Transformations of a crackling jet noise waveform and potential implications for quantifying the “crackle” percept

S. Hales Swift, Kent L. Gee, and Tracianne B. Neilsen

Citation: Proc. Mtgs. Acoust. **22**, 045005 (2014); doi: 10.1121/2.0000468
View online: http://dx.doi.org/10.1121/2.0000468
View Table of Contents: http://asa.scitation.org/toc/pma/22/1
Published by the Acoustical Society of America

Proceedings of Meetings on Acoustics

Volume 22 http://acousticalsociety.org/

168th Meeting of the Acoustical Society of America

Indianapolis, Indiana

27-31 October 2014

Physical Acoustics: Paper 1aPA10

Transformations of a crackling jet noise waveform and potential implications for quantifying the "crackle" percept

S. Hales Swift, Kent L. Gee and Tracianne B. Neilsen

Department of Physics and Astronomy, Brigham Young University, Provo, UT; hales.swift@gmail.com, kentgee@byu.edu, tbn@byu.edu

In the 1975 paper by Ffowcs-Williams et al. on jet crackle, there are several potentially competing descriptors–including a qualitative description of the sound quality or percept, a statistical measure, and commentary on the relation of the presence of shocks to the sounds quality. These descriptors have led to disparate conclusions about what constitutes a crackling jet, waveform, or sound quality. This presentation considers modifications of a jet noise waveform that exhibits a crackling sound quality and initially satisfies all three definitions. These modifications alter the statistical distributions of primarily the pressure waveform or its first time difference in order to demonstrate how these modifications do or do not correspond to changes in the sound quality of the waveform. The result, although preliminary, demonstrates that the crackle percept is tied to the statistics of the pressure difference waveform instead of the pressure waveform itself.

Published by the Acoustical Society of America

1. INTRODUCTION

In 1975, Ffowcs Williams *et al.*[1] published their analyses of jet "crackle," describing it as an "annoying" component of jet noise. Qualitatively, they depicted it as "sudden spasmodic bursts of a rasping fricative sound not dissimilar to that made be the irregular tearing of paper... ...a badly connected loud speaker... [or] ... the spitting of water added to extremely hot fat. It is a startling staccato of cracks and bangs and its onomatope, 'crackle', conveys a subjectively accurate impression." As characterized by Ffowcs Williams, crackle was important because of its capacity for causing annoyance that could not be detected through examination of the spectrum and spectrum-dependent noise metrics.

In their further analysis of jet crackle, Ffowcs Williams *et al.*[1] make a number of observations regarding characteristics in the waveforms associated with the audible sound quality that each may suggest a definition of or approach to detecting or quantifying the crackle percept. These include:

1. The presence of shocks: "intense spasmodic short-duration compressive elements of the wave form"

2. Skewed pressure amplitude probability density function: "...signals with a normalized skewness less that 0.3 do not crackle while those with a skewness in excess of 0.4 crackle distinctly."

3. Undetectable by purely spectral means: "'Crackle' cannot by characterized by the normal spectral description of noise."

4. Primarily a source phenomenon: "Crackle is formed (we think) because of local shock formation due to nonlinear wave steepening at the source and not from long-term nonlinear propagation." though long term nonlinear effects apparently also result in crackle, particularly, "in flight, where they are additive"

Among these four definitions, we approach this study agreeing with 1 and 3; their relationship with 2 is examined in this paper. Direct investigation of 4 is beyond the scope of this perceptually focused paper.

Based on Ffowcs Williams' suggested metric, much research has focused on crackle as quantified using the skewness (Sk) of the pressure probability density function (PDF). As he correctly observed, "A quantitative measure of the effect is an essential prerequisite for its systematic study." Thus, in keeping with the measure that he provided in his paradigmatic paper, the skewness of the pressure waveform is regularly reported in crackle research and this equivalence is often assumed in subsequent studies, without specifically verifying its perceptual significance.

Other metrics have been proposed as possible indicators of jet crackle. Attention to the importance of shocks in producing the crackling sound quality has led some, including McInerny and Olcmen[3] and Gee *et al.*,[4] to consider the PDF of the time derivative of the pressure waveform $\partial p/\partial t$ and its statistics as a possible quantifier for crackle. Reichman *et al.*[5] has suggested on the basis of analytical and empirical evidence that a skewness of $Sk\{\partial p/\partial t\} > 5$ is indicative of significant shock formation.

In evaluating these two competing quantifiers, it is important to note that they are, in many practical cases, inconsistent. Gee *et al.*[4] have shown that regions of high pressure skewness ($Sk\{p\}$)

and regions of high derivative skewness $Sk\{\partial p/\partial t\}$ are not always spatially coincident with one another. They further argue that to the extent that shocks are an important contributor to a crackling sound quality, the use of derivative skewness better identifies this characteristic on physical grounds. Based on the combined hypotheses that shock content is directly important to the crackle percept and is often distinct from pressure skewness, and that $Sk\{\partial p/\partial t\} > 5$ can be thought of as a threshold for significant shock formation, the resultant "Reichman/Gee criterion" will also be considered as a potential predictor of crackle.

Gee *et al.*[2] have also challenged Ffowcs Williams' crackle criterion on perceptual grounds. They have shown that pressure skewness of $Sk\{p\} = 0.4$ is not a sufficient condition for a crackling sound quality. In that study, the authors began with a Gaussian white noise waveform and filtered it to match the spectral content of noise from an afterburning F/A-18E Super Hornet. The Gaussian waveform was then nonlinearly transformed in order to produce a pressure PDF and thus $Sk\{p\}$ similar to the afterburner waveform. The afterburner waveform has a skewness of $Sk\{p\} = 0.6$ and crackles distinctly, as one would expect based on the Ffowcs Williams criterion. The transformed Gaussian noise has essentially the same spectrum and PDF as the afterburner waveform, however, it does not crackle. This study provides evidence of a significant inconsistency between the sound quality identified by Ffowcs Williams and the metric he used in an attempt to quantify it.

In considering the grounding definition of crackle as a perceivable and reportedly annoying sound quality, it is important to evaluate which, if either, of the proposed physical criteria ($Sk\{p\}$ or $Sk\{\partial p/\partial t\}$) consistently identifies a signal with a crackling sound quality. Relationships between these conflicting definitions can be clarified by deliberately modifying one of more of the characteristics while leaving other characteristics unchanged. This paper outlines a series of targeted waveform modification, their results in terms of spectra, PDFs of the pressure and derivative time series, and their implications for predicting a crackling sound quality.

