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Higher-order estimation of active and reactive acoustic intensity

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The phase and amplitude gradient estimator (PAGE) method can be used to increase the bandwidth of complex acoustic intensity estimates obtained with multi-microphone probes. Despite the increased bandwidth, errors arise when using this method, which is based on linear least-squares gradients, in non-planar fields. Examples of non-planar fields include the acoustic near field of a radiating source or near a null in a standing-wave field. The PAGE method can be improved by increasing the number of microphones and using a Taylor expansion to obtain higher-order estimates of center pressure, pressure amplitude gradient, and phase gradient. For one-dimensional active intensity in a simulated monopole field, a four-microphone probe is shown to converge to less than 0.2 dB error at a closer distance than a two-microphone probe with the same inter-microphone spacing. For reactive intensity in a standing wave field, increasing the number of microphones improves the bandwidth, and applying a higher-order method to traditional reactive intensity estimation outperforms higher-order PAGE.

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1. INTRODUCTION

Obtaining accurate estimates of acoustic intensity is vital for intensity-based sound power measurements, as well as source characterization and location. A method for estimating intensity using two microphones and their cross-spectra was developed in the 1970s and is still in use today.[1-3] This method, referred to in this article as the traditional method, has a limited bandwidth highly dependent on microphone spacing. At high frequencies, when the distance between the microphones becomes larger compared to a wavelength, the method has inherent bias errors due to inaccuracies in the finite-difference and finite-sum formulas.[4] On the other hand, phase mismatch of non-ideal microphones causes error when the microphone spacing is small compared to a wavelength, at low frequencies. These two constraints limit the estimation bandwidth of a two-microphone intensity probe.

In order to overcome the high-frequency bias errors and extend estimation bandwidth, the Phase and Amplitude Gradient Estimator method (PAGE) has been developed.[5] By separating the frequency-dependent complex pressures into amplitude and phase, calculations of active and reactive intensity take a new form. The PAGE method has extended the bandwidth of plane wave measurements by at least an order of magnitude for broadband sources.[6]

For an ideal plane wave, the PAGE method estimation of active and reactive intensity has zero bias errors at higher frequencies.[7] Thus, the remaining error is the low-frequency phase mismatch error, which occurs as the probe spacing is small relative to a wavelength. Therefore, probes with larger microphone separation are preferred over probes with smaller separation. However, with a larger probe, it can be harder for the PAGE method to function accurately in non-planar fields, such as a monopole field or a standing wave field. In this work, we explore how the use of additional microphones and higher-order estimates of the first derivatives can improve intensity estimation accuracy in these high-curvature fields.

2. METHODOLOGY

The multiple-microphone approach to estimating intensity has been explored by Cazzalato and Hansen[8] and Pascal and Li.[9] Under these formulations, gradients are estimated using a least-squares method across the microphones. This traditional method can be extended with the PAGE formulation resulting in a process referred to as least-squares PAGE.

An overview of the traditional and PAGE methods is given here. The general formulas for frequency-domain calculations of active and reactive intensity are

$$\boldsymbol{I} = \frac{1}{2} Re\{p\boldsymbol{u}^*\} \tag{1}$$

$$\boldsymbol{J} = \frac{1}{2} Im\{p\boldsymbol{u}^*\} \tag{2}$$

where p is the complex pressure at the center of the probe, $\boldsymbol{u}$ is the complex particle velocity, and $*$ denotes a complex conjugate. In the traditional method, p is either obtained from a microphone at the center of the probe or estimated as a weighted average from multiple microphones at surrounding locations. These weights are determined for the probe configuration either according to a least-squares estimation scheme or by using the higher-order method that will be explained later in this article. The traditional method estimates particle velocity using Euler's equation,

$$\boldsymbol{u}^{TRAD} = \frac{j}{\rho_0 \omega} \widehat{\nabla p}, \tag{3}$$

where ρ_0 is the ambient air density, ω is the angular frequency, and an overhat indicates that the quantity is estimated. The estimated gradient of the complex pressure, $\widehat{\nabla p}$, comes from a finite-difference between the probe microphones. In the traditional method, the complex forms of p and $\boldsymbol{u}^{TRAD}$ are used in Eqs. (1) and (2) to estimate active and reactive intensity.

