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# Teaching the descriptive physics of string instruments at the undergraduate level

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At Brigham Young University a general education course introduces students to the basic descriptive acoustic principles of music, speech, and audio. A third of this course focuses on the physics of musical instrument families. Three of these families include bowed string, plucked string, and struck string instruments. The concepts of driven systems and freely vibrating systems are taught, including the consequences of these excitation conditions. A hands-on lab for the course enables students to explore how the length, density, and tension of the string change the fundamental frequency. An in-class demonstration highlights the role of inharmonicity on the partial frequency values for these string instrument families. The combination of hands-on activity and demonstration aids the students in comprehending the basic acoustic principles behind string instruments and the reason for varying levels of inharmonicity between these instrument families.

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# 1. INTRODUCTION

Physics 167: Descriptive Acoustics of Music and Speech is a 3.0 credit hour course that is offered as a General Education course for various majors, mostly non-science majors, at Brigham Young University (BYU) in Provo, Utah. The course focuses on concepts and only requires algebra level mathematic skills. Student majors include music, communication disorders, sound recording, physics, engineering, and many others. The course was created by William J. Strong at BYU and first offered in 1969. Since then 6,270 students have taken the course as of the end of the Winter 2016 semester with an average of 132 students per year. The course is normally offered twice a year. The textbook for the course is *Music Speech Audio, 4th Edition*, written by William J. Strong and George R. Plitnik;[1] this text is used at other universities besides BYU.

The course is divided up into three units. The first unit on introductory acoustics covers simple harmonic motion, compound oscillators, resonance, wave motion, wave phenomena, complex waves, transduction, sound levels, and room acoustics. The second unit on hearing and speech covers the human hearing mechanism, psychoacoustics, hearing impairment, noise, the human speech production mechanism, vocal tract effects, speech sounds, speech defects, and degraded speech. The third unit on musical instruments covers the human singing voice and then covers several musical instrument families including: bowed strings, plucked strings, struck strings, brass, mechanical reeds, air jets, percussion, and electronic instruments and music.

This paper focuses on some of the ideas and methods used to teach the string instrument families. In Unit 1 we introduce modes on strings and the idea of a fundamental frequency and partials. In Unit 2, in talking about the speech mechanism, we introduce the idea of a source, resonator, and filter, which is then extended to the musical instrument families. In Unit 3 we spend 3 class periods on bowed, plucked, and struck string instrument families. This paper will discuss the key concepts we wish for the students to learn, a laboratory exercise that is used to explore string instruments, and we will explain several demonstrations used in class to illustrate the principles covered.

# 2. KEY CONCEPTS FOR STRING INSTRUMENTS

This section will discuss various key concepts covered in the units on string vibrations of musical instruments in BYU's general education course on the Descriptive Acoustics of Music, Speech, and Audio. This material represents concepts the students learn during nearly two weeks of the course scope. The purpose of including this material is to provide an introductory review to the physics of string motion and radiation. This material may be useful for students in this course and other similar courses as a review.

When a string vibrates a wave travels up and down the length of the string at a certain wave speed. This wave speed is essentially independent of the frequency of vibration. When the string is displaced from its rest position, the tension in the string is the principle restoring force present. The wave speed in the string is equal to the square root of the tension in the string divided by the linear density (density divided by the string's cross sectional area). Thus if the string is tightened, waves on the string will travel faster, whereas if the string is made thicker or made of a more dense material, waves on the string will travel slower.

Resonances in string vibration occur at certain vibration frequencies whose wavelengths are related to the string's length according to how the string is supported at its ends. Principally there are three combinations of string end conditions: when the string is fixed at both ends, free at both ends, or fixed at one end and free at the other end. When the end conditions are the same, the lowest frequency resonance for that string occurs when the wavelength of vibration equals twice the string's length. Because the speed of sound is equal to the product of the frequency times the wavelength, the resonance frequency equals the string's wave speed divided by twice the string's length. This resonance corresponds to the fundamental mode of vibration, it has a fundamental frequency, and it is the first partial. For simple strings with the same end conditions, additional resonances occur at frequencies corresponding to all integer multiples of the fundamental frequency:

$$f_1 = \frac{v}{2L}, \text{ All Harmonic Multiples .} \tag{1}$$

When the string's end conditions are different, the lowest frequency resonance for that string occurs when the wavelength is four times the length of the string. Thus its fundamental frequency is the string's wave speed divided by four times the string's length. This string cannot support modes of vibration at even multiples of the fundamental frequency and thus the additional higher frequency modes occur at odd integer multiples of the fundamental frequency:

$$f_1 = \frac{v}{4L}, \text{ Odd Harmonic Multiples} \tag{2}$$

The third mode shape of the fixed-fixed string and the fixed-free string are displayed in Figure 1.


***Figure 1. Third mode shape of a fixed-fixed string (left) and a fixed-free string (right).***

String vibrations can be constantly driven or the vibration can be induced and the string is allowed to freely vibrate. When a string is driven into vibration with a repeating driving motion (a periodic driving force), it might initially resist the driving force, but eventually the string must vibrate at whatever rate of constant motion is driving it. Normally the rate at which the motion repeats is the fundamental frequency because it is the easiest frequency to drive the string at. How the motion deviates from a sine wave determines which partial frequencies are present and their relative amplitudes. A repeating driving motion means that the partial frequencies can only be integer multiples of the fundamental frequency. This also implies that as the string vibrates, it moves in the exact same fashion over the period of the fundamental frequency.

A plucked string is pulled at some location, giving the string an initial displacement but at the moment the string is released the string has zero velocity. On the other hand, a struck string is struck at some location, giving the string an initial velocity when normally the string is at rest, meaning it initially has zero displacement. In either of these situations, the string is then left to vibrate on its own. Since the string vibrates the easiest at it resonance mode frequencies, the string will normally have the highest amplitude at the fundamental frequency and, in general, lower amplitude at each successive partial frequency above the fundamental. The key difference when a real string is left to vibrate on its own is that the partial frequencies are almost never at integer multiples of the fundamental frequency, on the contrary, normally the partial frequencies are slightly higher frequency values than would be expected for integer frequency multiples of the fundamental. Free systems' vibration amplitude decays over time and the shape of the waveform changes from period to period of the fundamental frequency.