2. EXPERIMENTAL PROCEDURE

A. JET NOISE WAVEFORM

To investigate the sound quality implication of the pressure skewness ($Sk\{p\}$) and derivative skewness ($Sk\{\partial p/\partial t\}$) criteria this study employs the afterburning waveform used previously by Gee *et al.*[2] The measurement conditions associated with this waveform were reported previously.[2] Pressure and derivative skewness values of $Sk\{p\} = 0.57$ and $Sk\{\partial p/\partial t\} = 5.59$, respectively, were recorded. The initial waveform crackles audibly, as predicted by either skewness criteria ($Sk\{p\} > 0.4$ or $Sk\{\partial p/\partial t\} \geq 5$) and, as suggested by the derivative skewness value, has significant shock content. The waveform has been scaled in magnitude for convenience in subsequent plots and transformations.

B. WAVEFORM ALTERATION METHODS

This initially crackling jet noise waveform is altered to produce daughter waveforms via a set of modifications. These modifications seek to minimally affect spectral characteristics while selectively changing either the pressure PDF and its associated skewness or the pressure derivative PDF and its associated skewness. By selectively altering these variables, waveforms are produced that isolate, to the extent possible—and thus help elucidate—the characteristics most relevant for and predictive of the crackle percept. The modifications performed include:

- Phase randomizing the waveform
- Transforming the pressure times series to have a Gaussian PDF ($Sk\{p\} \approx 0$)
- Transforming the derivative time series to have a Gaussian PDF ($Sk\{\partial p/\partial t\} \approx 0$)
- "Slowing" shocks by interpolating points into the time series at points of rapid pressure increase

Limited variations of these methods are also reported to help clarify the spectral, statistical or sound quality effects of these transformations; each modification process will now be discussed.

i. Phase randomization

Ffowcs Williams noted that crackle cannot be identified by the magnitude spectrum associated with a waveform or from any metric derived therefrom. To illustrate this, the jet noise waveform is transformed to the Fourier frequency domain. Here, each positive and negative frequency pair is given a randomized phase. The conjugate relationship within each pair is maintained so that when the spectrum is inverse Fourier transformed the resultant waveform is real valued. The net effect of this modification is that individual frequency components are moved back and forth relative to one another, disrupting time-domain behavior that arises from unique temporal relationships of these components. Shocks and other unique features are thus dispersed, while precisely maintaining the original magnitude spectrum. This method was discussed previously by Gee *et al.*[6] as applied to nonlinearly propagated noise. In that work, it was shown to remove a crackling sound quality from a nonlinearly propagated signal.

ii. Constructing a tailored nonlinear transformation

Because the skewness of a PDF is at the center of both of the currently proposed criteria for identifying crackle, the capability to achieve the desired statistical property is essential to evaluating the perceptual significance of each criterion. Thus, a transformation is desired that can map a skewed PDF f_1 to f_2, a Gaussian (G) PDF as shown in Figure 1. This transformation is accomplished through the use of the cumulative distribution function (CDF). The CDF is a probabilistic measure used to describe the probability P that a value chosen randomly from a given random process, X, is less than or equal to a particular value, x, e.g., $P(X \leq x)$. Values of the CDF fall, by definition, in the range $P \in [0, 1]$. By using the MATLAB `sort` function, the values of a time series can be arranged in ascending order and placed in a one to one correspondence with the natural numbers from 1 to the number of elements in the time series. Ordered in this way, the index values indicate how many elements of the time series have a value less than or equal to the value of a given element. Dividing the index numbering by the length of the time series normalizes the values such that a sampled approximation of the CDF is obtained. The normalized index values are uniformly distributed between zero and one, such that relating amplitude values to their normalized index values maps the time series to a uniform distribution between zero and one $f_1(t) \rightarrow P(t) \in (0, 1]$.

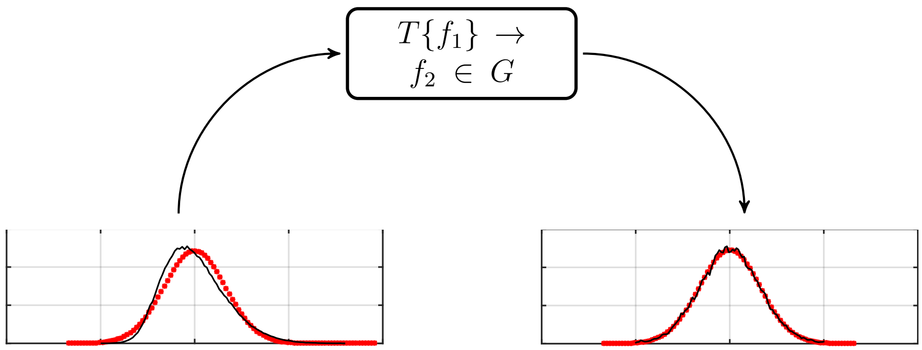

Figure 1: A transformation (T) mapping an arbitrary PDF f_1 (black, left) to f_2, a Gaussian (G) PDF (black, right). A comparison Gaussian of the same mean and standard deviation (red).

A transformation is desired that produces a Gaussian PDF, primarily because this PDF has zero-skewness. Accordingly, the Gaussian inverse CDF can be used to map the domain from $P \in [0,1]$ to a set of values which will be Gaussian-distributed. In practice, a truncated Gaussian distribution is used in order to avoid infinite values and the inverse Gaussian CDF, CDF_G^{-1}, is sampled and interpolated. The sample values are obtained in MATLAB using `normcinv` or `-sqrt(2)*erfcinv(2*p)` (depending on the available version) to get inverse CDF values corresponding to inputs between 0.001 and 0.999 in increments of 0.0001. The resultant truncated Gaussian is then scaled by the standard deviation of the input time series so that it has comparable variability. This completes the mapping $f_2(t) = \mathrm{CDF}_G^{-1}\{P(t)\} \rightarrow f_2 \in G$. This process is applied separately to both the pressure time series ($T_p\{p\}$) and the derivative time series ($T_d\{\partial p/\partial t\}$) in order to examine the effects of each. In practice only a portion of the waveform is used to construct the transformation in the interest of efficiency.

