.

Experimental validation of acoustic intensity bandwidth extension by phase unwrapping

Kent L. Gee, Tracianne B. Neilsen, and Scott D. SommerfeldtMasahito Akamine and Koji Okamoto

Citation: The Journal of the Acoustical Society of America **141**, EL357 (2017); doi: 10.1121/1.4979604

View online: http://dx.doi.org/10.1121/1.4979604

View Table of Contents: http://asa.scitation.org/toc/jas/141/4

Published by the Acoustical Society of America

Articles you may be interested in

A point-based scattering model for the incoherent component of the scattered field
The Journal of the Acoustical Society of America **141**, (2017); 10.1121/1.4976584

Streaming and sound localization with a preceding distractor
The Journal of the Acoustical Society of America **141**, (2017); 10.1121/1.4979167

Estimating the spectral tilt of the glottal source from telephone speech using a deep neural network
The Journal of the Acoustical Society of America **141**, (2017); 10.1121/1.4979162

Robust source localization from wavefield separation including prior information
The Journal of the Acoustical Society of America **141**, (2017); 10.1121/1.4979258

Fingerprint imaging of dry finger using photoacoustics
The Journal of the Acoustical Society of America **141**, (2017); 10.1121/1.4976193

A finite element study on the cause of vocal fold vertical stiffness variation
The Journal of the Acoustical Society of America **141**, (2017); 10.1121/1.4978363

Experimental validation of acoustic intensity bandwidth extension by phase unwrapping

Kent L. Gee,[a] Tracianne B. Neilsen, and Scott D. Sommerfeldt
Department of Physics and Astronomy, Brigham Young University, Provo, Utah, 84602, USA
kentgee@byu.edu, tbn@byu.edu, scott_sommerfeldt@byu.edu

Masahito Akamine and Koji Okamoto
Graduate School of Frontier Sciences, University of Tokyo, Kashiwa, Chiba, 277-8561, Japan
akamine@thermo.t.u-tokyo.ac.jp, k-okamoto@k.u-tokyo.ac.jp

Abstract: The phase and amplitude gradient estimator (PAGE) method for active acoustic intensity uses pairwise microphone transfer functions to obtain the phase gradient, which improves the calculation bandwidth over the traditional weighted quadspectral method. Additionally, for broadband sources, the PAGE theory indicates that the transfer function phase can be unwrapped to further extend the usable frequency range to beyond the spatial Nyquist frequency. Here, two experiments demonstrate intensity bandwidth extension by more than an order of magnitude using phase unwrapping. First, plane-wave tube results show accurate one-dimensional intensity calculations with the microphones separated by five wavelengths, 30 times the traditional limit. Second, two-dimensional measurements of a laboratory-scale jet with a four-microphone probe yield physically reasonable calculations at frequencies 15 times the traditional limit.

© 2017 Acoustical Society of America

[PEB]
Date Received: November 15, 2016 **Date Accepted:** March 21, 2017

1. Introduction

Measurement of active acoustic intensity has numerous applications, including obtaining radiated sound power,[1] noise source identification,[2] sound field reconstruction,[3,4] and determining building sound insulation.[5] A limitation of using multimicrophone intensity probes[6] to characterize broadband sources and fields has been a requirement of repeating the measurement with a number of microphone spacings to overcome the significant bias errors[7] associated with the traditional quadspectrum-based intensity method.[8,9] This process increases measurement time and cost. Recently, the challenge of characterizing jet noise fields[10,11] motivated formulation of a new intensity calculation method that extends multimicrophone probes' bandwidth. The phase and amplitude gradient estimator (PAGE) method,[12] which calculates a phase gradient via pairwise microphone transfer functions, greatly reduces bias errors[13] up to the probe's spatial Nyquist frequency, f_{Nyq}. Furthermore, the PAGE method allows the possibility of unwrapping the transfer function phase for broadband signals before finding the phase gradient, thereby extending the intensity calculation bandwidth beyond f_{Nyq}. Initial applications of phase unwrapping to PAGE intensity calculations of solid rocket motor[14] and military jet[15] noise sources increased the upper frequency limit of the vector calculations by ∼3–4 times that of the traditional method.

