

# Quantifying Nonlinearity in the Propagation of Noise from Military Jet Aircraft

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## ABSTRACT

Because of the high noise levels radiated by military jet aircraft, it has been hypothesized that nonlinearity influences the propagation of the noise. A numerical model, which accounts for second-order cumulative nonlinearity, atmospheric absorption and dispersion, and geometrical spreading, has been developed to propagate jet noise waveforms. Numerical propagation of recorded waveforms from recent static engine run-up measurements demonstrates significant waveform steepening and an accompanying transfer of spectral energy to high frequencies that agrees well with measured spectra. Furthermore, the measured and nonlinearly-predicted waveforms are perceived to be significantly “louder” or “more annoying” than linearly-predicted waveforms, despite the fact that standard metrics such as overall sound pressure level (with flat, A, and C weighting) and Mark-VII perceived loudness show little difference between linearly- and nonlinearly-predicted spectra. The results of this study demonstrate the need for additional investigations with alternate metrics that more closely relate to perceived annoyance or loudness of high-amplitude jet noise.

## 1. INTRODUCTION

Part of an effort to improve community and environmental impact models for the noise radiated by military jet aircraft has comprised the inclusion of nonlinear propagation effects in the models<sup>1</sup>. This paper compares the results of a numerical model developed to nonlinearly propagate a noise waveform against recent F/A-22 Raptor run-up measurements. Various power-based, single-number metrics are then calculated and discussed in the context of their poor correlation with subjective response. In order to more effectively demonstrate the perceived difference between nonlinear and linear propagation, the results of numerically propagating a shaped random noise waveform are also included.

## 2. NUMERICAL MODEL SUMMARY

The numerical model used to nonlinearly propagate a time waveform has been previously documented in Refs. 2-4; consequently, its details are only briefly summarized. The model solves a generalized Burgers equation, which incorporates effects of quadratic nonlinearity, atmospheric absorption and dispersion, and geometrical spreading via a hybrid time-frequency domain algorithm that is based on methods described in Refs. 5-7. For a sufficiently small step size, the nonlinear and small-signal processes may be considered independent and treated

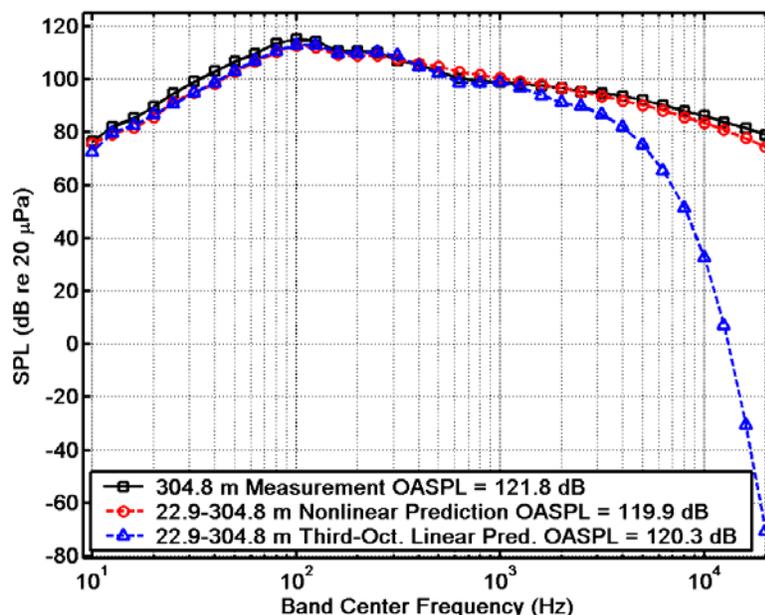
separately, allowing the physical phenomena to be considered in the domain in which they are most readily applied. Nonlinearity is treated in the time domain and absorption, dispersion, and geometrical spreading are applied to the Fourier pressure spectrum in the frequency domain. By using the fast Fourier transform and its inverse to alternate between the two domains, the pressure waveform may then be marched forward in range.