A transformation, T, determined in this way has several advantages worth mentioning:

- Preserves order in time and relative magnitude $f_1(t_1) > f_1(t_2) \Rightarrow T\{f_1(t_1)\} > T\{f_1(t_2)\} \rightarrow f_2(t_1) > f_2(t_2)$
- Preserves continuity and maps discontinuities to discontinuities
- Preserves temporal locations of key features such as maxima, minima and, if present, shocks
- Always returns finite values

The resultant transformation is typically nonlinear. Code for its calculation is given in appendix A.

iii. Slowing the shocks

In addition to modifications that affect the entire time series, the last modification considered affects (at least directly) only the shocks. In this modification, shocks are first identified. To locate shocks, the rate of rise between subsequent points in the time series is examined and if a pair of subsequent points exceeds a given rate of rise (informally, a “speed limit”) then the pair of points is identified as a shock. When a shock is identified, the minimum number of points necessary to slow the rate of rise to a value less than the speed limit are introduced and given values determined by linear interpolation. Slowing the shocks in this way truncates the large positive derivative values that lead to elevated values of derivative skewness by imposing a maximum value on the derivative time series. This method will, necessarily, increase the length of the signal. Introduction of additional points at the shocks necessarily increases the length of the signal. However, if the threshold or speed limit is chosen judiciously the increase in length need not be great in order to have a significant effect on the sound quality. The MATLAB code which implements this method is given in appendix A.

3. RESULTS

The alterations outlined above yield modified waveforms. The resultant spectra, time series and PDFs with their associated statistics are evaluated. Additionally, the perception of a crackling or non-crackling sound quality is determined by informal listening tests. This enables analysis of the relationship between the crackle percept and its two candidate metrics ($Sk\{p\}$ and $Sk\{\partial p/\partial t\}$). The waveforms will be made available for direct examination in a future paper in order to allow external verification. Statistical results from all of the experiments are aggregated in Table 1 for convenience. In addition to the measures primarily discussed, kurtosis (Kt) measures are also included, as these have also been considered as possible physical quantifiers of shock content.[7] In order to help interpret the skewness and kurtosis values seen in the table, please note that for a perfect Gaussian the skewness is 0 and the kurtosis is 3.

Table 1: Statistical measures of interest for each of the waveforms as well as informal subjective crackle assessments for each waveform.

	original	re-phased	$T_p\{p\} \to G$ transformed	$T_d\{\frac{\partial p}{\partial t}\} \to G \to \int_L dt$	slowed shocks	$\int_L \frac{\partial p}{\partial t} dt$
$Sk\{p\}$	0.57	0.00	0.02	-0.05	0.56	1.27
$Kt\{p\}$	3.31	2.97	2.97	2.79	3.28	5.56
$Sk\{\partial p/\partial t\}$	5.59	0.00	5.45	-0.11	0.79	5.30
$Kt\{\partial p/\partial t\}$	67.24	3.00	66.55	3.02	3.61	66.10
Crackle?	Y	N	Y	N	N	Y

A. RE-PHASED WAVEFORM

The modification of the Fourier phase of positive and negative frequency pairs in complex conjugate pairs successfully removes a crackling sound quality. This is implicitly predicted in Ffowcs Williams' assertion that a crackling sound quality cannot be predicted by the spectrum alone. Statistical results for this waveform modification are given in Table 1. As reflected in the skewness and kurtosis statistics, phase randomization resulted in both pressure and derivative time series becoming almost perfectly Gaussian in character. The spectral levels, as expected, are identical to the original.

B. NONLINEARLY TRANSFORMING THE PRESSURE TIME-SERIES ($T_p\{p\} \rightarrow G$)

In transformation T_p, the original afterburner waveform is nonlinearly transformed such that the mapped pressure waveform has a Gaussian PDF. The transformation begins by computing the CDF of p (Figure 2 lower left) and a truncated inverse Gaussian CDF of the same standard deviation (Figure 2 upper left). The resultant desired nonlinear transformation, T_p, mapping the afterburner waveform to a Gaussian distribution is shown in the center left panel of Figure 2. The degree of nonlinearity in the transforming function is relatively low as can be determined from the fact that the transformation could be reasonably approximated by a straight line through most of its domain and range.

The transformation T_p has relatively modest effects on the spectrum. Spectra corresponding to the input waveform and the transformed waveform are shown in the right portion of Figure 2, along with the exact and smoothed differences between the spectra. These latter measures are included to simplify comparison because of some degree of visual noise in the spectra. The smoothed spectrum is obtained from the original spectrum by averaging (after calculation of levels) across sets of 21 adjacent narrowband spectral bins. This practice is followed throughout this paper whenever spectra are displayed. As can be seen in this figure, there are only minimal differences in the spectra associated with original afterburner waveform and pressure transformed waveforms.

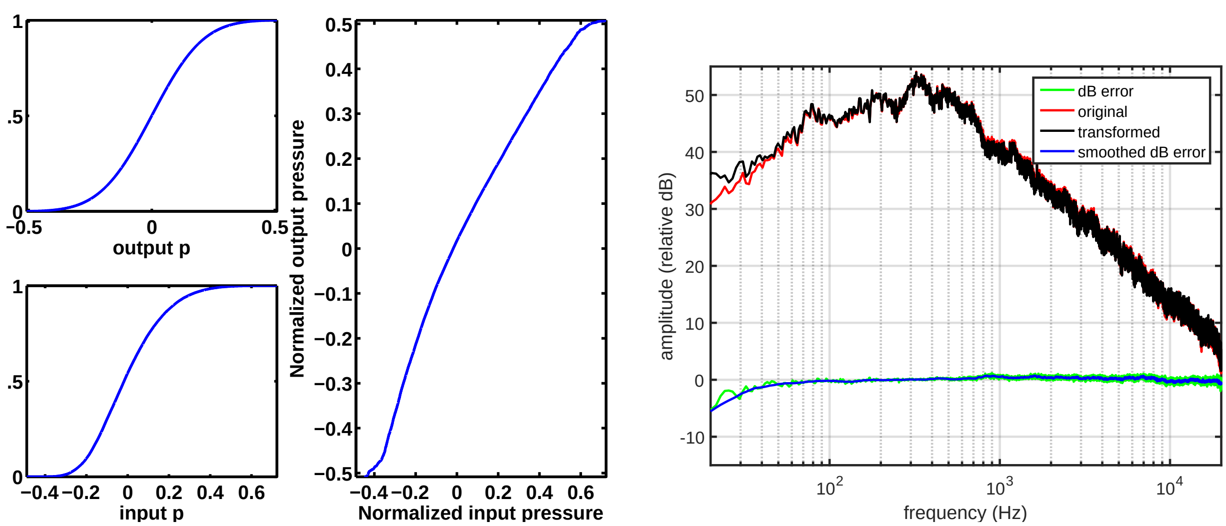

Figure 2: (Left) The CDFs of the jet pressure time series (top) and the target Gaussian distribution (bottom). (Center) The transformation, ($T_p\{p\} \rightarrow G$), mapping the jet pressure time series to a Gaussian. (Right) The original jet (red) and transformed (black) spectra (right)