Although these initial applications demonstrated the promise of applying phase unwrapping with the PAGE method, these measurements were made on large, complex sources in the field. Experiments that investigate phase unwrapping and PAGE performance in more controlled settings are required. This letter further investigates extension of intensity calculation bandwidth by phase unwrapping for broadband, radiated fields: first, an anechoic plane-wave tube experiment that represents a near-ideal environment with a known benchmark, and second, a laboratory-scale supersonic jet noise experiment. The results show that, for broadband sources, transfer function phase unwrapping with PAGE increases the upper frequency limit of the intensity probe by more than an order of magnitude.

---

[a]Author to whom correspondence should be addressed.

2. PAGE method summary

The PAGE method's improvements stem from a least-squares gradient of the pressure phase across microphones to calculate the active intensity. The method builds from the work of Mann and colleagues,[16,17] who investigated radiated intensity and other energy-based quantities. They expressed complex pressure at position, $\boldsymbol{r}$, and frequency, ω, as a magnitude and phase, $p(\boldsymbol{r}) = P(\boldsymbol{r})e^{-j\phi(r)}$, from which the particle velocity vector is obtained via Euler's equation as

$$\boldsymbol{u}(\boldsymbol{r}) = \frac{j}{\rho_0 \omega}\nabla p = \frac{1}{\rho_0 \omega}[P(\boldsymbol{r})\nabla\phi(\boldsymbol{r}) + j\nabla P(\boldsymbol{r})]e^{-j\phi(r)}. \quad (1)$$

The time-average active intensity is written as

$$\boldsymbol{I} = \frac{1}{2}\mathrm{Re}\{p\boldsymbol{u}^*\} = \frac{1}{2\rho_0\omega}P^2\nabla\phi = \frac{1}{\rho_0\omega}\overline{P^2}\nabla\phi, \quad (2)$$

where * signifies complex conjugate, ρ_0 is ambient density, and $\overline{P^2}$ is mean-square pressure. Although Mann *et al.*[16] considered Eq. (2) theoretically, Mann and Tichy[17] stepped from this expression directly to the traditional quadspectral approach when describing experiments.

In the PAGE method,[12] $\overline{P^2}$ can be found by locating a microphone at the probe center or, alternatively, by finding a least-squares estimate of the pressure magnitude. The least-squares phase gradient, $\nabla\phi$, is calculated from N microphones located at positions, $\boldsymbol{r}_{1..N}$, as

$$\nabla\phi \approx (\mathbf{R}^T\mathbf{R})^{-1}\mathbf{R}^T\boldsymbol{\Delta\phi}, \quad (3)$$

where $\mathbf{R}$ is a position difference matrix written as

$$\mathbf{R} = [\boldsymbol{r}_2 - \boldsymbol{r}_1|\ldots|\boldsymbol{r}_N - \boldsymbol{r}_{N-1}]^T, \quad (4)$$

and the phase differences, $\boldsymbol{\Delta\phi}$, are found via microphone pairwise transfer functions

$$\boldsymbol{\Delta\phi} = -\left[\arg\{H_{1,2}\}, \ldots, \arg\{H_{N-1,N}\}\right]^T. \quad (5)$$

Whereas the traditional method is limited to well below f_{Nyq}, the use of the transfer function phase in PAGE generally reduces active intensity bias errors up to f_{Nyq}.[13] Furthermore, it allows for the direct possibility of phase unwrapping for broadband signals for which $\nabla\phi$ varies smoothly as a function of frequency,[12] thereby extending the upper frequency limit of the PAGE intensity calculation. The smooth phase variation is necessary because the phase unwrapping process required to obtain an accurate $\nabla\phi$ at a given frequency depends on the phase variation at lower frequencies. Phase discontinuities that occur in $\boldsymbol{\Delta\phi}$ because of poor signal coherence or strong, tonal signals arriving from widely different directions could cause unwrapping errors, thus affecting $\nabla\phi$ and the PAGE-calculated intensity in Eq. (2) at all higher frequencies.

Although phase unwrapping needs to be further investigated for additional environments, the plane-wave tube and laboratory jet noise experiments described in this letter demonstrate the success of phase unwrapping for two important broadband source cases: (a) a propagating wavefront that may be described as locally planar across the probe, and (b) an extended radiator whose properties vary smoothly with frequency.

