

AN ACTIVE CONTROL STRATEGY FOR MINIMIZING THE ENERGY DENSITY IN ENCLOSURES

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INTRODUCTION

Active noise control has been applied as a method of attenuating unwanted noise in enclosures. Research has shown that a global reduction of the sound field can be obtained by minimizing the potential energy in an enclosure [1]. As a practical application of this theory, squared pressure is minimized at a discrete number of locations. However, for an enclosure, a standing wave field exists so that the pressure field has distinct locations of maximum and minimum pressure. Minimizing the squared pressure will provide a global attenuation of the sound field if the error sensors are located at pressure maxima; however, if the error sensors are not located at pressure maxima, in some cases the sound field with control can be higher than that without control. As an alternative control strategy, minimizing the energy density at a discrete location is considered. This new control strategy provides significant attenuation of the sound field with little dependence on the location of the error sensor, since the error sensor is sensitive to both pressure and velocity components.

The energy density based controller is applied to a closed duct, representing a one-dimensional sound field. The duct cross section is chosen such that all cross modes have cutoff frequencies greater than any frequency of interest. Results are presented for both an analytical model and an experimental configuration.

ENERGY DENSITY FORMULATION

The acoustic energy density, w(t), is given by the equation

$$w(t) = \frac{\rho v^2(t)}{2} + \frac{p^2(t)}{2\rho c^2} , \qquad (1)$$

where ρ is the fluid density, v(t) is the acoustic particle velocity, p(t) is the acoustic pressure, and c is the propagation speed. Using Eq. 1 and Euler's Equation, the energy density can be theoretically calculated given any analytical expression for the spatial pressure distribution. Experimentally, the velocity is measured using a two microphone

technique, such as is used to measure acoustic intensity. For one dimension, the pressure gradient can be approximated as

$$\nabla p = \frac{p_2 - p_1}{\Delta x} , \qquad (2)$$

where p_2 and p_1 are the measured pressure signals of two microphones separated by an axial spacing of Δx . For this application the time integral is experimentally evaluated by passing both measured pressure signals through an analog circuit that subtracts the two signals and subsequently performs the time integration of the difference signal to produce one signal proportional to velocity. The pressure term in Eq. 1 is approximated by the average of the two measured pressure signals.

THEORETICAL REPRESENTATION

The pressure in the duct model can be represented by its normal modes given as

$$p(x) = \sum_{n=0}^{\infty} (A_n + B_n Q_c) \Psi_n(x) , \qquad (3)$$

where the eigenfunctions are given by $\Psi_n(x) = \cos(k_n x)$, Q_c represents the source strength of the control source, and the A_n and B_n are defined according to

$$A_{n} = \frac{-j\omega\rho}{L(k^{2}-k_{n}^{2})}Q_{p}\Psi_{n}(x_{p}); \qquad B_{n} = \frac{-j\omega\rho}{L(k^{2}-k_{n}^{2})}\Psi_{n}(x_{c}) .$$
(4)

In Eq. 4, L is the length of the enclosure, k_n are the eigenvalues given by $n\pi/L$, k is the disturbance wavenumber, ω is the angular frequency, Q_p is the primary source strength, and x_c and x_p are the locations of the control and primary sources, respectively.

As reported previously [2], the control source strengths for minimizing the potential energy, squared pressure, and energy density are given by

$$Q_{c,pe} = -\frac{\sum_{n=0}^{\infty} B_n^* A_n}{\sum_{n=0}^{\infty} B_n^* B_n}; \qquad Q_{c,p} = -\frac{\sum_{n=0}^{\infty} A_n \Psi_n(x_e)}{\sum_{n=0}^{\infty} B_n \Psi_n(x_e)}; \qquad Q_{c,e} = -\frac{\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} B_n^* A_l F_{n,l}}{\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} B_n^* B_l F_{n,l}},$$
(5)

where x_i is the discrete location at which the squared pressure or energy density are to be minimized and $F_{n,l}$ is given as

$$F_{n,l} = \Psi_n(x_e) \Psi_l(x_e) + \frac{1}{k^2} \frac{\partial \Psi_n(x_e)}{\partial x} \frac{\partial \Psi_l(x_e)}{\partial x} .$$
 (6)

The expressions for each control source strength can be substituted into Eq. 3, so that the simulated pressure field inside the duct model can be computed for each type of control. To calculate the field without control, Q_c is set equal to zero.