Comparison between the input and output PDFs in Figure 3 confirms that $T_p\{p(t)\}$ results in a Gaussian pressure distribution. As shown on the right side of the figure, the PDF of the derivative time series maintains its initially skewed form, though small changes in the distribution are apparent, e.g., at its peak. The statistics associated with the two criteria have values of $Sk\{p(t)\} = 0.02$ and $Sk\{\partial p(t)/\partial t\} = 5.45$. For this waveform, the Ffowcs Williams criterion would, therefore, predict no crackle, and the Reichman/Gee criterion would predict observation of a crackling sound quality. The sound quality of the transformed waveform is nearly identical to the original, with both displaying fairly clear crackle.

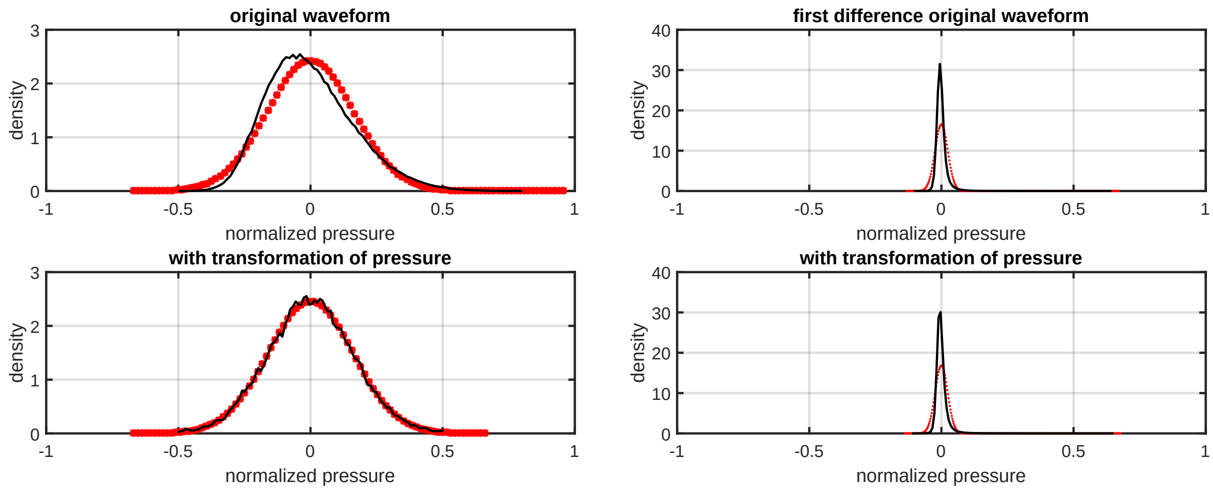

Figure 3: The PDF associated with the acoustic pressure time series (left) and its derivative (right), before (top) and after (bottom) a transformation ($T_p\{p\} \rightarrow G$) mapping the pressure time series (black) to a Gaussian target PDF (red).

C. NONLINEARLY TRANSFORMING THE PRESSURE DERIVATIVE TIME-SERIES FOLLOWED BY LEAKY INTEGRATION $T_d\{\frac{\partial p}{\partial T}\} \rightarrow G \Rightarrow \int_L dt$

The transformation T_d changes the waveform such that the PDF of $\partial p/\partial t$ is Gaussian. In order to do this some additional processes are involved:

1. The time-derivative (obtained in scaled form using a first difference via the MATLAB `diff` command, with the scalar factor of dt is neglected throughout) is evaluated.
2. The CDF of $\frac{\partial p}{\partial t}$ is used to construct the first part of the transformation (from the original range and distribution to a uniform distribution in the range $[0, 1]$) and these values are interpolated into the sampled Gaussian inverse CDF as described above.
3. The transformed derivative time series $T_d\{\frac{\partial p}{\partial t}\}$ is reintegrated using leaky integration.

The final step involving leaky integration is necessary because the transformed $\partial p/\partial t$ acts like a noisy signal relative to normal integration causing the integral to depart from locally zero-mean behavior in the manner of a random walk. The leaky integration acts as a high-pass filter.

The transformation $T_d\{\partial p/\partial t\} \rightarrow G$ shown in Figure 4 (center) is more strongly nonlinear than the pressure transformation T_p shown in Figure 2. This is because the PDF of $\partial p/\partial t$ is initially much more skewed due to the presence of shocks. The CDFs of the original $\partial p/\partial t$ and the Gaussian to which it is mapped can be seen in the bottom left and top left portions of Figure 4.

Unlike T_p, T_d causes a significant change in low-frequency spectral content. The spectra shown in the right panel of Figure 4 agree well above 500 Hz, with passable agreement between 60 and 500 Hz. Below 60 Hz, increased levels are an artifact of reintegrating the transformed waveform.

Considering the PDF of the pressure and derivative time series before and after the transformation T_d, shown in Figure 5, it is clear that the derivative time series PDF (right) is now Gaussian as desired, with a skewness of $Sk\{\partial p(t)/\partial t\} = -0.05$. The pressure time series PDF (left) is

