Effects of errors in the plant model on the performance of algorithms for adaptive feedforward control," IEE Proc.-F **138**(4), 313-319 (1991).

[13] S. J. Elliott and P. A. Nelson, "Active Noise Control," IEEE Signal Processing Mag. **10**, pp.12-35 (1993).

[14] L. A. Sievers and A. H. von Flotow, "Comparison and extensions of control methods for narrow-band disturbance rejection," IEEE Trans. on Signal Processing **40**(10), (1992).

[15] S. D. Snyder and C. H. Hansen, "The effect of transfer function estimation errors of the filtered-x LMS algorithm," IEEE Trans. on Signal Processing **42**(4), 950-953 (1994).

[16] S. D. Snyder and C. H. Hansen, "The influence of transducer transfer functions and acoustic time delays on the implementations of the LMS algorithm in active noise control systems," J. of Sound Vib. **141**(3), 409-424 (1990).