### 3. F/A-22 RAPTOR RESULTS

F/A-22 ground engine run-up measurements were conducted at Edwards Air Force Base on 15 Sept., 2004 between 6:30-8:30 PDT. Temperature gradients and wind speeds were slight to non-existent, resulting in an environment conducive to making propagation measurements. Bruel and Kjaer and GRAS condenser microphones were located behind the aircraft along the 90-145° radials (relative to the jet inlet) from 22.9-304.8 m (75-1000 ft) at an approximate height of 1.8 m. Additional measurement details are available in Ref. 2.

#### A. Predictions versus Measurement

A pressure waveform recorded at 22.9 m and 125° for one engine at idle and the other at afterburner (AB) has been numerically propagated to 304.8 m according to the measured ambient conditions. The measured AB third-octave spectrum at 304.8 m and 125° is shown in Fig. 1, along with nonlinear and linear predictions. The linearly-predicted spectrum is a free-field extrapolation of the third-octave spectra at 22.9 m with atmospheric absorption for each band calculated at the band center frequency. Agreement between the nonlinear prediction and measurement is quite good, especially when compared to the linear prediction. The measured spectrum at 304.8 m for AB is about 3 dB higher than predicted below 100 Hz, which is currently thought to be due to atmospheric effects, and not nonlinearity, in that similar behavior was observed for a low-amplitude measurement with both engines at idle<sup>2</sup>. The nonlinearly-predicted spectral levels are about 1 dB lower than the linear prediction between 100-300 Hz. However, the obvious disparity between nonlinear and linear theory occurs with the energy transfer to frequencies above 1 kHz, which agrees quite well with the measurement. Additional comparisons of the model against the ground run-up measurement are found in Ref. 2.



**Figure 1:** Measured and predicted third-octave spectra at 304.8 m and one engine at afterburner.

## B. Metric Calculations

A number of metrics have been calculated from the third-octave spectra: overall sound pressure level (OASPL), A-weighted OASPL (OASPL<sub>A</sub>), C-weighted OASPL (OASPL<sub>C</sub>), and Mark-VII perceived loudness<sup>8</sup> (Mark-VII PL). The calculated metrics, as well as the difference between nonlinear and linear predictions, are shown in Table 1 for the AB measured and predicted spectra at 304.8 m and 125°. With the exception of OASPL<sub>A</sub>, agreement between the measured and predicted spectra is not extremely good, which is likely due, at least in part, to the discrepancy between the measured and predicted spectra below 100 Hz (see Fig. 1). Nonlinearity is seen to reduce the OASPL relative to linear propagation, which is perhaps intuitively obvious because energy is transferred from the peak-frequency region, which dominates an OASPL calculation, to primarily higher frequencies where the effect of atmospheric absorption is greater. This nonlinear transfer of energy results in greater overall dissipation of the acoustical energy. As opposed to the non-weighted OASPL, the nonlinearly-predicted OASPL<sub>A</sub> shows a 1.2 dBA increase relative to linear propagation because of the increased weighting given to frequencies in the 1-6 kHz range. The OASPL<sub>C</sub>, however, mirrors the behavior of the OASPL calculation because the C-weighting curve is much flatter than the A-weighting curve at low frequencies. Finally, the Mark-VII PL calculation shows the greatest relative difference between nonlinear and linear predictions in that the perceived level due to nonlinear propagation is 1.7 dB greater than for linear propagation. These differences are rather marginal, however, given the typical experimental uncertainty associated with a field-type measurement.

**Table 1:** Calculated metrics for F/A-22 spectra at 304.8 m. Predicted spectra were obtained by numerical propagation of the 22.9-m waveforms.

Spectrum Type	OASPL (dB)	OASPL <sub>A</sub> (dBA)	OASPL <sub>C</sub> (dBC)	MARK-VII PL (PLdB)
Measured	121.8	111.2	121.5	118.5
Nonlinear Pred.	119.9	111.4	119.7	117.6
Linear Pred.	120.4	110.2	120.1	115.9
Non – Lin	-0.5	1.2	-0.4	1.7

