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A COMPARISON OF CONTROL STRATEGIES FOR MINIMIZING THE SOUND FIELD IN ENCLOSURES

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INTRODUCTION

To globally minimize the sound pressure level in an enclosure, it has generally been accepted that the appropriate quantity to minimize is the potential energy in the field [1]. However, this presents problems for practical implementation, since the potential energy is obtained by spatially integrating the field, which is generally not feasible using discrete transducers. Thus, the general procedure has been to minimize the squared pressure signal from a number of discrete transducers, as an approximation to minimizing the potential energy [1-4]. Indeed, if an infinite number of transducers were used, the sum so obtained would approach the true value for the potential energy obtained by spatially integrating the field.

For regular enclosures, where it is possible to determine an optimal location for the microphone, the method of minimizing the squared pressure has worked well. However, for applications where the enclosure is not regular, and it is not easy to determine the best microphone location, this method has not been as effective. The reason for this is that the method simply minimizes the squared pressure at a number of discrete points, rather than the potential energy in the field. This paper outlines an alternative control strategy for minimizing the potential energy in the field. This strategy comes closer to providing optimal control than does the method of minimizing the squared pressure.

REPRESENTATION OF THE FIELD

The pressure in any enclosure can be represented in terms of its normal modes. For simplicity, a one-dimensional enclosure will be considered to develop the concepts in this paper. Such an enclosure would be represented by a closed duct, where the cross-section is such that all cross-modes of the duct lie well above the frequency of interest. For an enclosure with a "primary" source creating the undesirable disturbance and a "secondary" source to control the sound field, the pressure field can be represented by

$$p = \sum_{n=0}^{\infty} \left(A_n + B_n Q_c\right)\psi_n(x) , \tag{1}$$

where $\psi_n(x)$ represent the modes of the enclosure (assumed to be normalized such that $\int_0^L \psi_n^2 \, dx = L$), Q_c represents the source strength of the control source, and the A_n and B_n are defined according to

$$A_n = \frac{-j\omega\rho}{L(k^2-k_n^2)} Q_p \psi_n(x_p) \quad ; \quad B_n = \frac{-j\omega\rho}{L(k^2-k_n^2)} \psi_n(x_c) . \tag{2}$$

Here, L is the length of the enclosure, k is the disturbance wavenumber, k_n is the wavenumber for the nth mode, ω is the angular frequency, ρ is the fluid density, Q_p is the primary source strength, and x_p and x_c are the locations of the primary and control sources, respectively.

For a closed duct with cross-section dimensions axb, the total potential energy in the volume of the enclosure, V, can be found using Eq. (1) as

$$E_p = \int_0^a \int_0^b \int_0^L \frac{|p|^2}{4\rho c^2} dx dy dz = \frac{V}{4\rho c^2} \sum_{n=0}^{\infty} (A_n + B_n Q_c)(A_n^* + B_n^* Q_c^*) . \tag{3}$$

If the A_n and B_n coefficients are represented in vector form, the potential energy can be expressed as

$$E_p = \frac{V}{4\rho c^2} \left[A^H A + A^H B Q_c + Q_c^* B^H A + Q_c^* B^H B Q_c\right] , \tag{4}$$

where the superscript H represents Hermitian. From this expression, the quadratic dependence of the potential energy on the secondary source strength can be clearly seen. Following standard minimization procedures, the optimal source strength is found to be

$$Q_{c,opt} = -\left(B^H B\right)^{-1} B^H A = -\frac{\sum_{n=0}^{\infty} B_n^* A_n}{\sum_{n=0}^{\infty} B_n^* B_n} . \tag{5}$$

This expression represents a result which has appeared a number of times in the literature [1]. The important point is that to arrive at this result, it is necessary to use the orthogonality property of the modes in the enclosure.

ALTERNATIVE CONTROL SCHEMES

Just as the modal structure of the field can be used to analytically determine the optimal solution for minimizing the potential energy, the modal structure can also be used to determine the control source strength and the field properties for other control schemes. Three control schemes have been investigated for a closed duct, and the results will be compared with the optimal solution in Eq. (5).