3. Plane-wave propagation experiment

The first validation of PAGE-method bandwidth extension by phase unwrapping is of broadband plane-wave propagation from a loudspeaker-driven, anechoically terminated acrylic tube. The tubes 10 cm inner diameter restricts plane-wave propagation to below ~2 kHz. For this experiment, pressure waveform data were synchronously recorded at four phase-matched G.R.A.S. 40AE 12.7 mm microphones with a 24-bit National Instruments PXI-4462 card. The microphone diaphragms were flush with the inside of the tube and were placed with separation distances (relative to the first microphone) of 5, 30, and 90 cm [see Fig. 1(a)]. For reference, the maximum microphone separation distance used in most commercial intensity probes is only 10 cm, with an upper frequency limit (-2 dB error) of 1.2 kHz.

In this experiment, the traditional intensity calculations, obtained from scaled pairwise quadspectra, can be compared to the benchmark propagating plane-wave case, for which the sound intensity and pressure levels are approximately equal, i.e., $L_I \approx L_P$. The L_I^{TRAD} for the three spacings and L_P are shown in Fig. 1(b). The error

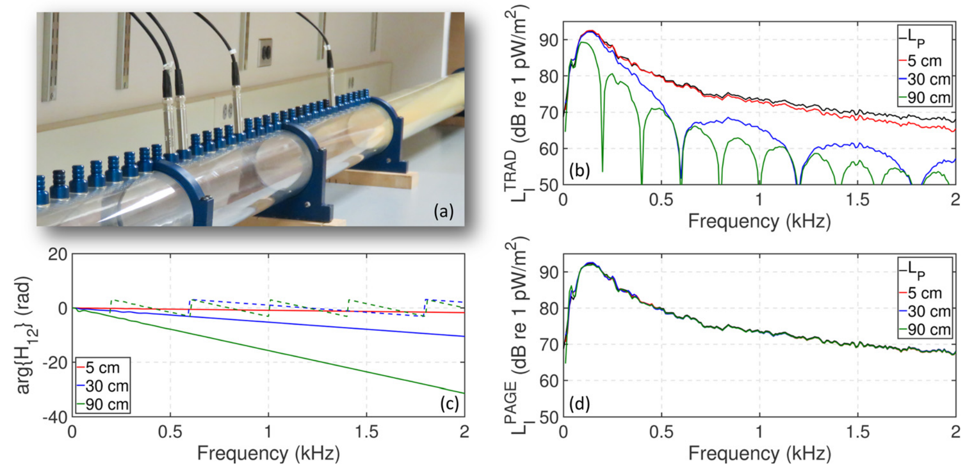

Fig. 1. (Color online) (a) Anechoic plane-wave tube experiment with downstream microphones placed at 5, 30, and 90 cm from the first microphone. (b) Traditional-method intensity levels, L_I^{TRAD} and L_P. (c) Wrapped (dashed) and unwrapped transfer function phases. (d) PAGE-calculated levels, L_I^{PAGE}, along with L_P.

for $d = 5$ cm is less than 1 dB below 1.3 kHz, but $d = 30$ and 90 cm—uncharacteristically large for intensity measurements—cause limited intensity bandwidth and nulls (with 180° changes in intensity direction) at integer multiples of f_{Nyq}.

The plane-wave tube experiment demonstrates intensity bandwidth extension by phase unwrapping. For a propagating plane-wave, $\Delta\phi = kd$ and $\nabla\phi = k$, such that Eq. (2) is written[12] as $I^{\mathrm{PAGE}} = \overline{P^2}/\rho_0 c$, the expected analytical result. In other words, if the phase gradient is accurate, the PAGE method reproduces the analytical intensity for the propagating plane-wave. The wrapped (dashed) and unwrapped transfer function phases for the different microphone spacings are shown in Fig. 1(c). The ability to unwrap $\arg\{H\} = \Delta\phi$ multiple times allows the PAGE intensity bandwidth to be extended past f_{Nyq} up to the tube's first cross-mode cutoff frequency. The convergence of L_I^{PAGE} for all three spacings to L_P is shown in Fig. 1(d), demonstrating the ability of the PAGE method to accurately obtain acoustic intensity for a broadband, propagating plane-wave, regardless of microphone spacing.