One possible, but potentially erroneous, conclusion at this point would be that although the nonlinear propagation of jet noise results in a substantial transfer of energy to high frequencies, the difference in perceptual impact relative to linear propagation is negligible because the differences in metric calculations are small and within the bounds of typical measurement uncertainty. However, the subjective sensation of sound often cannot be quantified by a simple metric<sup>9</sup>, despite the preference of scientists and engineers to adopt single-number metrics such as those calculated here. Playback of the nonlinearly- and linearly-propagated waveforms, which is not possible here given the potential sensitivity of the F/A-22 time-series data, reveals a clear subjective distinction between linear and nonlinear propagation despite its failure to appear in the overall power-based metric calculations. The nonlinearly-propagated waveforms sound noticeably louder and are more annoying than the waveforms derived from linear theory. Furthermore, the nonlinearly-propagated waveform sounds very similar to the measured waveform and much more so than the linearly-propagated waveform. A hypothesis for the cause of the additional loudness associated with nonlinear propagation is now discussed.

### C. Waveform Statistical Analysis

There is an impulsive or crackle-like<sup>10</sup> quality of the nonlinearly-propagated and measured waveforms that is not present in a linear prediction from 22.9 to 304.8 m, which causes the nonlinear prediction to be perceived as louder and more annoying than the linear waveform. It is believed that this impulsive quality is the result of significant waveform steepening and shock formation. Consideration of the probability density function (PDF) for each of the predicted and measured AB waveforms and their time-derivative estimates at 304.8 m strengthens this hypothesis.

Examination of the waveform distributions in Fig. 2 shows that the measured and predicted PDFs are very similar. In other words, the statistics of the waveforms themselves do not reveal major distinguishing characteristics of nonlinear and linear propagation. On the other hand, a comparison of the distributions of the measured and predicted waveform time derivatives, estimated using forward rather than centered differencing<sup>12</sup>, reveals an entirely different scenario in Fig. 3. The measured and nonlinearly-predicted distributions agree very well and differ considerably from the linearly-predicted PDF. The nonlinear and measured distributions have maximum values at waveform slopes that are slightly negative, above which the distributions decay into a long tail of positive values, which correspond to the positive slopes associated with the rapid rise of shock-like wavefronts of varied pressure amplitudes. This extreme difference between the waveform time derivative estimates is what likely characterizes the perceived distinction between nonlinear and linear propagation. In order to provide further evidence of this hypothesis, an additional example that may be described in more detail is now considered.

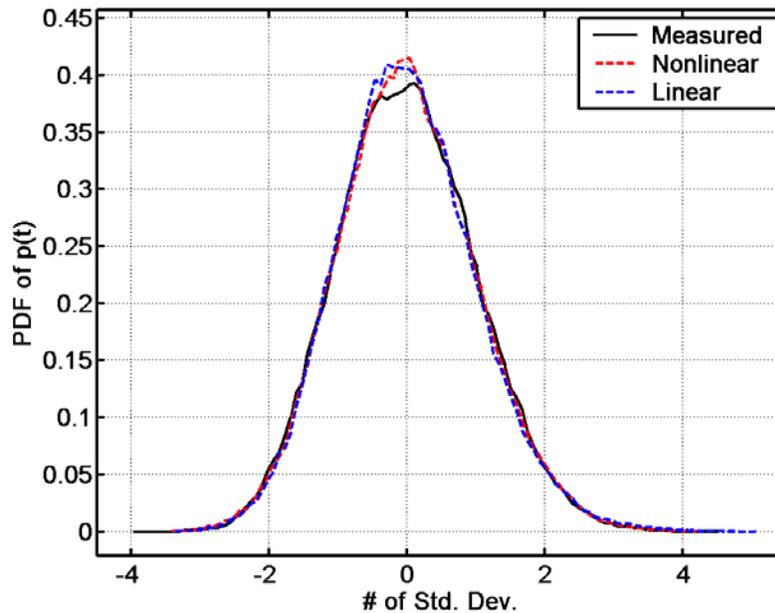
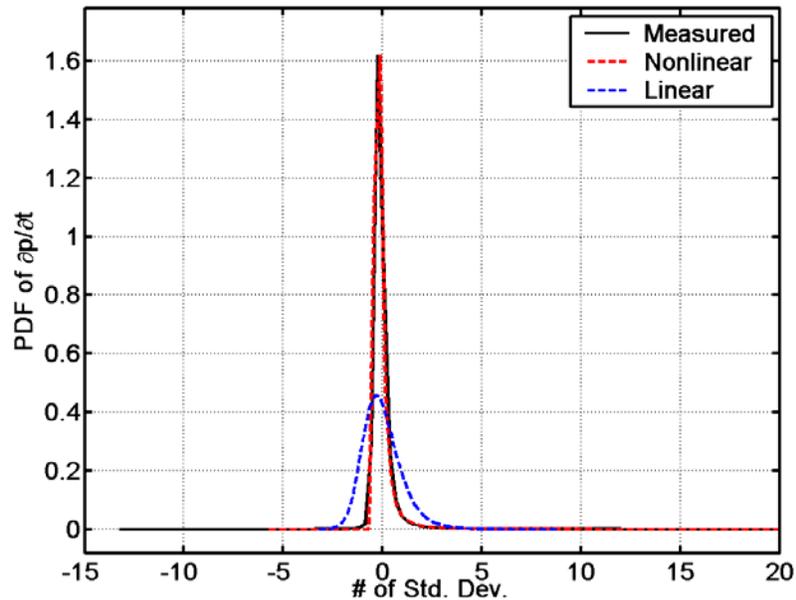