THE CONTROL ALGORITHM

The control algorithm implemented in this research was developed by Sommerfeldt [3,4], and is similar to the filtered-x algorithm developed by Widrow and Stearns [5] except that the control path transfer function is adaptively computed in real time by the controller; whereas the filtered-x algorithm generally implements a fixed model of the control path transfer function. In this research, an LMS update is used to calculate the control filter and the control path transfer function.

EXPERIMENTAL APPARATUS

The sound field is controlled in a PVC pipe, of length 5.6 m and diameter 0.102 m. One end of the duct is rigidly capped. An enclosed speaker is placed at the other end to provide a primary source. This configuration approximates a one dimensional standing wave field. A "T" junction is placed in the duct to provide a location for the control speaker. Holes are drilled in the pipe to provide several locations for the error sensor. An additional small microphone is mounted on a cart and placed inside the duct to scan the pressure field with and without control.

Control is provided using a Spectrum DSP96002 System Board (based on Motorola's DSP96002 digital signal processor) with a Spectrum Four Channel Analog I/O board. The controller requires a reference input and an error signal and outputs the control signal. A schematic of the experimental apparatus is shown in Fig. 1.

SIMULATION RESULTS

A computer program is written to calculate the control source strength for each control strategy. For a single driving frequency, the program calculates the sound pressure level (SPL) at discrete locations inside the duct model for four cases: no control, potential energy control, squared pressure control, and energy density control.

Two cases are presented with the error sensor at x/L=0.47 and the control source located at x/L=0.34. In the first case, the duct model is excited at a frequency of 213 Hz, which corresponds to a pressure maximum at the error sensor. At this frequency,



Figure 1: Experimental Setup



Figure 2: Predicted SPL at 213 Hz with the control at x/L=0.34 and error sensor at x/L=0.47.

the velocity component is small and it would be expected that the two methods give similar control. Fig. 2 confirms this expected result as both methods provide a significant amount of reduction in the region from the control source to the capped end. At first glance, it would appear that the potential energy control is not providing the most attenuation of the sound field. However, the potential energy control is reducing the sound field in the area between the two sources more than the other methods. This feature is masked when the data is displayed on a dB scale.

In the second case, the duct model is excited at a frequency of 200 Hz, yielding a pressure minimum at the error sensor, so that both pressure and velocity components exist. For this case, the energy density control is expected to provide a larger amount of attenuation than the squared pressure control. As seen in Fig. 3, the energy density control reduces the sound field more than the squared pressure control throughout most of the duct model.

A second configuration consists of the error sensor at x/L=0.43 and the control source at x/L=0.69. As seen in Fig. 4, the energy density approximates the potential energy control and provides significant attenuation compared to no control, while the squared pressure control amplifies the sound field throughout most of the duct model.



Figure 3: Predicted SPL at 200 Hz with the control at x/L=0.34 and error sensor at a x/L=0.47.



Figure 4: Predicted SPL at 180 Hz with the control at x/L=0.69 and error sensor at x/L=0.43.

EXPERIMENTAL RESULTS

The three cases considered in the simulation are measured experimentally by exciting the duct at the desired frequency, applying control, and measuring the pressure field inside the duct with the track mounted microphone. Fig. 5 shows the 213 Hz case with the error sensor located near the middle of the duct. The experimental results agree well with the predicted results with both methods of control providing an average of 35 dB reduction throughout the region from the control source to the capped end.

For the 200 Hz case, the measured data is shown in Fig. 6. As predicted, the energy density control provides an additional 20 dB of reduction when compared to the squared pressure control and nearly 40 dB of reduction compared to no control.

Fig. 7 shows the second experimental configuration with the error sensor at x/L=0.43 and the control source at x/L=0.69. As predicted, the squared pressure control makes the sound field higher than no control. The shape of the energy density curve is as predicted; however, its level is about 10 dB higher than predicted. This is thought to be due to the driving frequency being an integer multiple of the power line frequency. Even with this higher than expected level, the sound field with energy density control is still lower than that created by the squared pressure control.



Figure 5: Measured SPL at 213 Hz with the control at x/l=0.34 and error sensor at x/L=0.47.



Figure 6: Measured SPL at 200 Hz for the control at x/L=0.34 and error sensor at x/L=0.47.



Figure 7: Measured SPL at 180 Hz with the control at x/L=0.69 and error sensor at x/L=0.43

CONCLUSIONS

If the locations of maximum pressure inside an enclosure are known, minimizing squared pressure at those locations will provide a significant amount of global reduction. However, minimizing the squared pressure at locations other than pressure maxima will not significantly reduce the sound field. As an alternative control method, minimizing the energy density will significantly reduce the sound field with little dependence on the error sensor location.

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