4. Supersonic jet experiment

The plane-wave tube experiment shows that two microphones may be separated by at least five wavelengths (an increase in bandwidth of a factor of ~30 over the traditional method[18]) and still obtain accurate PAGE intensity estimates for a broadband propagating wave. A comparison of two-dimensional (2D) vector intensity with smaller microphone spacing and a more complex broadband source, an unheated jet, reveals similar bandwidth improvement while extending the intensity calculations well into the ultrasonic range. The jet is an extended, frequency-dependent source with waves radiated in different directions, but with a principal radiation angle that is also frequency-dependent. The ideally expanded, 20-mm nozzle-diameter (D_j), Mach 1.8 jet is located at the Kashiwa Campus of the University of Tokyo. Although the facility is not anechoic, nearby reflecting surfaces were wrapped in fiberglass, resulting in good agreement[19] with anechoic far-field sound level measurements.[20] Calibrated acoustic pressure waveform data were synchronously acquired at five 2D intensity probes and a 40 D_j-radius arc with National Instruments PXI-4498 cards sampling at 204.8 kHz. The intensity probes[14] were constructed of 6.35 mm G.R.A.S. 46BG microphones without grid caps located at the center and vertices of an equilateral triangle with circumradius 25.4 mm (1.27 D_j). A probe and four intensity measurement locations (I–IV) are displayed with respect to the jet shear layer in Fig. 2(a). These near-field locations were selected because the 30° and 35° angles formed by II/IV and I/III, respectively, match the overall sound pressure level (OASPL) peak directivity range in Fig. 2(b). In addition, 16 G.R.A.S. 40BE microphones were located on a 40 D_j arc (centered at $x = 10\,D_j$) to obtain the estimated directivity of the dominant noise region.

The behavior of the pairwise $\arg\{H\}$ needed for $\nabla\phi$ establishes the feasibility of using PAGE with phase unwrapping for this experiment. For example, Fig. 3(a) shows the wrapped and unwrapped phase difference between the vertex microphone pairs used to obtain $\nabla\phi$ at location I. The B:D and B:C pairs reveal near linear behavior with multiple unwrappings, while $\arg\{H_{\mathrm{CD}}\}$ suggests the noise travels nearly

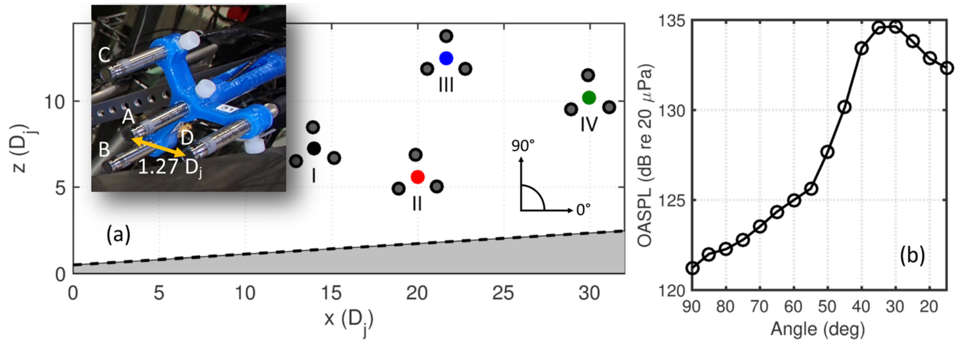

Fig. 2. (Color online) (a) Four-microphone, 2D intensity probe and probe locations. The center microphone is labeled A, and the outer microphones B–D. The probe orientation is such that B and D are closest to the shear layer. (b) OASPL (dB re 20 μPa) at 40 D_j, with angle referenced to a jet centerline point 10 D_j downstream of the nozzle exit.

perpendicular to the line between C:D. Figure 3(b) also shows smooth variation in unwrapped pairwise arg$\{H\}$, this case for Location IV. The differences in arg$\{H\}$ result in different propagation angles for the two locations as a function of frequency. Figure 3(c) displays the traditional and PAGE intensity levels, along with L_P for locations I–IV in Fig. 2(a). As expected for a radiated field from a broadband source, the PAGE method follows L_P to 40 kHz for all four locations, whereas the traditional method exhibits >1 dB errors above 2.5–3 kHz, in agreement with a preliminary loudspeaker experiment.[21] Because the dominant acoustic radiation occurs at $\geq$3 kHz, the PAGE method is essential in obtaining accurate acoustic intensity measurements. The slight separation of L_P and L_I below 2 kHz at locations I and II is likely caused by some combination of acoustic near-field errors—both in calculating large gradients with a first-order least-squares estimate[9,13] and because the near-field intensity is likely a superposition of radiated and hydrodynamic pressure fluctuations.[22] These low-frequency effects are important physically and are the subject of on-going investigations, but are not central to this letter's core message.