The PAGE method, on the other hand, treats complex pressure not in terms of real and imaginary parts, but as amplitude and phase, where $p = Pe^{-j\phi}$. The amplitude of the pressure at the center of the probe, P, is either measured by a microphone at the center of the probe or estimated as a weighted average of the amplitudes of the outer microphones. The PAGE equations for active and reactive intensity are[5]

$$\boldsymbol{I}^{PAGE} = \frac{\hat{P}^2 \widehat{\nabla\phi}}{2\rho_0\omega} \tag{4}$$

and

$$\boldsymbol{J}^{PAGE} = -\frac{\hat{P}\,\widehat{\nabla P}}{2\rho_0\omega}. \tag{5}$$

The estimate of the gradient of the phase across the probe, $\widehat{\nabla\phi}$, is obtained via the phase of the transfer functions between microphone pairs. In a plane wave field, the PAGE formulation for active and reactive intensity has zero bias errors up to the spatial Nyquist frequency. If the source is broadband, and there is sufficient coherence between the microphones, a transfer function's phase can be unwrapped, allowing an estimate of active intensity to be accurate above the spatial Nyquist frequency. The success of the PAGE method in extending the bandwidth of reliable intensity estimates has been shown for propagating (active) sound fields.[6] However, performance of the PAGE method in standing wave (reactive) sound fields has been relatively unexplored.

To hopefully obtain more accurate intensity estimates in reactive sound fields, the higher-order PAGE method has been developed, following the work of Jensen.[10] This allows for higher-order estimation of $\hat{P}$, $\widehat{\nabla P}$, and $\widehat{\nabla\phi}$, which are needed for $\boldsymbol{I}^{\text{PAGE}}$ and $\boldsymbol{J}^{\text{PAGE}}$. In his work, Jensen developed a method to obtain higher-order estimates of a function and its derivatives using an arbitrary grid of measurement points. Following Jensen's notation, the function f (which could represent either P or ϕ) is sampled at several measurement positions each as g_i, where i is the index of the grid position. Each value of g_i can be expressed as a linear combination of f and its derivatives evaluated at the origin, according to the Taylor series expansion. In two dimensions, g_i can be expressed as

$$\begin{aligned} g_i &\equiv f(\alpha_i, \beta_i) \\ &= f(0,0) + \left(\alpha_i \frac{\partial}{\partial x} + \beta_i \frac{\partial}{\partial y}\right) f(0,0) + \cdots + \left(\alpha_i \frac{\partial}{\partial x} + \beta_i \frac{\partial}{\partial y}\right)^m f(0,0)\frac{1}{m!} - \delta_i \end{aligned} \tag{6}$$

where α_i and β_i are the x and y coordinates, respectively, of grid position i relative to the origin, m is the desired accuracy order, and δ_i is an error constant of order $m+1$. The origin can be defined arbitrarily, although it is usually placed at the center of the probe. Equation (6) can be written in matrix form to include all the grid positions as

$$T_m F_m = G_m + \varepsilon_m. \tag{7}$$

The vector F_m contains the derivatives in the Taylor series (along with their appropriate factorial coefficients):

$$F_m = \left(f \;\; \frac{\partial f}{\partial x} \cdots \frac{1}{i!}\frac{\partial^i f}{\partial x^{i-j}\partial y^j} \cdots \frac{1}{m!}\frac{\partial^m f}{\partial y^m}\right)^T, \tag{8}$$

where T denotes a vector transpose. The matrix T_m contains a row for each grid position, where each row consists of the grid coordinates raised to appropriate powers matching the Taylor series derivatives in F_m,

$$T_m = \begin{bmatrix} 1 & \alpha_1 & \beta_1 & {\alpha_1}^2 & 2\alpha_1\beta_1 & \cdots & {\beta_1}^m \\ 1 & \alpha_2 & \beta_2 & {\alpha_2}^2 & 2\alpha_2\beta_2 & \cdots & {\beta_2}^m \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_{n_m} & \beta_{n_m} & {\alpha_{n_m}}^2 & 2\alpha_{n_m}\beta_{n_m} & \cdots & {\beta_{n_m}}^m \end{bmatrix}, \tag{9}$$

and n_m is the number of grid positions. If F_m includes all the derivatives in the two-dimensional Taylor series, then the number of grid positions needed to achieve the desired accuracy order, m, is

$$n_m = (m+1)(m+2)/2. \tag{10}$$

The right side of Eq. (7) consists of a vector of the grid measurements

$$G_m = \left(g_1 \ g_2 \ \cdots \ g_{n_m}\right)^T \tag{11}$$

and an error vector

$$\varepsilon_m = \left(\delta_1 \ \delta_2 \ \cdots \ \delta_{n_m}\right)^T. \tag{12}$$