As mentioned previously, the most common control scheme involves minimizing the squared pressure at a discrete point. Using Eq. (1), an expression for the squared pressure at a discrete point can be obtained, which will also be quadratic with respect to Q_c. Minimizing the squared pressure leads to an expression for the control source strength given by

$$Q_{c,p} = -\frac{\sum_{n=0}^{\infty} A_n \psi_n(x_e)}{\sum_{n=0}^{\infty} B_n \psi_n(x_e)}, \tag{6}$$

where x_e represents the "error" location, where the squared pressure is being minimized. By comparing Eq. (6) with Eq. (5), one can clearly see that the control source strength obtained by minimizing the squared pressure is not in general the same as the optimal control source strength to minimize the potential energy. The reason for this lies in the fact that the optimal solution utilizes the orthogonality of the eigenfunctions in performing the spatial integration, while no such orthogonality is involved in minimizing the squared pressure. Thus, the question arises as to whether some other minimization criterion exists which will come closer to emulating the orthogonality which leads to the optimal solution.

An alternative control scheme to minimize the potential energy in the field would be to minimize the total intensity at a discrete location. The rationale in choosing such a function is that the intensity represents an energy quantity, and hence one might expect to minimize energy by minimizing the intensity. As with the previous schemes, the intensity is a quadratic function of the control source strength, so that standard minimization techniques can be used. For a one-dimensional enclosure, the result of minimizing the intensity is that the optimal control source strength is found to be the same as that given in Eq. (6) for minimizing the squared pressure. Thus, for such an enclosure minimizing the intensity at a discrete location will result in the same performance as simply minimizing the squared pressure. It should also be pointed out that if standard intensity measurement techniques are used, two sensors will be required to obtain the intensity measurement, whereas only a single sensor is required to minimize the squared pressure.

A third control scheme involves minimizing the total energy density at a discrete location. Minimizing the squared pressure really minimizes the potential energy density at a discrete location, whereas minimizing the total energy density minimizes both the kinetic and potential energy densities. The energy density is also a quadratic function of the control source strength and, if standard procedures are used, the resulting optimal source strength is found to be

$$Q_{c,e} = -\frac{\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} B_n^* A_l F_{nl}}{\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} B_n^* B_l F_{nl}}, \tag{7}$$

where F_{nl} is defined according to

$$F_{nl} = \psi_n(x_e)\psi_l(x_e) + \frac{1}{k^2}\frac{\partial\psi_n(x_e)}{\partial x}\frac{\partial\psi_l(x_e)}{\partial x} . \tag{8}$$

This result reveals that the solution obtained by minimizing the total energy density would agree with the optimal solution in Eq. (5) if $F_{nl} = \delta_{nl}$, where δ_{nl} represents the Kronecker delta function. Thus, the properties of the F_{nl} function will determine how closely this scheme can approximate the optimal solution.

The control schemes presented above have been investigated numerically for a rigid one-dimensional enclosure, and some of the results will be presented below.

RESULTS

For the results presented here, a rigid-walled duct of length L has been assumed, so that the eigenfunctions for the duct can be represented by

$$\psi_n(x) = \cos(k_n x) , \tag{9}$$

where k_n is given by $n\pi/L$. Here it is assumed that the duct cross-section is sufficiently small so that the cross-modes do not contribute significantly. A single primary source is located at $x = 0$, and a single control source is located at $x = 0.359\ L$. To obtain accurate convergence, a sufficient number of modes must be included from the infinite sums. For two- and three- dimensional enclosures, it has been found that this generally involves retaining at least several thousand modes. For the one-dimensional enclosure here, it was found that good results were obtained by including 50 modes. As a check, several cases were tested including 100 modes, and it was found that the results agreed to within less than 0.1 dB.

For each configuration investigated, the optimal source strength was calculated using Eqs. (5)-(8) for the case of minimizing the potential energy, the squared pressure at a discrete location, and the total energy density at a discrete location. With these source strengths, the total potential energy in the enclosure was then determined by means of Eq. (3) for the case of no control and the cases of minimizing the potential energy, the squared pressure, and the energy density.

For a regular enclosure such as is represented here, it has been shown that effective control can be obtained by minimizing the squared pressure at a sensor located in the corner of the enclosure [1], which corresponds to the end of the duct in the present case. Figure 1 shows the results obtained with the error sensor located near the end of the closed duct ($x = 0.98\ L$) as a function of excitation wavenumber (frequency) for the various control schemes discussed above. For these conditions, minimizing the squared pressure should produce near optimal results, and for most frequencies, it can be seen that this is the case. It can also be seen that for all frequencies, minimizing the energy density at the sensor gives essentially identical control as minimizing the squared pressure.