The PAGE-calculated intensity vector angle is shown in Fig. 3(d) for all four locations. Although not shown for clarity, the traditional method calculates angles within 0°–10° of PAGE below 4–5 kHz, beyond which they diverge rapidly. This limited bandwidth of the traditional method is insufficient to study the radiation characteristics at maximum L_I frequencies, approximately 3–10 kHz [see Fig. 3(c)]. However, the extended bandwidth of the PAGE-calculated intensity angles for those frequencies [see Fig. 3(d)] reveals that the near-jet intensity angles fall between 25° and 40°, thus

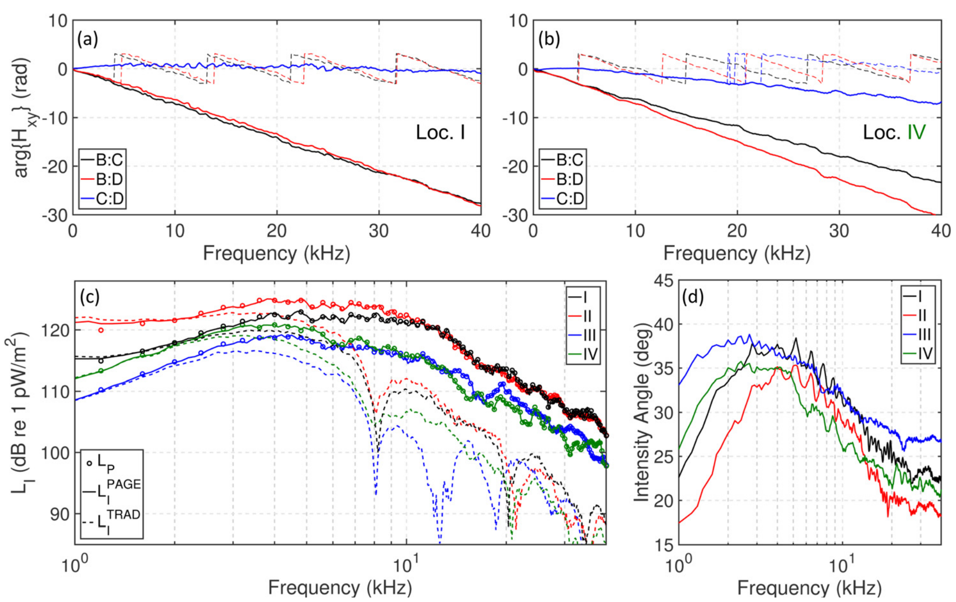

Fig. 3. (Color online) (a) Wrapped and unwrapped transfer function phases between microphone pairs shown in Fig. 2(a) for probe location I near a Mach 1.8 unheated jet. (b) Same as (a) but for location IV. (c) Intensity and pressure levels at probe locations I–IV. (d) PAGE-calculated intensity angles at locations I–IV.

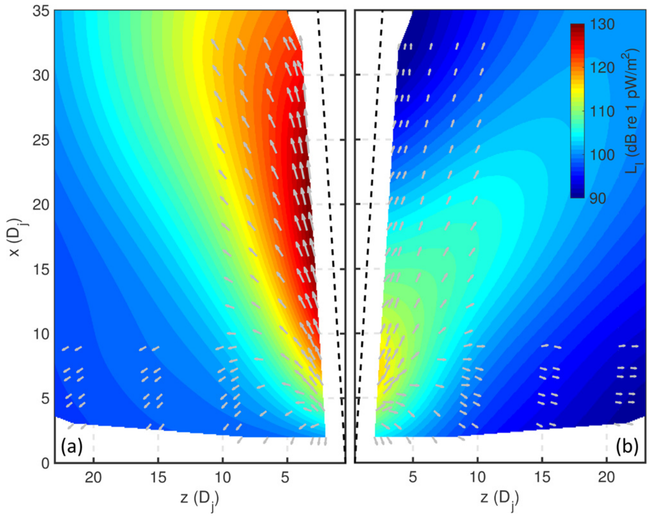

Fig. 4. (Color online) PAGE-calculated vector intensity maps for a Mach-1.8 unheated jet at (a) 2 kHz and (b) 40 kHz. The 40 kHz map is mirrored across the jet centerline (x axis). Vector lengths have been scaled by the eighth root of the intensity magnitude for visibility.

bounding the 40 D_j OASPL peak directivity range of 30°–35°. Furthermore, these intensity angles match within 1°–2° at location pairs I/III and II/IV over this frequency range, thus establishing the near-field nature of the dominant radiation lobe well beyond the traditional intensity limit for a probe of this size. Although there is no true benchmark like the plane-wave tube experiment, the combination of (a) the concordance of L_P and L_I across frequency, which is expected for source radiation problems, and (b) the tracking of peak-frequency-region intensity angles across the near-field location pairs that correspond with the far-field peak OASPL directivity angles (from the 40 D_j arc) provides a reasonable indication that the PAGE-predicted intensity estimates are physical when phase unwrapping is employed.