Figure 2: Probability density function of 304.8-m AB waveforms.



**Figure 3:** Probability density functions of the 304.8-m AB waveform time derivatives. The positive tails for both the measured and nonlinearly-predicted distributions extend out to approximately 63 standard deviations.

#### 4. SHAPED RANDOM NOISE EXAMPLE

In order to provide an example that can be discussed in more detail, a shaped Gaussian random noise waveform was first created with an OASPL of 150 dB re 20  $\mu$ Pa. The spectral shape has a nominal  $f^2$  slope below a peak frequency of 100 Hz, and a  $f^{-2}$  slope beyond. The power spectral density (PSD) at an assumed initial distance of 10 m is shown (labeled “Input”) in Fig. 4. This waveform was numerically propagated with nonlinear and linear propagation to 500 m through a homogenous atmosphere with ambient pressure, temperature, and relative humidity of 1 atm, 20° C, and 50 %, respectively.

##### A. Predicted Waveforms and Spectra

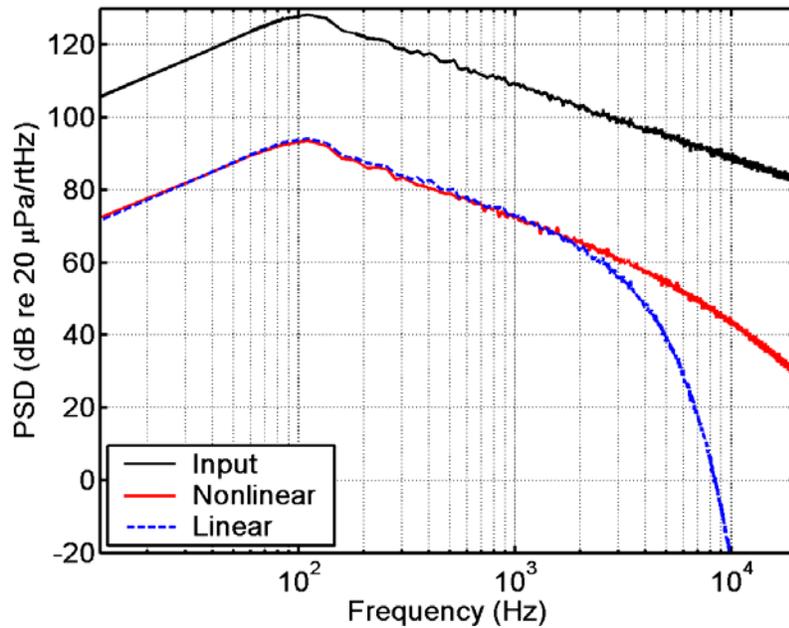
Before proceeding to metric calculations and a PDF-based analysis, it is appropriate to first discuss the predicted waveforms and spectral densities for this example. Small segments of the nonlinearly- and linearly-predicted waveforms at 500 m are shown in Fig. 5, in which waveform steepening and the presence of shocks are apparent for the nonlinear case. The corresponding spectral predictions are displayed in Fig. 4. The linearly-predicted PSD is less than -200 dB re 20  $\mu$ Pa/ $\sqrt{\text{Hz}}$  at 20 kHz; however, the figure's ordinate has been truncated to show the slight loss of power due to nonlinearity from the peak-frequency region (1-2 dB from 100 Hz to 1 kHz).