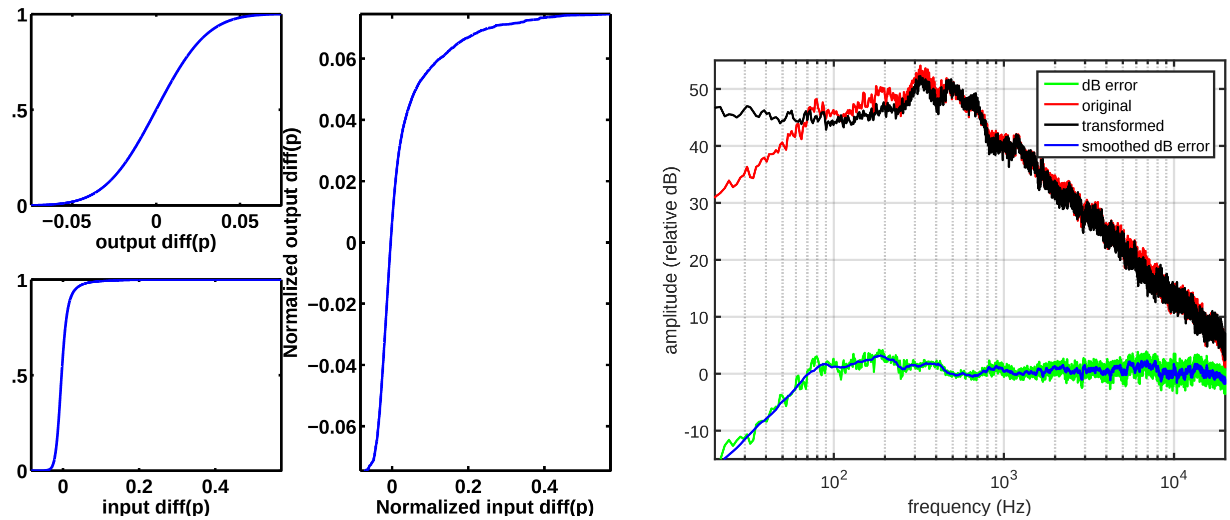

***Figure 4:** (Left) The CDFs of the jet pressure derivative time series (top), the target Gaussian distribution (bottom). (Center) the transformation ($T_d\{\frac{\partial p}{\partial t}\} \rightarrow G$) mapping the jet derivative time series to a Gaussian. (Right) The original jet spectrum (red), and the derivative transformed spectra after leaky integration ($T_d\{\frac{\partial p}{\partial t}\} \rightarrow G \Rightarrow \int_L dt$) (black).*

also transformed to a nearly Gaussian form by this transformation, with $Sk\{p(t)\} = -0.11$. Both the Ffowcs Williams and the Reichman/Gee criterion predict no crackle based on the reported statistics. Consistent with both criteria, the informally observed sound quality is non-crackling.

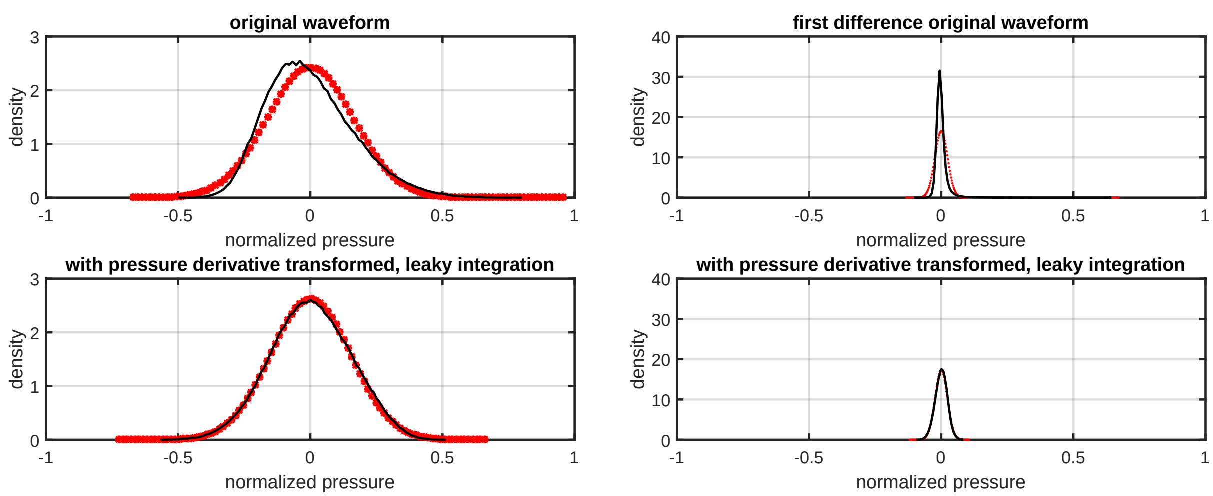

***Figure 5:** The PDFs associated with the acoustic pressure time series (left) and its derivative (right), before (top) and after (bottom) transformation of the pressure derivative time series to a Gaussian target PDF, followed by leaky integration ($T_d\{\frac{\partial p}{\partial t}\} \rightarrow G \Rightarrow \int_L dt$).*

To show the effects of both transformations—$T_p\{p(t)\} \rightarrow G$ and $T_d\{\partial p/\partial t\} \rightarrow G$—on the time series, the original time series is plotted together with the two transformed time series in Figure 6. The behavior through time of the original afterburner waveform (top) and and the waveform transformed by $T_p\{p\}$ (middle), are similar as would be expected from a monotonic transformation in amplitude. When $\partial p/\partial t$ is nonlinearly transformed by T_d, however, the shocks are noticeably reduced in amplitude (bottom), though they occur at the same positions. Small changes occur in the locations of increasing or decreasing intervals under T_d, but a high degree of general similarity remains.

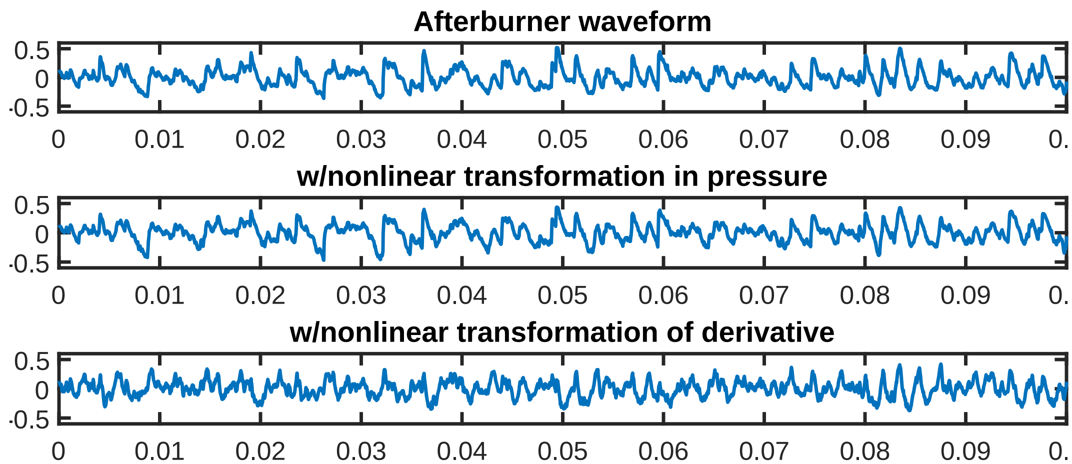

Figure 6: Time series snapshot of original afterburner waveform (top), after mapping the pressure time series to a Gaussian distribution ($T_p\{p\} \to G$, middle) and after mapping the derivative time series to a Gaussian distribution ($T_d\{\frac{\partial p}{\partial t}\} \to G \Rightarrow \int_L dt$, bottom).