Estimates of the function f and its derivatives, evaluated at the origin, are found by solving Eq. (7) for F_m,

$$F_m = {T_m}^{-1} G_m + E_m \tag{13}$$

where E_m is an unknown error vector of order $m+1$,

$$E_m = {T_m}^{-1}\varepsilon_m. \tag{14}$$

Because the error vector E_m is unknown, the accuracy order of estimates in Eq. (13) is m, the order of the terms used in the Taylor series. For any probe geometry, the rows of ${T_m}^{-1}$ give sets of finite difference coefficients that, when combined with the quantities measured at the grid locations, calculate the function f and its derivatives evaluated at the origin. Once these finite difference coefficients are obtained, they can be reused for any data set, provided the probe geometry and estimation location remain constant.

In order to estimate active and reactive intensity using the PAGE method [Eqs. (4) – (5)], $\hat{P}$, $\widehat{\nabla P}$, and $\widehat{\nabla \phi}$ are needed. $\hat{P}$ and $\widehat{\nabla P}$ can be obtained by following the above procedure with measurements of P at each microphone making up G_m. Since phase is a quantity wrapped between $-\pi$ and π, measurements of ϕ at each microphone are not absolute phases, and cannot be used to accurately obtain $\widehat{\nabla \phi}$. To circumvent this issue, one microphone is chosen as a reference microphone, and relative phases are obtained at each microphone by unwrapping the phase of the transfer function relative to the reference microphone. These relative phases can be used in G_m to obtain $\widehat{\nabla \phi}$. It is often preferable to calculate the transfer function relative to a center microphone to minimize distances between microphone pairs, as this minimizes unwrapping errors.

By choosing which derivatives to include in F_m, as well as the corresponding coordinates in T_m, the higher-order method for estimating f can be employed for one, two, or three dimensions. However, the matrix T_m must be non-singular or the inverse cannot be obtained. For example, if the three-dimensional

Taylor series is used, the grid points cannot lie in a line or in a plane. The impact of including higher-order estimates of $\hat{P}$, $\widehat{\nabla P}$, and $\widehat{\nabla\phi}$ in the PAGE method are now discussed for the case of a one-dimensional probe near a monopole and in a standing wave field, followed by an examination of how to apply this technique for two-dimensional probes.

3. MONOPOLE

In order to understand how higher-order PAGE performs in a field with curvature, we have analyzed the method's performance for active intensity estimation in a monopole field. Figure 1 shows three one-dimensional probes used in this simulated field, consisting of two, four, and six microphones evenly spaced along a line pointing to the source, where d is the distance between microphones and r is the distance from the center of the probe to the source.

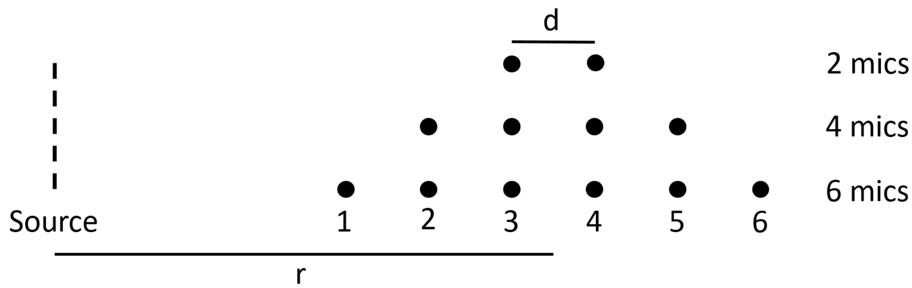

Figure 1. Schematic of one-dimensional probes consisting of two, four and six microphones. The distance between microphones is d, and the distance from the center of the probe to the source is r.

