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ADAPTIVE VIBRATION CONTROL USING AN LMS-BASED CONTROL ALGORITHM

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INTRODUCTION

The fundamental principles governing the reduction of energy in an acoustic or vibration field have been understood for decades. In recent years, the concept of active control has been investigated as a means of controlling the acoustic or vibration field [1–3]. To implement active control, a number of "secondary" sources are introduced into the system which interact with the original "primary" source in such a way as to achieve the desired control. Active control has been found to be most effective for low frequency attenuation. Thus, it is often desirable to use active control in combination with passive techniques since effective high frequency attenuation is possible using conventional passive control.

Recent advances in high-speed digital signal processors have made adaptive implementations of active control attractive. In a number of applications, the frequency or spatial distribution of the primary field may change with time. In addition, the parameters of the system to be controlled may also change with time. Adaptive systems have the ability to track such changes and provide optimal control over a much broader range of conditions than conventional fixed control systems. This paper will discuss a control system based on the least-mean-squares (LMS) algorithm which has been developed for active control applications. The control algorithm is applicable for time varying systems, including systems whose response to the secondary source input varies with time. Thus, the algorithm must simultaneously perform system identification and control. Both of these processes are achieved adaptively, which results in a control system which is fully adaptive and requires no *a priori* measurements or training.

DEVELOPMENT OF THE ALGORITHM

Consider a system, to be adaptively controlled using a control actuator, an error sensor, and a sensor to provide an input training sequence. For an LMS-based algorithm, the control signal, $y(t)$, is given by

$$y(t) = \sum_{i=0}^{I-1} w_i(t)x(t-i), \tag{1}$$

where the $w_i(t)$ are the filter coefficients for the Ith order LMS control filter, $x(t-i)$ is the input training sequence, and t is a discrete time index. If the transfer function from the control actuator to the error sensor is modeled as a time-varying FIR filter, with coefficients $h_j(t)$, the error signal can be written as

$$e(t) = d(t) + \sum_{j=0}^{J-1} h_j(t) \sum_{i=0}^{I-1} w_i(t-j)x(t-i-j), \tag{2}$$

where $d(t)$ is the "desired" signal to be cancelled, given by the response of the system to the primary input alone. The error signal can be seen to consist of the response of the system to both the primary input and the secondary control input. Using (2), both a system identification algorithm and a control algorithm can be developed.

<u>System Identification</u>: For an LMS-based system, the desired signal, $d(t)$, is assumed to be correlated with the input signal, $x(t)$. If this relationship is modeled using a FIR filter,

$$d(t) = \sum_{j=0}^{J-1} c_j(t)x(t-j), \tag{3}$$

where the $c_j(t)$ represent the transfer function between the input signal and the desired signal. Introducing vector notation and using (1), (2), and (3) allows the error signal to be written in the simple form

$$e(t) = \mathbf{\Theta}^T(t)\mathbf{\Phi}(t), \tag{4}$$

where

$$\begin{aligned} \mathbf{\Theta}^T(t) &\equiv \left[h_0(t)\ h_1(t) \cdots h_{(J-1)}(t)\ c_0(t) \cdots c_{(J-1)}(t)\right] \\ \mathbf{\Phi}^T(t) &\equiv \left[y(t)\ y(t-1) \cdots y(t-J+1)\ x(t) \cdots x(t-J+1)\right]. \end{aligned}$$

All values contained in $\mathbf{\Phi}(t)$ are available, either as measured or computed data. Thus, a number of adaptive estimation algorithms are available for equations in the form of (4). The projection algorithm [4] was chosen for this task in the present application. If the estimate of the filter coefficients is denoted by $\hat{\mathbf{\Theta}}(t)$, the coefficients are updated according to

$$\hat{\mathbf{\Theta}}(t+1) = \hat{\mathbf{\Theta}}(t) + \frac{a\mathbf{\Phi}(t)}{b + \mathbf{\Phi}^T(t)\mathbf{\Phi}(t)} \left[e(t) - \hat{\mathbf{\Theta}}^T(t)\mathbf{\Phi}(t)\right], \tag{5}$$

where $b > 0$ prevents division by 0 and $0 < a < 2$ to ensure convergence.

LMS Control Filters: To develop the form of the LMS control filters, it is useful to consider the case in which the filter coefficients, w_i, are time-invariant. In this case, (2) can be rearranged as

$$e(t) = d(t) + \sum_{i=0}^{I-1} w_i \sum_{j=0}^{J-1} h_j(t)x(t-j-i). \tag{6}$$

The term $\sum_{j=0}^{J-1} h_j(t)x(t-j-i)$ is a filtered version of the input signal and will be denoted by $r(t-i)$. The form of (6) corresponds to inverting the order of the two transfer functions involved. Equation (6) can now be written as

$$e(t) = d(t) + \mathbf{r}^T(t)\mathbf{W}, \tag{7}$$

where

$$\mathbf{r}^T(t) \equiv [r(t)\; r(t-1) \cdots r(t-I+1)]$$
$$\mathbf{W} \equiv \left[w_0\; w_1 \cdots w_{(I-1)}\right].$$

The LMS-based algorithms are designed to minimize the mean-square-error performance criterion, given by

$$J = E\{e^2(t)\},$$

where $E\{\;\}$ denotes the expectation operator. If standard techniques are used [5], the resulting update equation for the LMS control filters is given by

$$\mathbf{W}(t+1) = \mathbf{W}(t) - \mu\mathbf{r}(t)e(t), \tag{8}$$

where μ is a convergence parameter chosen to maintain stability. It can be shown that the algorithm is stable for $0 < \mu < 2/\lambda_{max}$, where λ_{max} is the largest eigenvalue of the filtered autocorrelation matrix, $E\{\mathbf{r}(t)\mathbf{r}^T(t)\}$.