As a final demonstration that the PAGE method may provide significant bandwidth extension for acoustic intensity, PAGE vector intensity maps calculated for all measurement locations are shown in Fig. 4 for 2 and 40 kHz. At 2 kHz, the PAGE and traditional intensity calculations are approximately equal, whereas at 40 kHz, only the PAGE method with phase unwrapping applied potentially yields physically realistic results. Relative to Fig. 2(a), the axes in Fig. 4 have been rotated 90° and the 40 kHz map in Fig. 4(b) is reflected about the jet centerline. To extend L_I^{PAGE} to regions where there are not vector measurements, L_P from the 40 D_j arc are included in the interpolation. At 2 kHz in Fig. 4(a), the intensity magnitudes and directions vary smoothly over the measurement locations. For $z > 3D_j$, the maximum magnitude vectors all point to approximately 20°, the peak radiation angle for 2 kHz at the 40 D_j arc. For $z < 3\,D_j$, there is a likely superposition of the hydrodynamic and radiated noise components, which slightly lessens the intensity angle. The transition between downstream and sideline-radiated intensities has been shown to agree with complementary Schlieren photography measurements.[23]

The 40 kHz PAGE intensity map in Fig. 4(b) contains results at approximately 15 times the upper limit for I^{TRAD} and this probe geometry. Remarkably, L_I^{PAGE} varies as smoothly at 40 kHz as at 2 kHz. The vectors largely follow the level trends, unlike I^{TRAD}, which underestimates the level by approximately 15 dB [see Fig. 3(b)] and has significant direction errors beyond ~5 kHz. The PAGE results are also physically consistent with jet noise theory and prior intensity analyses: high frequencies radiate at a greater angle than low frequencies and originate from a more compact, upstream source region.[14,15] A few anomalously directed vectors, such as near (6, 8.5) D_j, may be due to microphones/probe holder scattering or reflections in the non-anechoic environment. Beyond 40 kHz, the number of questionable vectors, particularly in the sideline direction and near the jet nozzle, increases. Thus, while the actual upper limit of the PAGE method for a broadband source varies with probe location, 40 kHz seems an appropriate practical limit for this probe geometry.

5. Concluding discussion

This letter has shown that the PAGE method with transfer function unwrapping can extend the calculation bandwidth of active acoustic intensity for broadband sources to several times the spatial Nyquist frequency, more than an order of magnitude greater than the traditional quadspectrum-based method limit. This finding should be extendable to intensity-based characterization of any acoustic source over the frequency range where the transfer function phase varies smoothly. Future work will investigate the ability to unwrap signals with tonal components above the spatial Nyquist frequency, signals with limited coherence, and signals from multiple sources arriving from widely different angles. This additional PAGE-related research for different measurement environments and sources will further establish the method's performance and may eventually influence measurement standards.

Acknowledgments

This work was supported by the National Science Foundation. The jet measurements were supported by the Japan Society for the Promotion of Science. The contributions of Daxton Hawks, Mylan Cook, Eric Whiting, and Darren Torrie are gratefully acknowledged.

References and links

[1]ISO 9614-1:1993, "Acoustics—Determination of sound power levels of noise sources using sound intensity—Part 1: Measurement at discrete points," (International Organization for Standardization, Geneva, Switzerland, 1993).

[2]Y. Oshino and H. Tachibana, "Noise source identification on rolling tires by sound intensity measurement," J. Acoust. Soc. Jpn. (E) **12**, 87–92 (1991).

[3]C. Yu, Z. Zhou, and M. Zhuang, "An acoustic intensity-based method for reconstruction of radiated fields," J. Acoust. Soc. Am. **123**, 1892–1901 (2008).