##### B. Waveform Playback

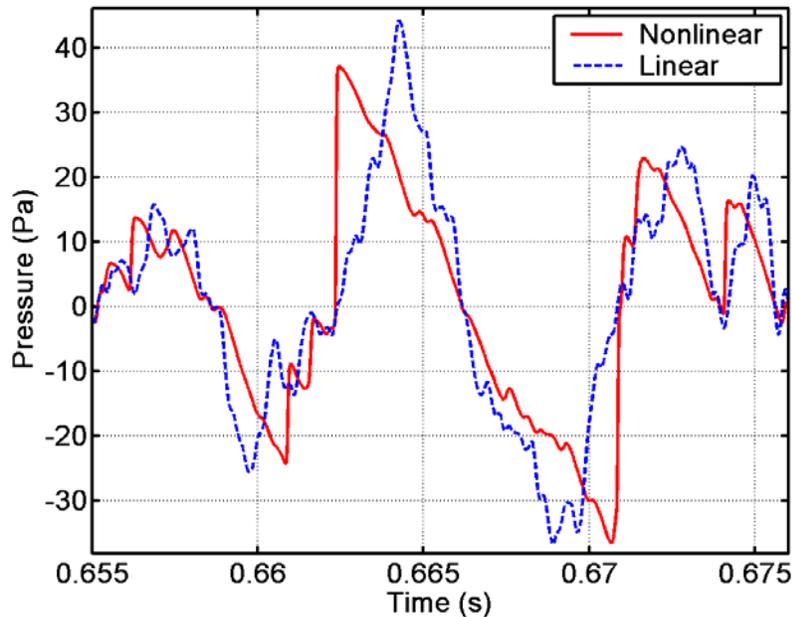
Before the calculations of the various single-number metrics are shown for the shaped random noise, the 500-m waveforms derived from nonlinear and linear propagation of the 10-m waveform may be heard by clicking on the “Nonlinear.wav” and “Linear.wav” links below. (Note that Adobe® Acrobat Reader 6.0 or later is needed for the links to function.) These two wave files qualitatively but effectively illustrate the perceived difference between nonlinear and linear propagation in the F/A-22 example described in the previous subsection.

[Nonlinear.wav](#)

[Linear.wav](#)



**Figure 4:** Power spectral density (PSD) of the input shaped random noise at 10 m and the linearly- and nonlinearly-predicted spectral densities at 500 m. The linear prediction exceeds -200 dB re 20 μPa/√Hz at 20 kHz.



**Figure 5:** Linearly- and nonlinearly-predicted random noise waveform segments at 500 m.

### C. Metric Calculations

Playback of the nonlinearly- and linearly-predicted wave files for the shaped random noise example has clearly demonstrated that there is a significant difference between the nonlinearly- and linearly-propagated waveforms. However, the calculated metrics do not capture this difference; the OASPL calculations with flat, A, and C weighting all indicate a loss of power of