For the control system developed, the system identification algorithm and LMS control algorithm operate simultaneously in real-time to provide both tasks necessary in achieving optimal control of the system.

EXPERIMENTAL RESULTS

The control system was implemented in real-time using the Motorola DSP56000ADS signal processing board. The controller was used to provide active control for a system consisting of a single two-stage vibration isolation mount (Fig. 1). The system was excited and controlled using Wilcoxin F4 shakers, and the input and error signals were obtained by means of PCB 303A11 accelerometers. The control shaker was suspended from the intermediate mass as a means of providing an inertial control force without creating a second force transmission path to the foundation.

The system was initially tested using a sinusoidal input excitation signal. In such an application, the adaptive filter must match the optimal

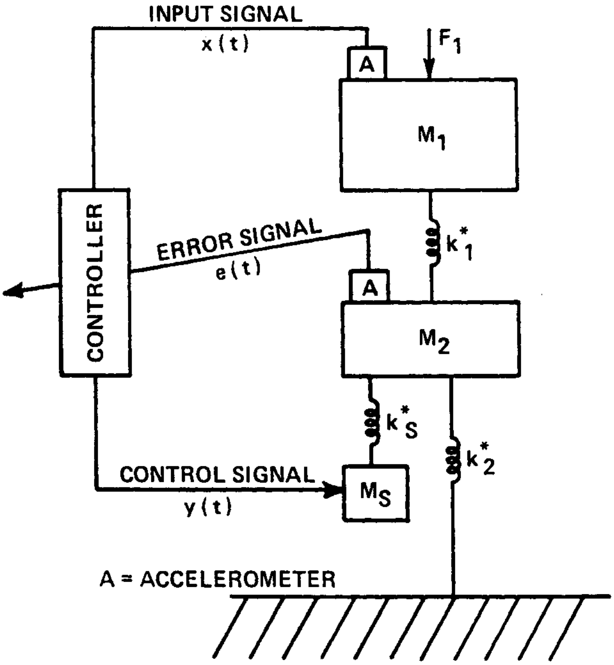

Fig. 1. Schematic diagram of the two-stage isolation mount.

magnitude and phase characteristics at only one frequency. Fig. 2 shows the frequency spectrum of the error signal, with the dashed curve showing the spectrum before control is applied and the solid curve showing the spectrum after control is applied and steady state achieved. For this particular case, about 34 dB attenuation was achieved. The maximum attenuation possible is limited in this case by the hardware of the control system.

The adaptive controller was also tested for an input excitation signal consisting of several discrete frequency components (Fig. 3). In this case, the controller must match the desired transfer function at several different frequencies, which it successfully did.

Finally, the adaptive controller was tested for the case of a random input excitation signal, bandpass limited to 0–200 Hz (Fig. 4). The controller attenuates some frequency regions, while it is ineffective for other frequency regions. The response of the control shaker falls off sharply below 30 Hz. Hence, it is ineffective for controlling the lowest resonance of the system. The isolation mount, as constructed, is a dispersive medium, with the higher frequency components propagating faster through the structure than lower frequency components. As a result, at sufficiently high frequencies, the desired signal to be cancelled, $d(t)$, is no longer correlated with the input signal, $x(t)$, corresponding to it. For such cases, it can easily be shown that the optimal control filter goes to zero, *i.e.* the control filter has zero frequency response at those frequencies. This effect can be seen in Fig. 4 for frequencies above 100 Hz.