Bowing a string is one example of a string being driven by a constant repeating motion, even if only for a quarter note or so. If we assume a quarter note is say one second long and we are playing a middle C note then this means that the string completes it cycle of motion 261 times during that second (middle C has a frequency of 261 cycles per second or Hz, and this rate is independent of the duration of the tone). Thus even one second produces vibrations that can be assumed to be constantly in repeating motion. As a string is bowed, the bow pulls the string due to the friction between the bow and the string. Eventually the string's tension builds up and overcomes the friction causing the string to slip back. The string is then grabbed again by the friction in the bow and the cycle repeats itself 261 times per second to play middle C. Figure 2 displays the displacement as a function of time of the string at the bowing location. Between points A and C the string is slipping along the bow and between points C and the next dashed line after D the string is sticking to the bow. The shape of the displacement waveform is sawtooth like and a typical spectrum of a violin tone's partial frequencies is shown in Figure 3.

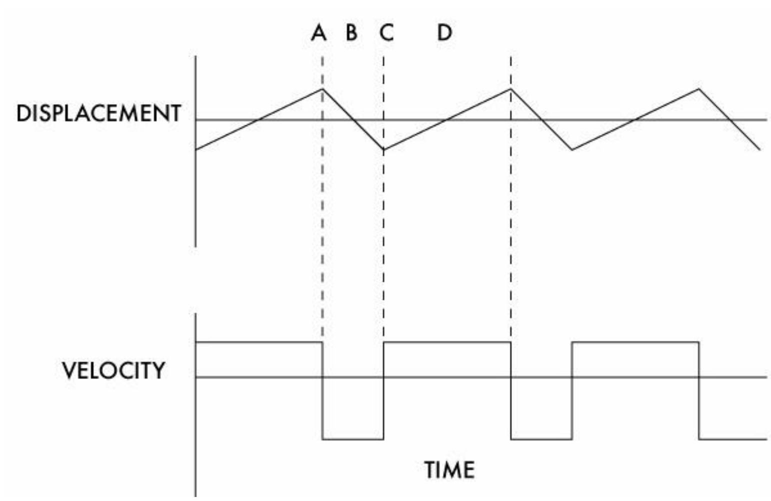


***Figure 2. Displacement and velocity of a bowed string's motion at the bowing location. This figure is Figure 38.3 from Ref. 1 and is used with permission.***

When a string is bowed at different locations, the modes are excited differently. When the bowing force is applied closer to the bridge of a violin, a larger amount of bowing force is necessary to excite the string. Bowing near the bridge produces a sharper bend in the string and therefore the higher partials are excited more strongly. Thus bowing near the bridge produces a brighter violin tone.

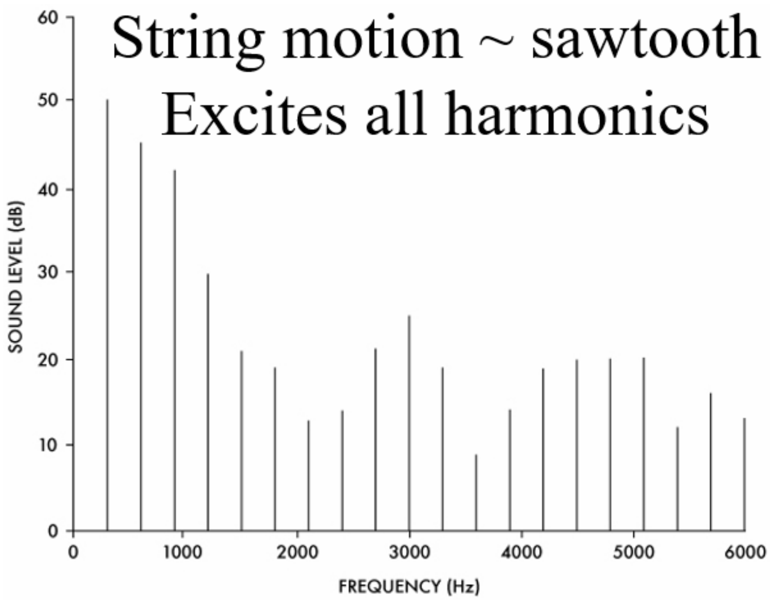


***Figure 3. Amplitudes of partials in a bowed string tone, indicating that all multiples of the fundamental frequency are present and the formant frequencies can been seen as the increases in groups of partials around 1000 Hz, 3000 Hz, and 4700 Hz. This figure is adapted from Figure 38.12 from Ref. 1 and used with permission.***

Ideal strings only have tension forces that oppose the motion of the string. Real strings though have some (normally small) amount of resistance in them that causes the free motion of the string to decay in amplitude over time. This resistance acts as a sort of restoring force that works to bring the string back to its rest position. Another difference between a real string and an ideal one is that real strings are not infinitely thin meaning that there is some amount of stiffness to them. One can hold a stick of wood or metal from one end of the stick and the other end of the stick won't droop. This is due to the stiffness inherent in the wood or metal stick. Strings do not normally have this high of a stiffness and usually fall limp to some degree when you try to hold them out horizontally like the stick. However, real strings don't fall perfectly limp and instead often have some level of inherent stiffness, particularly metal strings and those with additional windings of metal around the string. Stiffness is what causes a freely vibrating string's vibration pattern to change over time and is the reason why the partial frequencies are not integer multiples of the fundamental frequency. This also implies that the higher partials travel at slightly different speeds than the fundamental frequency and this causes the variations in the waveform from period to period of the fundamental.

Both the location of the string's excitation and the shape of the plucking or striking mechanism affect the spectrum from a string vibration. When a string is plucked or struck at a given location, if that location happens to be at a nodal position for one of the string's modes, then that mode cannot be efficiently excited. For a given instantaneous shape of a string, the strings' modes can be added up with different amplitudes to create that shape. The characteristics of the plucking/striking device, or plectrum/striker, impact the radiated sound. The sharper the plectrum is, the brighter the tone is. The reason for this is similar to the reason why bowing closer to the bridge on a violin causes its tone to be brighter. A sharp plectrum creates a sharper bend in the string and in order to create that string shape, many strong higher partial frequencies must be added together to produce that sharp string shape. Figure 4 shows an illustration of the string shape with a sharp and a smooth plectrum and the corresponding partial frequencies. Note that the 4th and 8th partials are missing since the plectrum plucked the string at 1/4th of the string length.