Because the effects of the leaky integration, if left unexamined, could appear to be a confounding factor for the removal of crackle by transforming the derivative, the influence of leaky integration of the derivative of the original afterburner waveform are reported as well. Statistical results are provided in Table 1, and spectra and PDFs of p and $\partial p/\partial t$ are shown in Figure 7. The form of the PDF of $\partial p/\partial t$ is relatively unchanged by leaky integration. The PDF of p is significantly increased by the high-pass filtering effect of the leaky integration. Attenuated spectral levels are seen below 400 Hz due to the high-pass filtering effect of differentiation plus leaky integration ($\int_L(\partial p/\partial t)dt$). The resultant signal exhibits clear crackle consistent with both criteria.

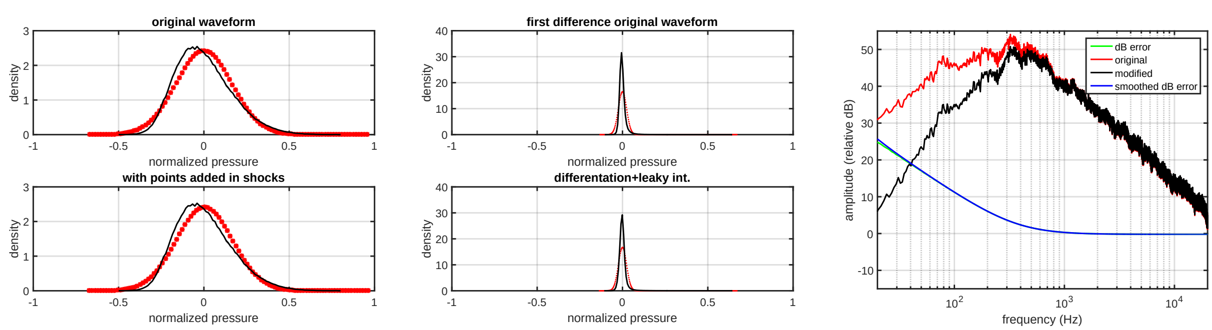

Figure 7: The PDFs associated with the acoustic pressure time series (left) and its derivative (middle), before (top) and after (bottom) and spectrum before and after $\int_L(\partial p/\partial t)dt$ (right).

D. INTERPOLATING POINTS INTO (OR "SLOWING") THE SHOCKS

The original afterburner waveform was next modified by "slowing" the shocks by interpolation of points as described in the methods section (2.2.3). This method has significant advantages in terms of modifying the derivative PDF while having only a minimal effect on the pressure PDF as seen in Figure 8. The key crackle assessment metrics have values of $Sk\{p(t)\} = 0.56$ and $Sk\{\partial p(t)/\partial t\} = 0.79$, which places this waveform above the threshold for crackle of Ffowcs Williams and below the threshold for crackle under the Reichman/Gee criterion. The sound quality observed in informal listening tests is non-crackling, consistent with the Reichman/Gee criterion.

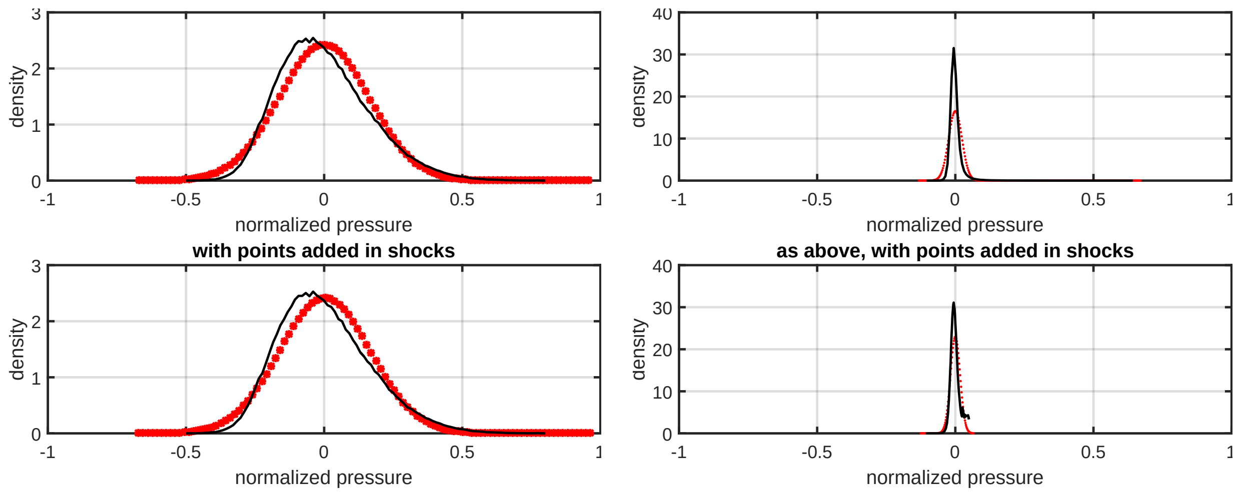

Figure 8: The PDFs associated with the acoustic pressure time series (left) and its derivative (right), before (top) and after (bottom) "shock slowing" (interpolation of points into the shocks).

The spectral effects of shock slowing are subtle, as shown in Figure 9 (left), which shows the spectrum after slowing shocks (black) compared with the original spectrum (red). Notably, the frequencies associated with various spectral features have been reduced due to the elongation of the waveform resulting from the introduction of additional points. A reduction of around 5-6 dB is seen at high frequencies, particularly between 3 and 8 kHz. In order to separate effects of elongation of the signal from effects due directly to slowing the shocks, the waveform is resampled to approximately the original length. A reduction in the spectral levels at high frequencies is still seen, though the agreement elsewhere is improved, as seen in Figure 9 (right). This loss at high frequencies is a reasonable result because slowing high-rate transients necessarily reduces the high frequency energy associated with such events. Local high-frequency reductions ultimately result in global high-frequency reductions.