For general enclosures, it may be difficult to determine a good location for

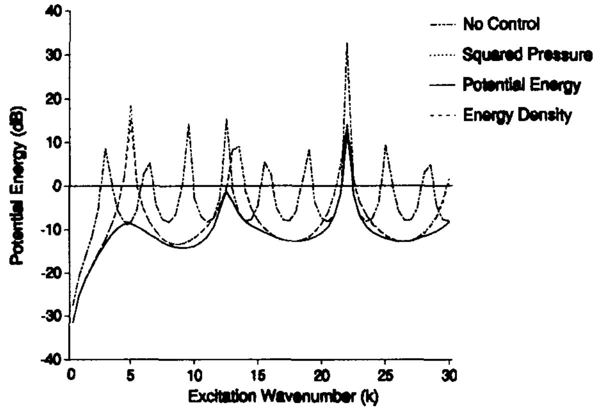

Figure 1. Total potential energy in the duct. For the squared pressure and energy density, the discrete sensor is located at the position x = 0.98L.

the sensor if one desires to minimize the squared pressure. Thus, it is also of interest to compare the performance of the control schemes for the case where a poor location for the error sensor is chosen. Figure 2 shows the results obtained with the error sensor located at the center of the duct ($x = 0.5\ L$) for the various control schemes. This represents a poor location for minimizing the squared pressure, as all of the odd modes have a pressure node at this location.

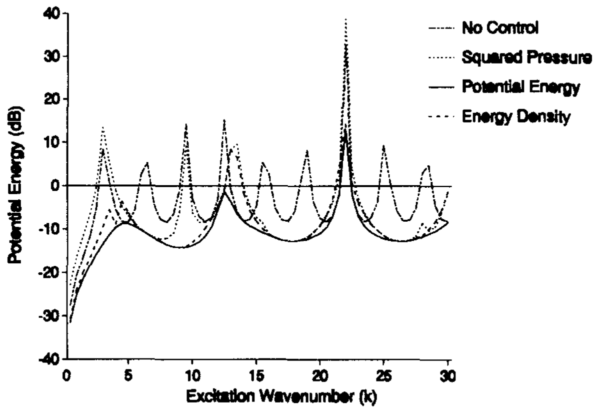

Figure 2. Total potential energy in the duct. For the squared pressure and energy density, the discrete sensor is located at the position x = 0.5L.

As can be seen, minimizing the squared pressure generally does a poor job in this case when the excitation frequency is close to the frequency of one of the odd modes. On the other hand, minimizing the energy density at the sensor produces results which are nearly optimal, except for the frequency range corresponding to a wavenumber of about 13. For this sensor location, minimizing the energy density leads to results which are clearly better than minimizing the squared pressure.

Figure 3 presents another example comparing the control schemes when only a single minimization sensor is used. In this case, the location of the error sensor was randomly selected to be at a location which was thought to be not necessarily good nor bad. In this case, the sensor is located at a position given by $x = 0.872\ L$. It can be seen that minimizing the squared pressure provides poor control for frequencies lying between the *1st* and *2nd* modes, as well as for frequencies below and at the *4th* mode. With the exception of the frequency

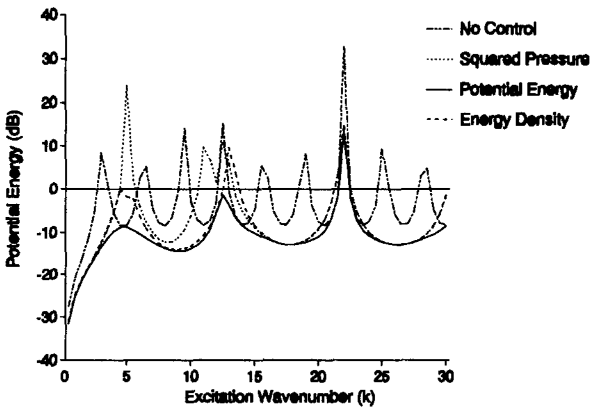

Figure 3. Total potential energy in the duct. For the squared pressure and energy density, the discrete sensor is located at the position x = 0.872L.

range just above the *4th* mode, minimizing the energy density provides considerably better control throughout the frequency range shown. The general trend which has been observed is that minimizing the energy density never seems to provide control which is significantly worse than minimizing the squared pressure, and in most cases provides control which is significantly better than minimizing the squared pressure. Furthermore, except for a few isolated frequencies, minimizing the energy density leads to control which is in general reasonably close to the optimal solution of minimizing the total potential energy in the duct. Thus, it seems that the function F_{nl} in Eq. (8) above performs approximately the same function as the orthogonality which led to the optimal solution for minimizing potential energy. This subject deserves further study in the context of three-dimensional enclosures.