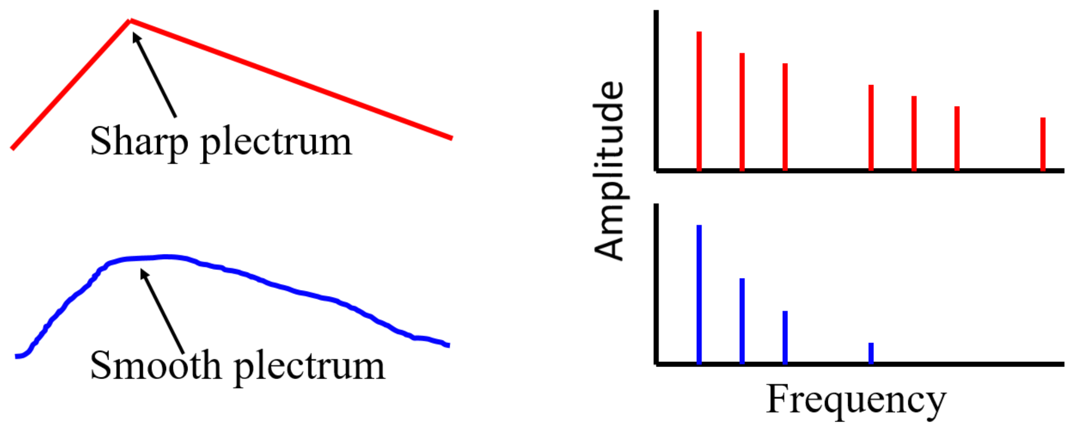


***Figure 4. An illustration of a string plucked with a sharp plectrum (upper left) and corresponding bright frequency spectrum (upper right), and of a string with a smooth plectrum (lower left) and corresponding frequency spectrum (lower right).***

In addition to the spectral characteristics of strings, the time-dependent characteristics differ depending on the method of excitation. The attack portion of a waveform is the initial excitation of the instrument as it begins to vibrate. The sustain portion of a waveform is when the instrument is being driven consistently. Finally the decay portion of a waveform occurs when the plucking, striking, or bowing of the instrument is completed. Plucked and struck string waveforms typically have a short attack as the string is pulled or hit and a long decay as the string is free to vibrate, with no sustain portion. Bowed string waveforms typically have a short attack as the bow starts to excite the string, a sustain portion (whose length depends on the duration of the bowing), and a short decay portion when the bowing ends and the string is left to vibrate.

String instruments cannot radiate sound efficiently from the vibration of their strings alone. Rather the string vibration is usually coupled to a sound board that is better able to radiate sound due to the large increase in vibrating area of the sound board. The sound board normally has its own structural resonances. The result is that the string vibrations are filtered by the radiation capabilities of the sound board. At resonances of the sound board the sound board radiates especially well, whereas at other frequencies away from sound board resonances the sound board does not radiate as well. These resonances behave similar to formant frequencies seen in the analysis of vocal production: the vocal tract has formant frequencies that filter the radiated sound from the vibration of the vocal folds. Figure 5 provides a simplified illustration of how the partial frequency spectrum of a bowed violin may be modified by the resonances of the violin's body and air cavity.

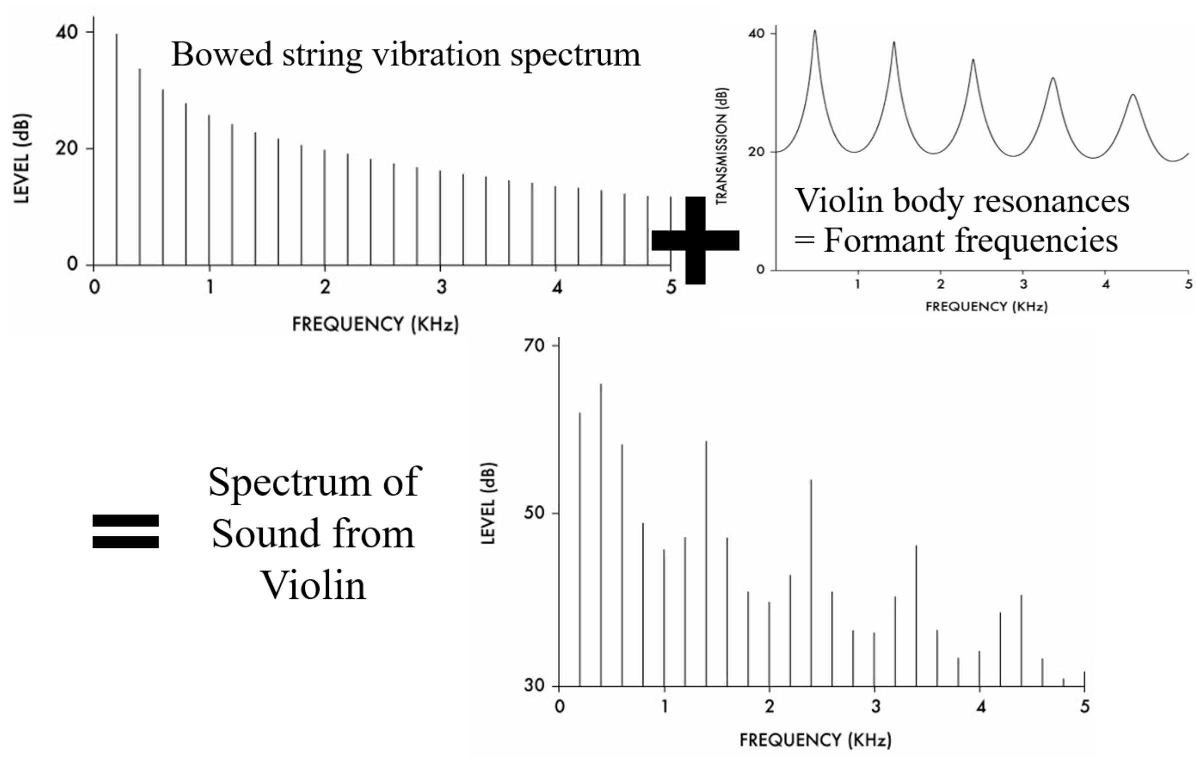


***Figure 5. An illustration of a bowed string's partial frequencies filtered by the violin's body resonances. Adapted from Figures 26.3, 26.6, and 26.7 from Ref. 1 and used with permission.***

## 3. LABORATORY EXERCISE ON STRING INSTRUMENTS

Each week during the course, an extra credit lab is offered. With around 100 total students we typically get around 30-40 of them regularly participating. We offer 10 time slots each week for lab attendance, led by a student teaching assistant (TA). The next three pages display the lab exercise as it currently stands. This lab is intended to be completed in an hour or less as students work together to complete the exercise with the assistance from the TA as needed.