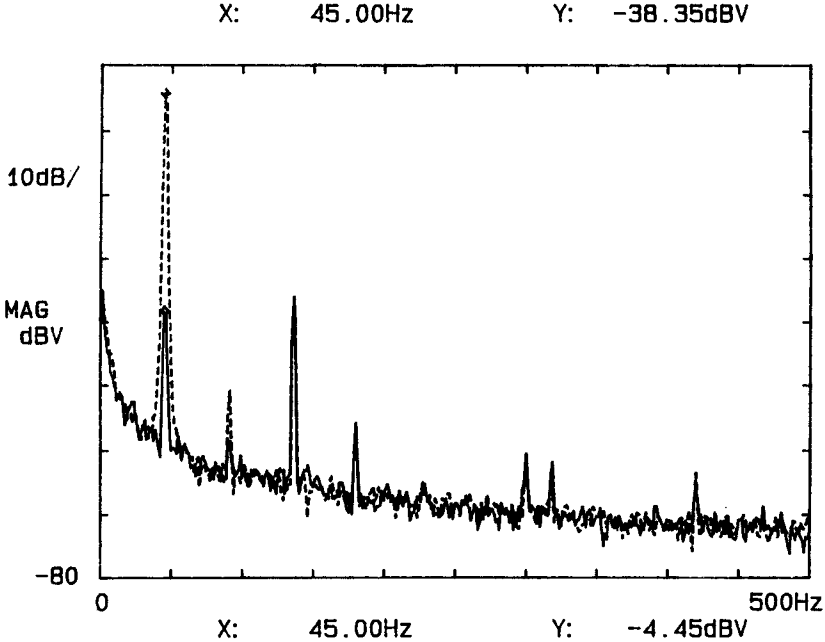

Fig. 2. Error signal spectrum – 45 Hz excitation signal: Without control, dashed line; with control, solid line.

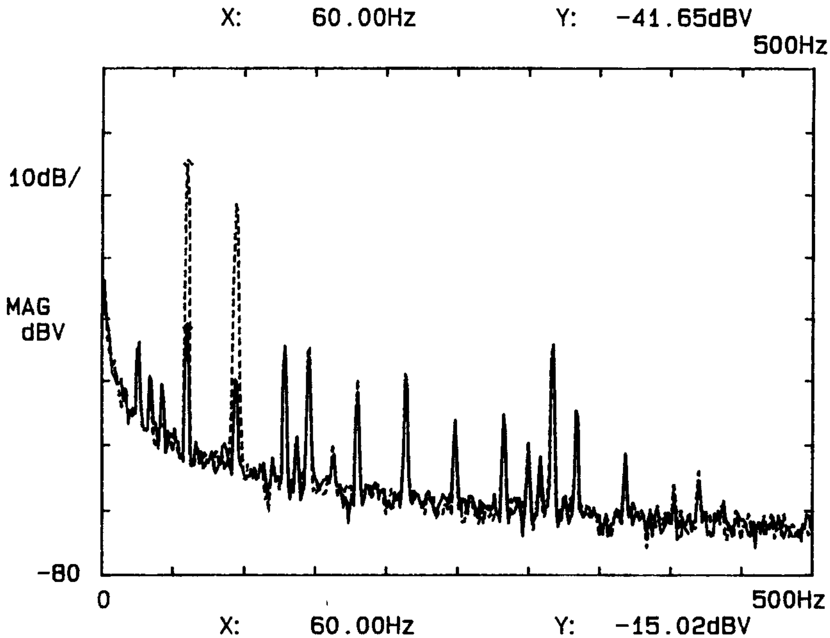

Fig. 3. Error signal spectrum – multiple frequency (60 Hz and 95 Hz) excitation signal: Without control, dashed line; with control, solid line.

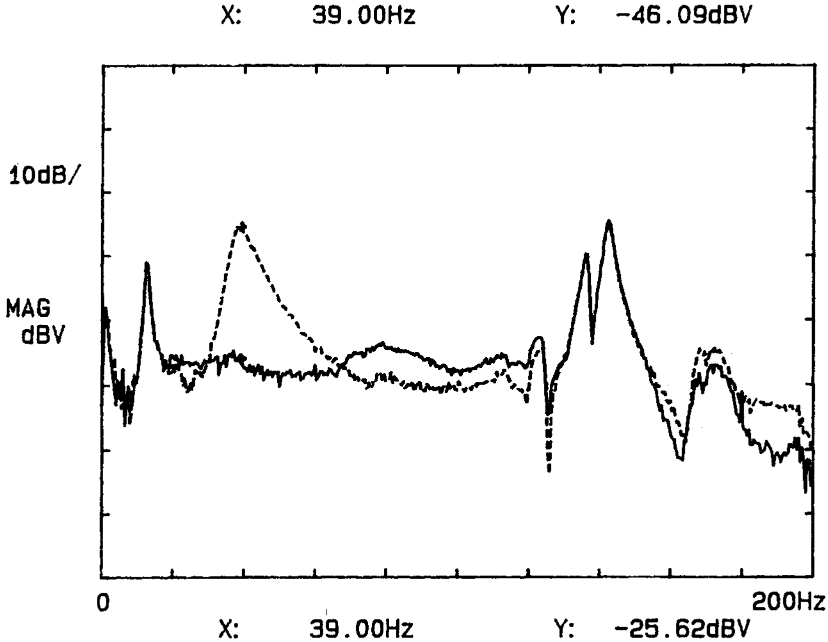

Fig. 4. Error signal spectrum – random excitation signal: Without control, dashed line; with control, solid line.

SUMMARY

The adaptive controller has been demonstrated to be effective in attenuating periodic signals, as well as some components of random excitation signals. The controller has also shown the capability of tracking changes in the parameters of the system to be controlled. As well, the control algorithm can readily be extended to a multi-input, multi-output system to deal with more complex multi-dimensional systems. In such a case, the controller acts so as to minimize the sum of the mean-square-errors.

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