The disadvantage of using energy density as the quantity to be minimized

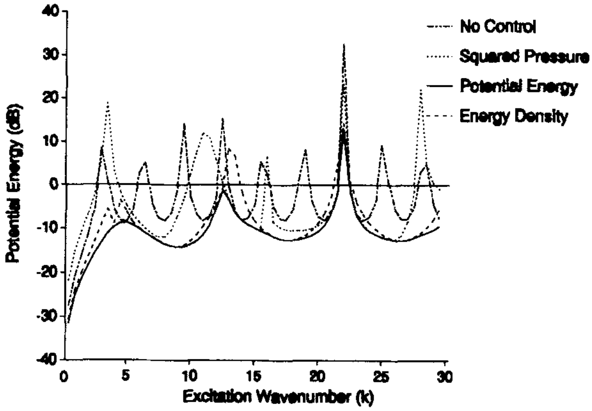

Figure 4. Total potential energy using multiple sensors. Squared pressure sensor locations: x = 0.02, 0.2, 0.4, 0.6, 0.8, 0.98L. Energy density sensor location: x = 0.5L.

is that it cannot be measured with a single sensor, as the squared pressure can. To investigate this idea, the controlled field was calculated using measurements at multiple discrete points to implement the control schemes. Figure 4 compares the results obtained by minimizing the squared pressure at six points with those obtained by minimizing the energy density at a single location. The results for the energy density parallel those shown in Figure 2. However, even with six pressure sensors being used, the potential energy in the controlled field is not minimized as well as when using the energy density. To measure the energy density using standard two-microphone techniques requires two microphones for a single point, so that these results would correspond to using six microphones for minimizing the squared pressure and two microphones for minimizing the energy density.

Another multiple sensor case is shown in Figure 5. In this configuration, eight sensors are used for minimizing the squared pressure, while two "energy density" sensors (i.e. four microphones) are used for minimizing the energy density. Minimizing the squared pressure at the eight sensor locations results in control which is reasonably close to optimal throughout the frequency range shown. However, in general the overall control using the squared pressure at eight locations is no better than using the energy density at two locations (four microphones). In addition, it can be recognized that for this case, the control obtained using two energy sensors is nearly optimal throughout the entire frequency range.

CONCLUSIONS

The expansion theorem, whereby the field is represented in terms of its modes, provides a powerful means of being able to investigate the active control of enclosures using various control schemes. Using this approach, it is possible

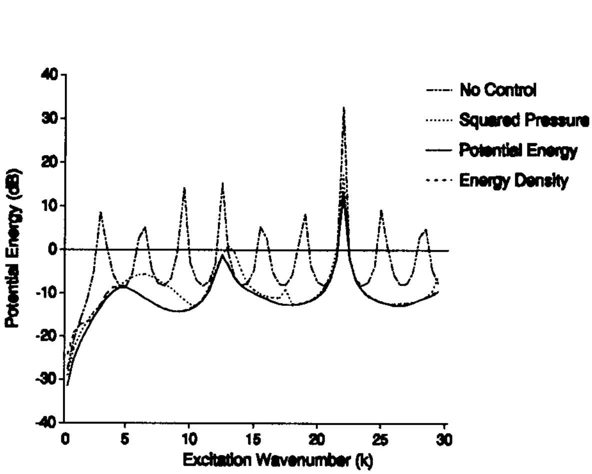

Figure 5. Total potential energy using multiple sensors. Squared pressure sensor locations: x = 0.02, 0.1, 0.2, 0.3, 0.4, 0.,5 0.75, 0.98L. Energy density sensor locations: x = 0.5, 0.98L.

to obtain analytical results for various control schemes which make it possible to understand how well various schemes might be expected to perform. The numerical results have demonstrated some of the problems which have been known relating to the use of the squared pressure as the quantity to be minimized in an active control system. The use of total energy density at discrete points has also been investigated, and it has been found that this scheme generally performs better than (and certainly no worse than) minimizing the squared pressure. As well, if multiple error sensors are used, it generally requires fewer sensor locations to achieve comparable control if the energy density is minimized rather than the squared pressure.

Several areas remain to be investigated. Since energy density cannot be measured directly, the effect of errors in estimating the energy density needs to be investigated, to determine how much the control could be expected to be degraded. In addition, these concepts can be extended to three-dimensional enclosures to develop alternative control schemes for minimizing the sound in general enclosures. In particular, the scheme of minimizing intensity should be looked at again in the context of three-dimensional enclosures, to determine if the control would be the same as for minimizing the squared pressure, as it is in one-dimensional enclosures.

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