# Physics 167 Extra Credit Lab 12
# Natural Modes of Strings

Name: ________________________________________ Date: ____________________

Objective: To understand how the length, tension and density of the string change the fundamental frequency of the string and the wave speed.

Equipment: ukulele, violin, tape measure, calculator, laptop with signal analyzer and microphone

**Part 1:** Length of the string

- Select a single string on either the ukulele or the violin
- Measure the length of the string. Record it in the table.
- Pluck the string and use the spectrum analyzer to find the fundamental frequency.
- Record it in the table.
- Press the string against the fingerboard/neck of the instrument such that the vibrating portion of the string is half the open-string length. Record the length and the fundamental frequency.
- Press the string against the fingerboard/neck of the instrument such that the vibrating portion of the string is two-thirds the open-string length. Record the length and the fundamental frequency.
- Compute the ratio of the two higher frequencies to the lowest frequency.
- Write down the musical interval that most closely corresponds to the frequency ratio.

| | Length - cm | Fundamental Frequency - Hz | Frequency Ratio: $f_{high}/f_{open}$ | Musical Interval |
|---|---|---|---|---|
| Open string, *L* | | | x | x |
| ½ *L* | | | | |
| $^2/_3$ *L* | | | | |

**Part 2:** Ukulele

The G and A strings on this cheap ukulele have approximately the same density and length. The reason each string has a different fundamental frequency is primarily due to the tension in the string. The notes on the ukulele (from the top to the bottom) are G4, C4, E4 and A4.

- List the musical interval between each string and the one below it.
- Measure the length of the strings. Record them in the table.
- Pluck the string and use the spectrum analyzer to find the fundamental frequency of each string. Record them in the table.
- Calculate the wave speed for each string: $v = f\lambda$.
- The speed of the wave on the string is $v = \sqrt{\frac{\text{tension}}{\text{density}}} = \sqrt{\frac{T}{\rho}}$. Assuming the density of the G and A strings are approximately equal, the ratio of the tensions in neighboring strings is equal to the square of the ratio of the wave speeds.
- Calculate the ratio of the tensions in G and A strings.

| | Musical Interval | Length - cm | Fundamental Frequency - Hz | Wave speed: $v = f\lambda$ | Ratio of Tensions: $\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)^2$ |
|---|---|---|---|---|---|
| G4 | G4:C4 | | | | x |
| C4 | E4:C4 | | | | x |
| E4 | E4:A4 | | | | x |
| A4 | G4:A4 | | | | G4:A4 |

**Part 3:** Violin

When in tune, each string on a violin should have approximately the same tension so the bow can grab each one with equal force. The reason each string has a different fundamental frequency is primarily due to the density in the string. The notes on the violin (from lowest to highest) are G3, D4, A4 and E5.

- List the musical interval between each string and the one next to it.

- Measure the length of the strings. Record them in the table.
- Pluck the string and use the spectrum analyzer to find the fundamental frequency of each string. Record them in the table.
- Calculate the wave speed for each string: $v = f\lambda$.
- Assuming the tensions in the strings are the same, the ratio of the densities of neighboring strings is inversely related to the ratio of the wave speeds.
- Calculate the ratio of the densities in neighboring strings.

| | Musical Interval | Length - cm | Fundamental Frequency - Hz | Wave speed: $v = f\lambda$ | Ratio of Densities: $\frac{\rho_1}{\rho_2} = \left(\frac{v_2}{v_1}\right)^2$ |
|---|---|---|---|---|---|
| G3 | G3:D4 | | | | G3:D4 |
| D4 | D4:A4 | | | | D4:A4 |
| A4 | A4:E5 | | | | A4:E5 |
| E5 | X | | | | X |

**Questions**:

1. What factors determine the speed with which a pulse travels along a plucked or bowed string?
2. How do the fundamental frequencies of strings with different length compare?
3. What happens to the fundamental frequency of the string when the tension is increased?
4. How does the fundamental frequency of string depend on its density?

A revision of the lab exercise for parts 2 and 3 might have the students measure the string lengths and fundamental frequencies and then compute the wave speed in the string. Students could then compute the expected ratio of the musical intervals for adjacent strings (based on the usual notes of the strings), the measured ratio of the fundamental frequencies, the ratios of the wave speeds, and then either the ratios of the tensions or the ratio of the densities depending on whether the Ukulele or the Violin is being used, respectively. This would allow the students to decide whether equal tempered tuning or just tempered tuning is appropriate for the instrument and then whether the wave speed ratio is the same as the musical interval ratio or whether the ratio of the tensions or densities are the same as the musical interval ratio.

## 4. DEMONSTRATIONS

The paper by Neilsen *et al.* identifies several demonstrations that have been created for this course.[2] Additional demonstrations may be found on YouTube. In particular, there are several high speed videos of the stick slip interaction in bowed string instruments such as those found in Refs. 3-6.

One particular visual demonstration discussed in the paper by Neilsen *et al.* is the driven string demonstration developed by Wayne Peterson at BYU.[2] This demonstration uses a variable speed jigsaw which drives a small women's belt. The belt provides just the right impedance matching to the saw. The other end of the belt is held still while the jigsaw excites the string into vibration. Figure 6 displays the first 5 modes of the string (belt) as the operation frequency of the jigsaw is increased. While driving the string at the frequency corresponding to the 2$^{nd}$ mode, the tension in the string is decreased until the 3$^{rd}$ mode shape appears. A decrease in string tension causes a decrease in the string's wave speed. Since the frequency is held constant, the wavelength must be decreasing with decreasing wave speed, hence why the 3$^{rd}$ mode appears when the tension is decreased from the 2$^{nd}$ mode frequency.