Time series resulting from the two variations of the "shock slowing" modification are shown in Figure 10. The approximate length increase due to the addition of points (center) as well as the resultant degree of distortion of features can be easily gaged from this figure by comparing the original (top) and shock slowed (center) waveforms. All features occur in the same order and at the same amplitude in this modification as in the original, but with more or less of a delay depending on how much of the previous waveform contains features exceeding a specified rate of rise. The resampled waveform, shown at the bottom of the figure, aligns well with the features of

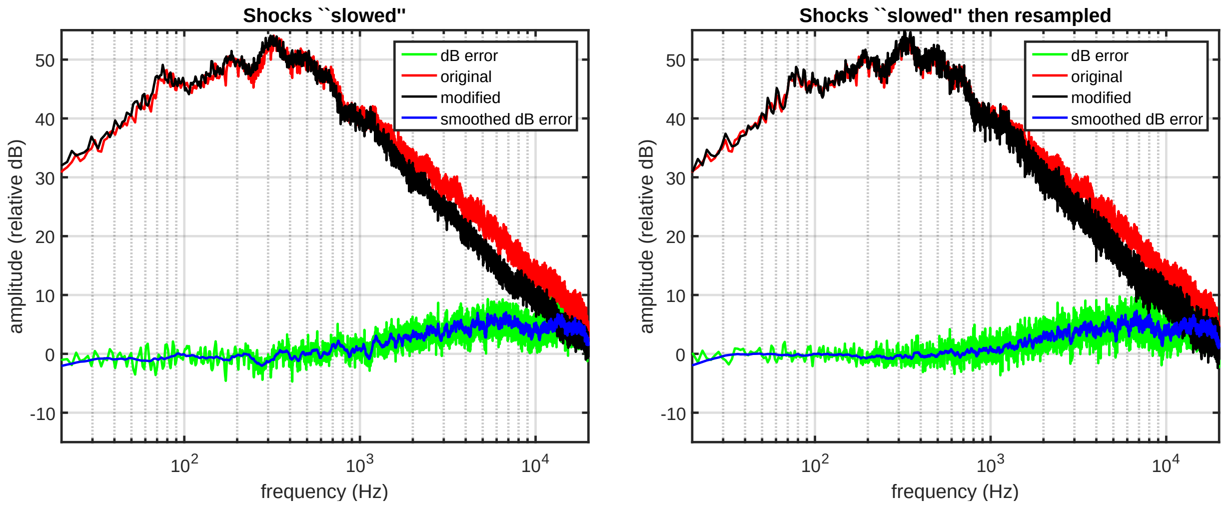

Figure 9: Spectrum before and after "slowing" shocks by interpolating points into regions with rise rates above a given threshold.

the original waveform indicating that the shocks are temporally distributed with some degree of inexact but approximate uniformity.

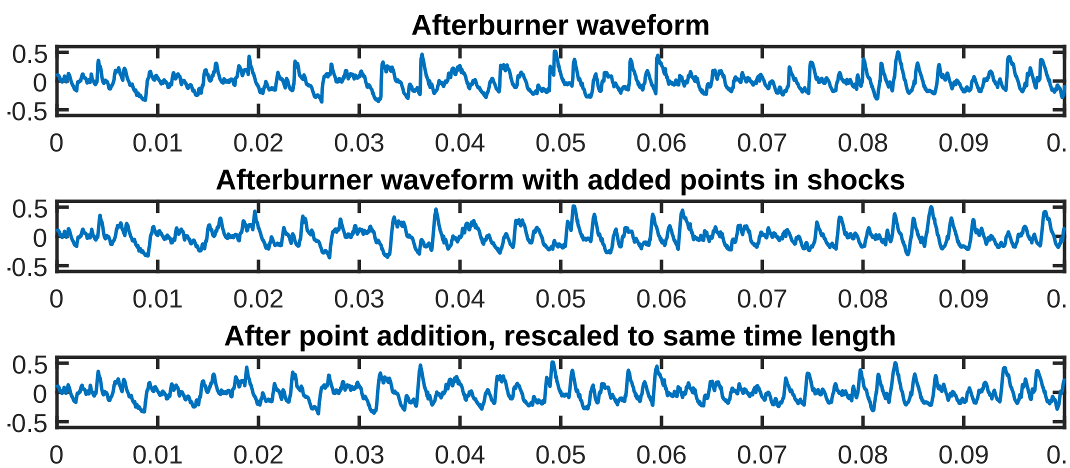

Figure 10: Time series snapshot before (top) and after (middle) "slowing" shocks, and after subsequent resampling (bottom).

4. DISCUSSION

A crackling afterburner waveform has been altered in order to create five daughter waveforms (re-phased, T_p, T_d, $\int_L (\partial p/\partial t)dt$, and slowed shocks). Re-phasing results in $Sk\{p\} = 0$ and $Sk\{\partial p/\partial t\} = 0$, and a non-crackling sound quality.

Transformation in pressure (T_p) reduces $Sk\{p\}$ but does not significantly reduce $Sk\{\partial p/\partial t\}$. The resultant signal crackles with little change in sound quality from the original. Thus, this experiment is strongly supportive of the Reichman/Gee crackle criterion over that of Ffowcs Williams because the first predicts crackle, while the second predicts no crackle.

When the $\partial p/\partial t$ is instead transformed (by T_d), both $Sk\{p\}$ and $Sk\{\partial p/\partial t\}$ are reduced in magnitude, with $Sk\{\partial p/\partial t\} \approx 0$. Because both the pressure PDF and the derivative PDF have been made more Gaussian by the transformation, both crackle prediction criteria agree that this signal should not exhibit crackle, and, indeed, it does not. Thus, this portion of the experiment is not independently conclusive as to which process causes the removal of perceived crackle. It is consistent, however, in connection with the pressure transformation experiment, with the importance of the derivative PDF rather than the pressure PDF: deliberate modification of the derivative PDF removed crackle, while deliberate modification of the pressure PDF did not.

Finally, when points are linearly interpolated into the shocks, thus slowing the rate of rise, the skewness of the pressure time series is almost identical to that of the original series. The resulting $Sk\{p\}$ leads to a crackle prediction by the Ffowcs Williams criterion, while the reduced $Sk\{\partial p/\partial t\}$ leads to a prediction of no crackle via the Reichman/Gee criterion. The non-crackling sound quality of this signal is highly supportive of the Reichman/Gee criterion over the Ffowcs Williams criterion as a crackle predictor.