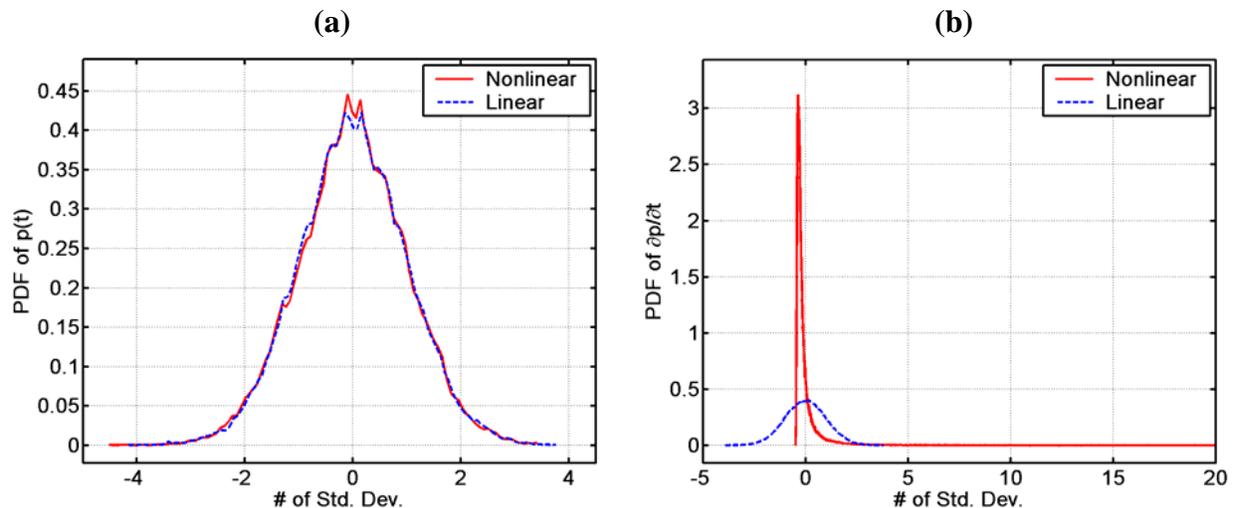
nonlinear relative to linear propagation but that the perceptual difference between the nonlinear waveform and the linear waveform is essentially negligible. The Mark-VII PL for this example, as was the case with the F/A-22, does show an increase in perceived loudness for nonlinear propagation relative to linear propagation, but again the difference is marginal.

**Table 2:** Calculated metrics for shaped random noise, numerically propagated from 10 to 500 m.

Spectrum Type	OASPL (dB)	OASPL <sub>A</sub> (dBA)	OASPL <sub>C</sub> (dBC)	MARK-VII PL (PLdB)
Nonlinear Pred.	114.7	106.4	114.4	111.6
Linear Pred.	115.4	106.7	115.1	111.2
Non – Lin	-0.7	-0.3	-0.7	0.4

#### D. Waveform Statistical Analysis

A PDF-based analysis of the waveforms and corresponding time derivative estimates in Fig. 6 reveals similar behavior to that seen in the F/A-22 example. The probability distributions for the waveforms in Fig. 6a demonstrate only minor differences between numerical propagation according to nonlinear and linear theory. On the other hand, examination of the distributions for the waveform time derivatives in Fig. 6b reveals an enormous difference between the linearly- and nonlinearly-predicted waveforms.



**Figure 6:** Probability density functions of the (a) waveforms and (b) waveform time derivatives at 500 m. Positive derivative values for the nonlinearly-propagated waveform continue out to nearly 79 standard deviations.

### 5. CONCLUSIONS

Conclusions based on the results of this study are three-fold. First, the comparison between predicted and measured afterburner spectra for the F/A-22 in Fig. 1 demonstrates that the propagation of high-amplitude jet noise can be significantly nonlinear. The second conclusion, which has been the primary focus of this paper, is that there is an important perceptual difference between nonlinear and linear propagation that is not apparent in single-number metrics, such as A-weighted overall sound pressure level. The use of these overall power-based metrics would

indicate that inclusion of nonlinear effects in community impact models is unimportant; however, playback of the shaped random noise example effectively indicates otherwise. The impulsive quality associated with a nonlinearly-propagating jet noise waveform that causes it to be perceived quite differently from a linearly-propagated waveform. This characteristic of the noise appears to be linked to the drastic differences between the shapes of the nonlinearly- and linearly-derived probability density functions of the waveform time derivatives.

Finally, the results of the present investigation have also clearly illustrated the need for additional research with alternative sound quality metrics that better agree with listener perception. The numerical model should be quite useful in obtaining nonlinearly-propagated jet noise waveforms for a variety of conditions that then may be presented to a jury of listeners for correlation of subjective response and metric calculations. This potential study would constitute a critical step forward in quantifying the role of nonlinearity in jet noise propagation and thereby improve community and environmental impact assessments.

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- <sup>10</sup> "Crackle" is the subjective phenomenon that occurs in heated, high-speed jets, which is typically quantified by the skewness (normalized third central moment) of the waveform. It was first studied by Ffowcs Williams *et al.*<sup>11</sup> in the context of the noise radiated by the Concorde's Olympus engines and although it is believed to be primarily a source phenomenon, its origin and relationship to finite-amplitude propagation effects are still not well understood at the present.
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