A demonstration that helps the students understand the relative inharmonicity of the spectra for driven, plucked and struck strings will now be presented. A single string on an upright piano (A0 with a fundamental frequency of 27 Hz), is excited using a shaker, plucking, and striking excitations. The struck string is excited by playing the piano string as usual with the piano hammer striking the string. The string can be plucked by removing a panel and displacing the piano string with one's finger before releasing it. Finally, the string can be driven by a shaker whose stinger has been temporarily glued onto the string. The shaker is then driven with a sawtooth waveform (to simulate a bowing excitation) with a fundamental frequency of 27.0 Hz. Figures 7-9 display the frequency spectra resulting from these different excitations of the piano string. Using a fundamental frequency of 27.0 Hz, dashed lines at harmonic partial frequencies have been added to the figures. Not only are the relative amplitudes of the partials obviously different for each of the excitation conditions, but one can also note the increasing deviation from harmonic frequencies (solid line peaks shifting to the right, with each successive partial frequency, relative to the harmonic-frequency dashed lines) for the two free vibration excitations (struck and plucked). Table 1 tabulates the first eight partial frequencies for these frequency spectra. Note that the driven string fundamental is slightly lower than the fundamental for the plucked and struck string; the cause of this is unknown. The seventh and eighth partials of both the plucked and struck strings end up being located 5 Hz higher in frequency than the eighth partial of the driven string.

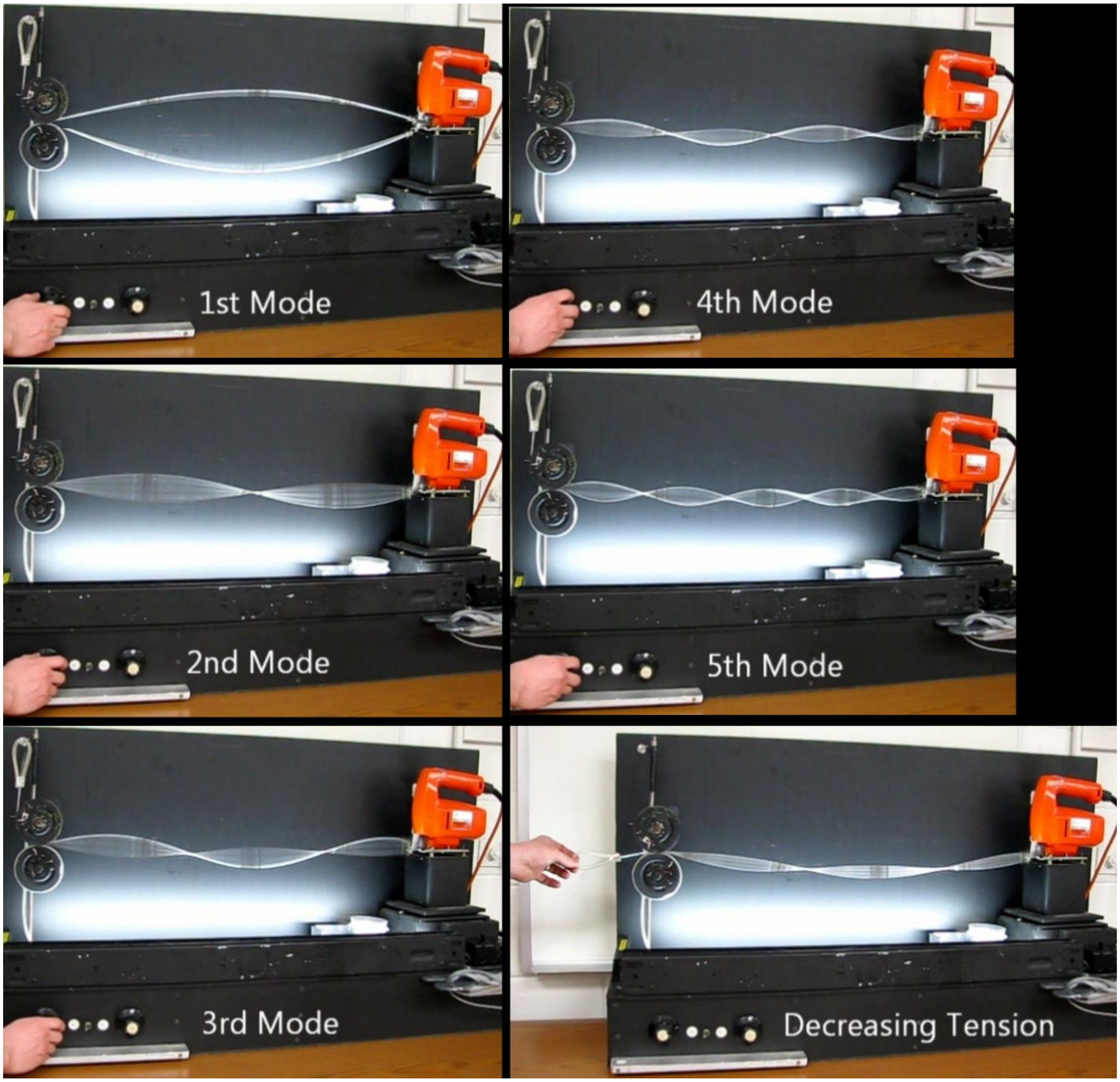


***Figure 6. Photographs of the jigsaw driven string demonstration. Modes 1-5 are displayed and identified with the white colored text. While the jigsaw is driving at the frequency corresponding to the 2nd mode, the tension is decreased and the 3rd mode shape appears (lower right photo). Note that the mode shape for the 1st mode has been edited to show the string's position at two different portions of the cycle, whereas the other images are actual still frame images extracted from a video of the demonstration.***

In order to quantify how inharmonic the higher partials are for the free vibration conditions relative to the driven condition, one can compute the difference between the measured partial frequencies and the expected harmonic partials. Figure 10 displays the first 35 partial frequency values extracted from spectra that include higher frequency content than the spectra shown in Figures 7-9. Note how the driven string partials increase exactly linearly with partial number but that the struck and plucked string partials are increasingly sharp (higher in frequency) with increasing partial number. This A0 string on this piano was determined to have an inharmonicity coefficient of 0.000453 (see Reference 7 for more information). The thickness of a string causes the string's restoring force to not only depend on tension, but also on the string's stiffness, like a bar's restoring force depends on its internal stiffness. This internal stiffness of the piano string, more apparent when the string has copper windings, causes inharmonicity in the tonal content. Inharmonicity in a string causes higher partials to be more and more sharp with increasing partial number. Figure 11 displays the subtraction result when the partial frequency value for the driven, struck, and plucked strings' partial frequencies are subtracted from harmonic partial frequencies of a 27 Hz fundamental. Because the driven string has all harmonic partials, its result in Figure 11 is all zeros. The struck and plucked strings' departure from harmonic partials is even more apparent in the subtraction results of Figure 11. This hammers home the idea that free systems inherently have inharmonic partials relative to their fundamentals, whereas driven systems inherently have harmonic partials relative to their fundamentals. Further, this demonstrates that it is not simply true that pianos always have inharmonic partials, nor do violins always have harmonic partials. What matters is the method of excitation. As an extension of this demonstration, we can zoom in on four cycles of the waveforms for the driven, struck, and plucked string, as shown in Figures 12-14. The driven string waveform has a large spike followed by several small spikes in each repeating period of the fundamental whereas the struck and plucked string waveforms change more visibly for each period of the fundamental. The fundamental's period is shown with the dashed lines. This can illustrate the constancy during a sustain portion of a waveform versus the changing nature of free vibration of a string. The driven string isn't perfectly repeating due to the presence of a low frequency and some other noise recorded. The accuracy of the recorded partial frequency values is within 1 Hz. The inharmonicity effect is much more apparent than the measurement uncertainty.