All of the transformations except re-phasing altered the spectrum. While re-phasing accomplished crackle removal with no spectral change, in both of the other transformations that led to crackle removal there was some spectral alteration. For T_p, the spectral changes are minimal, and no significant change in crackle content is noted. When T_d was applied to the derivative and the resultant transformed derivative reintegrated there was an increase in low frequencies below 60 Hz, however, the increased level in this regime was still lower than the levels seen in the peak frequency region and it seems unlikely that the observed reduction in crackle can be accounted for by an increase in masking or low frequency noise. Instead, it seems that the increase in low-frequency noise is merely an artifact of the reintegration process. Reintegration, applied by itself to the untransformed $\partial p/\partial t$, does not remove crackle, but acts as a high-pass filter. When shocks are "slowed", there is a decrease in high frequency spectral content. This effect is directly attributable to the attenuation of the rise rate of shocks and is not merely an artifact of elongation. While crackle is not identifiable using the spectrum alone, the spectral impact of waveform modifications that affect crackle may be visible upon comparison. It is important to track and evaluate these changes to rigorously evaluate their significance and make reasonable decisions about future directions of research.

5. CONCLUSIONS

Waveform transformations have been used to compare two criteria for crackle. The two metrics discussed each quantify physical characteristics of a waveform that have been suggested as influencing perceptual characteristics. Targeted modification of a crackling waveform has enabled the evaluation of these purported associations. The relationship between the observed sound quality and the two criteria is summarized in Table 2. For these waveforms, the Ffowcs Williams criterion does not predict a crackling sound quality while the Reichman/Gee criterion does. Further studies should consider the use of modified jet noise waveforms as important tools in characterizing human response to jet noise, determining its causes and mitigating its effects. The Reichman/Gee criterion ought to be the subject of further investigation and development designed to determine an optimal physical statistical predictor of a crackling sound quality in jets or other similar noise sources.

Table 2: Crackling (red) or non-crackling (blue) sound quality organized by Skewness magnitude of the pressure and derivative time-series.

	$Sk(p) > 0.4$	$Sk(p) < 0.3$
$Sk(dp/dt) \geq 5$	original	$T_p\{p\} \rightarrow G$
$Sk(dp/dt) < 5$	shock slowing	rephased
	Gee *et al. (2007)* $T_{Sk}\{G\} \rightarrow Sk$	$T_d\{dp/dt\} \rightarrow G$

Following Ffowcs Williams, some crackle research has assigned primary importance to crackle as a source phenomenon producing a skewed pressure distribution. This phenomenon is interesting in its own right. However, the pressure skewness of a jet noise waveform has been shown to have minimal impact on sound quality and characterizing it as "crackle" does a disservice to the advancement of research on both topics by conflating an inaudible distributional characteristic with an audible sound quality with direct and noticeable human impacts.

ACKNOWLEDGMENTS

Portions of this work were performed at Brigham Young University, ERDC-CERL as an ORISE Fellow, and at Purdue University.

REFERENCES

[1] J. E. Ffowcs Williams, J. Simson, V. J. Virchis, "'Crackle': an annoying component of jet noise", *J. Fluid Mech.* **71**, 251-271 (1975).

[2] K. L. Gee, V. W. Sparrow, A. A. Atchley, T. B. Gabrielson, "On the perception of crackle in high-amplitude jet noise", *AIAA J.* **45**, 593-598 (2007).

[3] S. A. McInerny and S. M. Olcmen, "High-intensity rocket noise: Nonlinear propagation, atmospheric absorption, and characterization" *J. Acoust. Soc. Am.*, **117**, 578-591 (2005).

[4] K. L. Gee, T. B. Neilsen, A. T. Wall, J. M. Downing, M. M. James, R. L. McKinley, "Propagation of crackle-containing jet noise from high-performance engines", *Noise Control Eng. J.* **64**, 1-12 (2016).

[5] B. O. Reichman, M. B. Muhlestein, K. L. Gee, T. B. Neilsen, D. C. Thomas, "Evolution of the derivative skewness for nonlinearly propagating waves", *J. Acoust. Soc. Am.*, **139**, 1390-1403 (2016).

[6] K. L. Gee, S. Hales Swift, V. W. Sparrow, K. J. Plotkin, J. Micah Downing, "On the potential limitations of conventional sound metrics in quantifying perception of nonlinearly propagated noise", *J. Acoust. Soc. Am.*, **121**, EL1-EL7 (2007).

[7] McInerny, S.A., “Launch vehicle acoustics. II-Statistics of the time domain data”, *J. Aircraft*, **33**, 518-523 (1996).

APPENDIX A: MATLAB CODE FOR THE CDF TRANSFORMATION MAPPING AND THE INTERPOLATION OF POINTS INTO THE SHOCKS

Figure 11: Code for tranforming the time series to have a Gaussian PDF

```
function output=spec_cdf2cdf(waveform)
%% spec_cdf2cdf accepts an array input and creates a monotonic nonlinear
 % transformation based on the cumulative distribution function of the
 % input variable. It then maps the one variable to a Gausian distribution.
try % gives the Gaussian contribution
    trace1=norminv(0.001:.0001:.999,0,1)*std(waveform);
catch
    trace1=-sqrt(2)*erfcinv(2*[0.001:.0001:.999])*std(waveform);
end
trace2=waveform(1:64000); % source for transformation generation
trace3=waveform; % waveform to be transformed

[B1,Ix1]=sort(trace1,'ascend');
[B2,Ix2]=sort(trace2,'ascend');
n1=[1:length(Ix1)]/length(Ix1);
n2=[1:length(Ix2)]/length(Ix2);
%% Remove any repeated values that would lead to discontinuous transform
[fix_B1,fix_n1]=remove_repeats(B1,n1);
[fix_B2,fix_n2]=remove_repeats(B2,n2);
%% Interpolate into the Gaussian
intermediate_variable=interp1(fix_B2,fix_n2,trace3,'linear','extrap');
intermediate_variable=fix_domain(intermediate_variable,n1);
output=interp1(n1,B1,intermediate_variable,'linear','extrap');
```

Figure 12: Slowing shocks by interpolation of points at instances with high rate of rise

```
for nn=length(y2)-1:-1:1;
    if (y2(nn+1)-y2(nn))>.05;
        num_pts=floor((y2(nn+1)-y2(nn))/.05);
        y2=[y2(1:nn) (1:num_pts)*(y2(nn+1)-y2(nn))/(num_pts+1)+y2(nn) y2(nn+1:length(y2))];
    end
end
```