[4]F. Jacobsen and V. Jaud, "Statistically optimized near field acoustic holography using an array of pressure-velocity probes," J. Acoust. Soc. Am. **121**, 1550–1558 (2007).

[5]ISO 15186-2:2010, "Acoustics—Measurement of sound insulation in buildings and of building elements using sound intensity—Part 1I: Field measurements," (International Organization for Standardization, Geneva, Switzerland, 2003).

[6]C. P. Wiederhold, K. L. Gee, J. D. Blotter, S. D. Sommerfeldt, and J. H. Giraud, "Comparison of multi-microphone probe design and processing methods in measuring acoustic intensity," J. Acoust. Soc. Am. **135**, 2797–2807 (2014).

[7]J. K. Thompson and D. R. Tree, "Finite difference approximation errors in acoustic intensity measurements," J. Sound Vib. **75**, 229–238 (1981).

[8]F. J. Fahy, "Measurement of acoustic intensity using the cross-spectral density of two microphone signals," J. Acoust. Soc. Am. **62**, 1057–1059 (1977).

[9]J.-C. Pascal and J.-F. Li, "A systematic method to obtain 3D finite-difference formulations for acoustic intensity and other energy quantities," J. Sound Vib. **310**, 1093–1111 (2008).

[10]K. L. Gee, J. H. Giraud, J. D. Blotter, and S. D. Sommerfeldt, "Near-field acoustic intensity measurements of a small solid rocket motor," J. Acoust. Soc. Am. **128**, EL69–EL74 (2010).

[11]T. A. Stout, K. L. Gee, T. B. Neilsen, and A. T. Wall, "Acoustic intensity near a high-powered military jet aircraft," J. Acoust. Soc. Am. **138**, EL1–EL7 (2015).

[12]D. C. Thomas, B. Y. Christensen, and K. L. Gee, "Phase and amplitude gradient method for the estimation of acoustic vector quantities," J. Acoust. Soc. Am. **137**, 3366–3376 (2015).

[13]E. B. Whiting, "Energy quantity estimation in radiated acoustic fields," M.S. thesis, Brigham Young University, Provo, UT, Sept. 2016.

[14]K. L. Gee, E. B. Whiting, T. B. Neilsen, M. M. James, and A. R. Salton, "Development of a near-field intensity measurement capability for static rocket firings," Trans. Jpn. Soc. Aeronaut. Space Sci. **14**, PO_2_9–PO_2_15 (2016).

[15]T. A. Stout, K. L. Gee, T. B. Neilsen, A. T. Wall, and M. M. James, "Source characterization of full-scale jet noise using vector intensity," Noise Control Eng. J. **63**, 522–536 (2015).

[16]J. A. Mann III, J. Tichy, and A. J. Romano, "Instantaneous and time-averaged energy transfer in acoustic fields," J. Acoust. Soc. Am. **82**, 17–30 (1987).

[17]J. A. Mann III and J. Tichy, "Near-field identification of vibration sources, resonant cavities, and diffraction using acoustic intensity measurements," J. Acoust. Soc. Am. **90**, 720–729 (1991).

[18]F. J. Fahy, *Sound Intensity*, 2nd ed. (The Spon Press, New York, 1995), pp. 89–108.

[19]M. Akamine, Y. Nakanishi, K. Okamoto, S. Teramoto, T. Okunuki, and S. Tsutsumi, "Acoustic phenomena from correctly expanded supersonic jet impinging on inclined plate," AIAA J. **53**, 2061–2067 (2015).

[20]B. Greska, "Supersonic jet noise and its reduction using microjet injection," Ph.D. thesis, The Florida State University, FAMU-FSU Collage of Engineering, 2005.

[21]D. K. Torrie, E. B. Whiting, K. L. Gee, T. B. Neilsen, and S. D. Sommerfeldt, "Initial laboratory experiments to validate a phase and amplitude gradient estimator method for the calculation of acoustic intensity," Proc. Mtgs. Acoust. **23**, 030005 (2017).

[22]P. Jordan and T. Colonius, "Wave packets and turbulent jet noise," Annu. Rev. Fluid Mech. **45**, 173–195 (2013).

[23]M. Akamine, K. Okamoto, K. L. Gee, T. B. Neilsen, S. Teramoto, S. Tsutsumi, and T. Okunuki, "Comparison of acoustic intensity vectors with SPL and phase distributions of supersonic jet," in *8th Asian Joint Conference on Propulsion and Power* (March 2016).