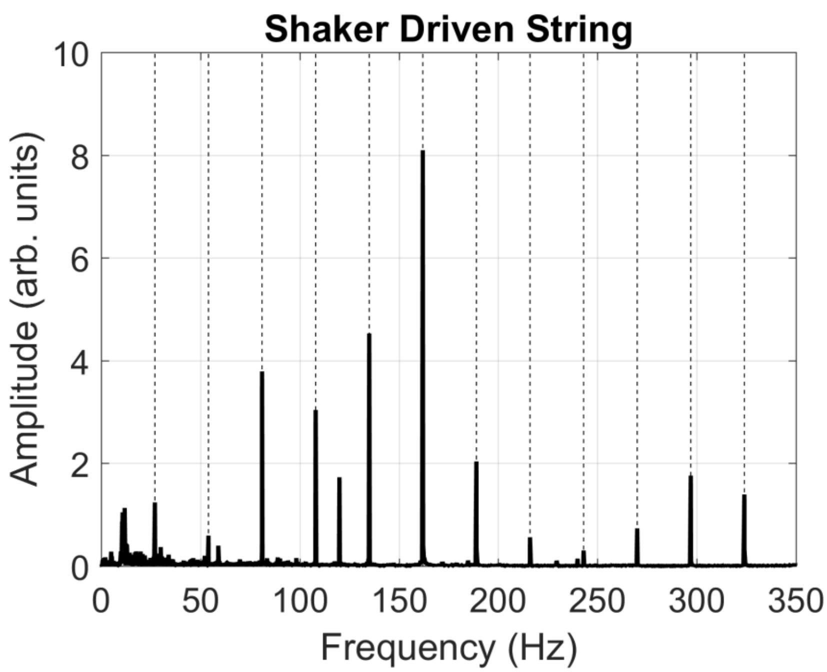


***Figure 7. Frequency spectrum of the sound recorded from an A0 piano string that has been driven by a shaker.***

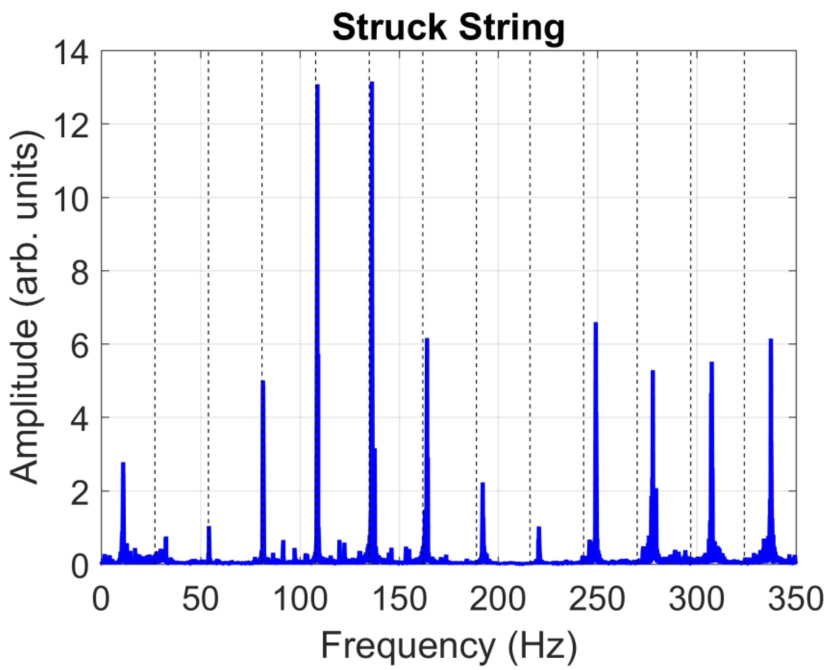


***Figure 9. Frequency spectrum of the sound recorded from an A0 piano string that has been struck with a hammer.***

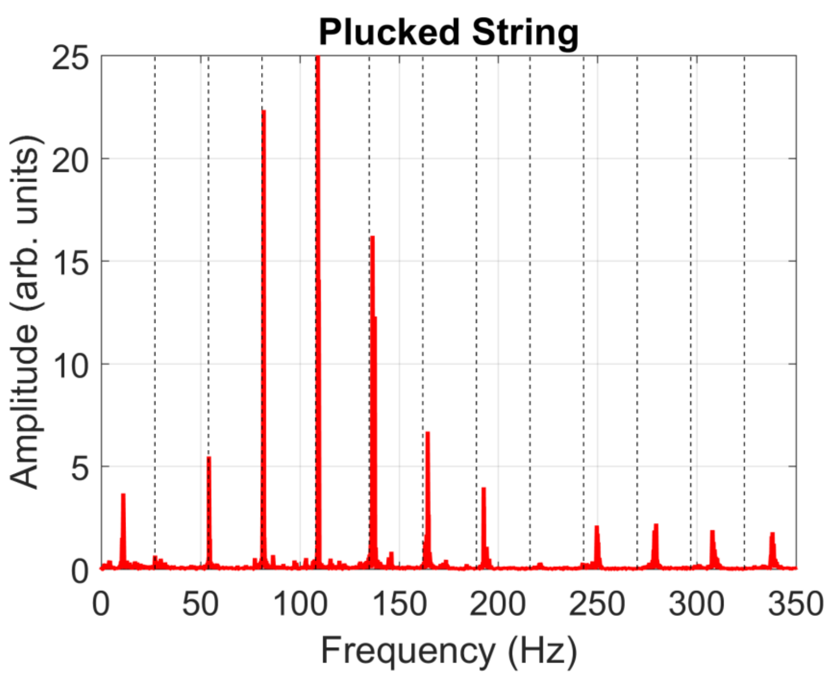


***Figure 8. Frequency spectrum of the sound recorded from an A0 piano string that has been plucked with a finger.***

Table 1. First eight partial frequencies (in Hz) of the sound radiated by an A0 piano string that has been driven, plucked, and struck. From Figures 7-9.

| Partial # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Harmonics | 27 | 54 | 81 | 108 | 135 | 162 | 189 | 216 |
| Driven | 27 | 54 | 81 | 108 | 135 | 162 | 189 | 216 |
| Struck | 32 | 54 | 82 | 109 | 137 | 164 | 193 | 221 |
| Plucked | 30 | 55 | 82 | 109 | 137 | 165 | 193 | 221 |

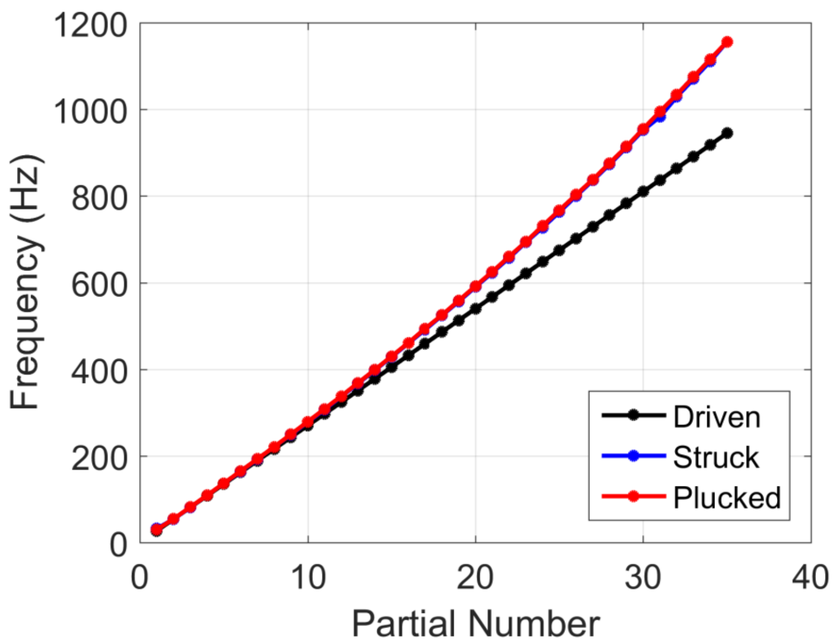


***Figure 10. Partial frequency values as a function of partial number for the first 35 partials of the driven, struck, and plucked string spectra. The struck and plucked string values are nearly identical.***

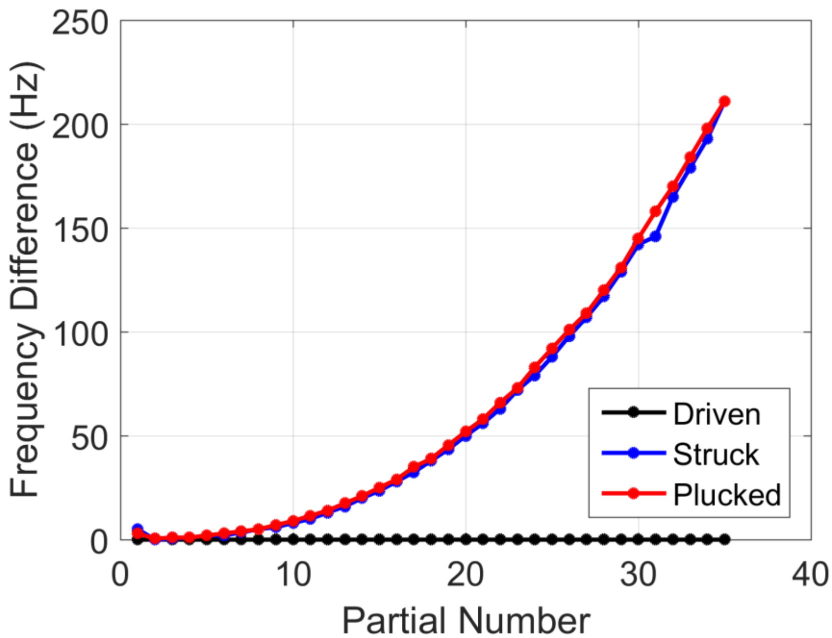


***Figure 11. Difference between extracted partial frequency values (those displayed in Figure 10) and harmonic partial frequencies based on a 27 Hz fundamental.***

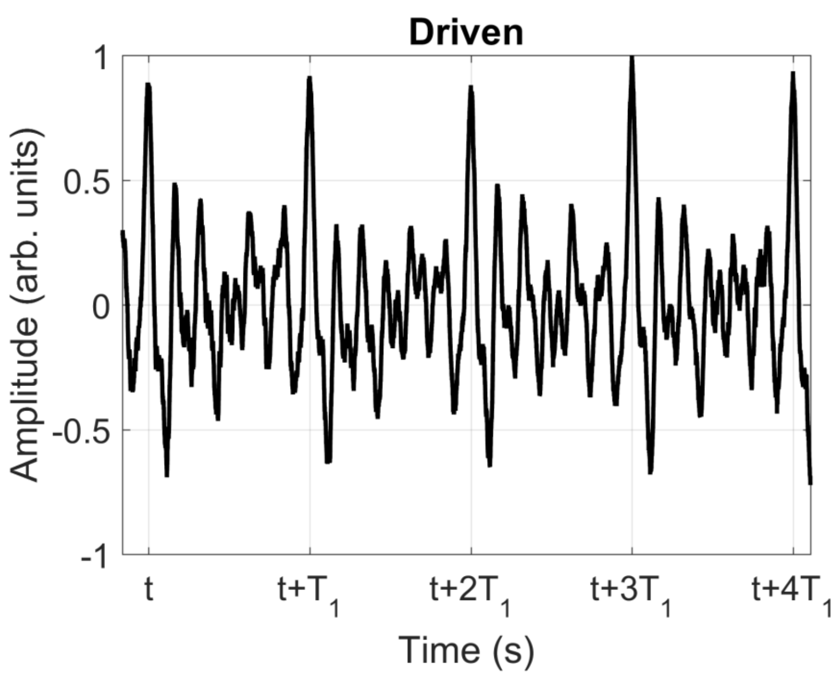


***Figure 12. Just over four cycles (cycles of 27 Hz) for the shaker driven string. Four periods, $T_1$, of the fundamental frequency, are shown, indicated by the dashed lines.***

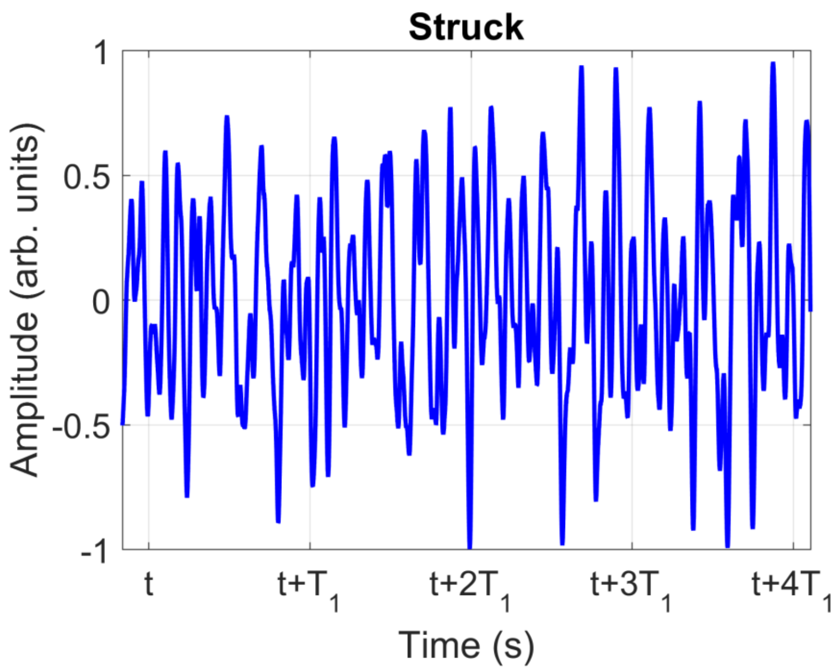


***Figure 13. Just over four cycles (cycles of 27 Hz) for the struck string. Four periods, $T_1$, of the fundamental frequency, are shown, indicated by the dashed lines.***

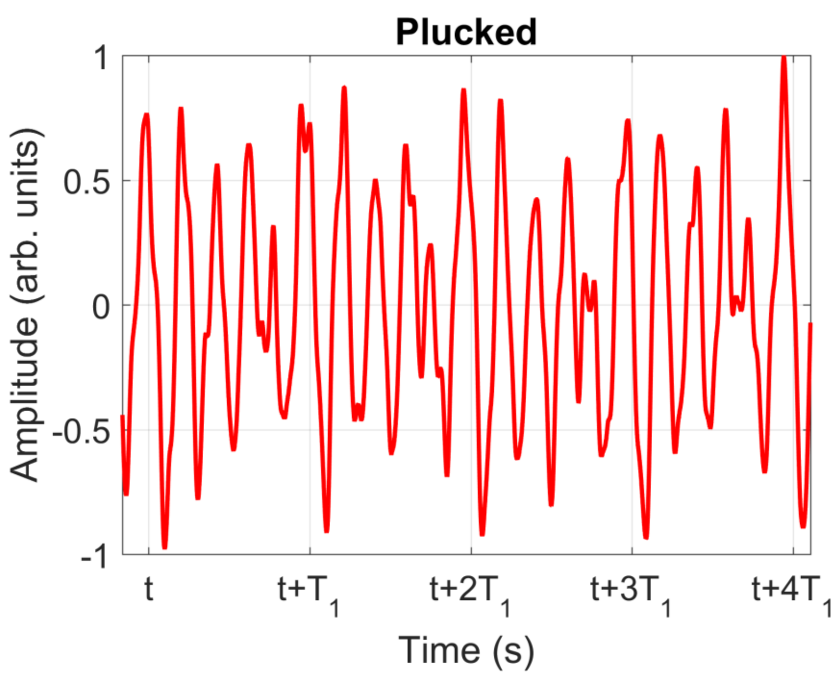


***Figure 14. Just over four cycles (cycles of 27 Hz) for the plucked string. Four periods, $T_1$, of the fundamental frequency, are shown, indicated by the dashed lines.***

# 5. CONCLUSION

This paper has described some of the key aspects of string instruments that students are exposed to in the general education, "Descriptive Acoustics" physics class offered at BYU. It has described key concepts that students are expected to understand for the acoustics of the bowed, plucked, and struck string instrument families. A laboratory exercise has been presented that provides students a hands-on opportunity to experience basic concepts in wave speed and resonances in strings. Finally, demonstrations have been discussed which the authors have found useful in teaching the students. In particular, the ability to demonstrate the tie between inharmonicity and string excitation method helps students to understand the spectral content of string instruments.

# ACKNOWLEDGMENTS

We gratefully acknowledge the work of William Strong, Scott Sommerfeldt, and Tim Leishman who have contributed greatly to this course over the years. We further acknowledge the work of Ben Christensen who helped design the lab contained herein and to Caleb Goates and Mark Berardi who have worked as key teaching assistants for this course and have provided edits to the lab assignments.

# REFERENCES

[1] W. J. Strong and G. R. Plitnik, *Music Speech Audio, 4th Edition*, (BYU Academic Publishing, 2013).

[2] T. B. Neilsen, W. J. Strong, K. L. Gee, B. E. Anderson, S. D. Sommerfeldt, and T. W. Leishman, "Creating an active-learning environment in an introductory acoustics course," J. Acoust. Soc. Am. **131**(3), 2500-2509 (2012).

[3] www.youtube.com/watch?v=FO5bq6x_Tws, last viewed on May 31, 2016.

[4] www.youtube.com/watch?v=KPpBvHXYWz4, last viewed on May 31, 2016.

[5] www.youtube.com/watch?v=6JeyiM0YNo4, last viewed on May 31, 2016.

[6] www.youtube.com/watch?v=ETVg7Gk6ayE, last viewed on May 31, 2016.

[7] B. E. Anderson and W. J. Strong, "The effect of inharmonic partials on pitch of piano tones," J. Acoust. Soc. Am. **117**(5), 3268-3272 (2005).