PAGE estimation of active intensity [Eq. (4)] relies on $\hat{P}$ and $\widehat{\nabla\phi}$, which are both estimated using the higher-order method. To illustrate how the higher-order method functions in this case, the following three equations show how $\widehat{\nabla\phi}$ is obtained using two, four, and six microphones, respectively:

$$\widehat{\nabla\phi} = \frac{1}{d}(-\phi_3 + \phi_4) \tag{15}$$

$$\widehat{\nabla\phi} = \frac{1}{d}\left(\frac{1}{24}\phi_2 - \frac{9}{8}\phi_3 + \frac{9}{8}\phi_4 - \frac{1}{24}\phi_5\right) \tag{16}$$

$$\widehat{\nabla\phi} = \frac{1}{d}\left(-\frac{3}{640}\phi_1 + \frac{25}{384}\phi_2 - \frac{75}{64}\phi_3 + \frac{75}{64}\phi_4 - \frac{25}{384}\phi_5 + \frac{3}{640}\phi_6\right) \tag{17}$$

In practice, each phase ϕ is the phase of an unwrapped transfer function relative to one of the microphones. Similar equations exist to estimate P with different coefficients, although they are not shown here.

The bias errors in active intensity in a simulated monopole field for two, four and six microphones, shown in Figure 2, illustrate the effect of higher-order PAGE. When the PAGE method with unwrapping is used, these errors are independent of frequency. Each of the probes has large error as one of the outer microphones approaches the source, which happens at a larger value of r/d for more microphones due to the larger probe size. However, since having more microphones allows for better sampling of the sound field, the bias error converges to zero faster. As the probe moves away from the source, the 4-microphone probe is the first to achieve less than 0.2 dB error at $r/d = 2.15$. The 2-microphone probe does not achieve less than 0.2 dB error until $r/d = 3.31$, making use of the 4-microphone probe advantageous over that range in the near-field. The six-microphone probe achieves less than 0.2 dB error at $r/d = 2.98$, which, due to the large size of the probe, is farther from the source than the four-microphone probe. Thus, the 4-microphone probe is generally preferable, achieving less than 0.2 dB error at the closest distance to the source.

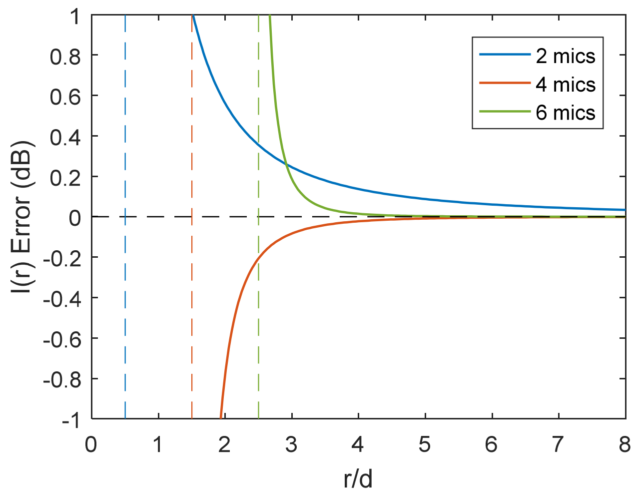

Figure 2. Active intensity error in a simulated monopole field for one-dimensional probes consisting of two, four and six microphones. The three dashed vertical lines correspond to values of r/d for each probe where an outer microphone is at the source.

4. STANDING WAVE

The performance of higher-order PAGE is further evaluated by considering a standing wave field, which is purely reactive. Figure 3(a) shows performance of the higher-order PAGE method reactive intensity estimation for probes consisting of two, four and six microphones in a simulated standing wave versus kd. The probe center for all three probes is placed at a point of maximum reactive intensity, halfway between a pressure node and a pressure antinode. Probes with higher numbers of microphones perform better; however, this benefit is reduced at high values of kd when the probe becomes large enough to span a null. As a purely real field, the complex pressure is either positive or negative, alternating across nulls. Since the PAGE expressions involve only the magnitude of the pressure, the entire field has positive amplitude under this method, interfering with the estimate of center pressure when the probe spans a null. PAGE with two microphones is within 0.2 dB error for $kd < 0.56$, whereas higher-order PAGE with six microphones is within 0.2 dB error for $kd < 0.97$, nearly doubling the frequency range of accurate reactive intensity estimation.

To compare with higher-order PAGE, the higher-order formulation in Sec. 2 can also be used to develop a higher-order traditional method. The higher-order traditional calculation uses the methods explained in Sec. 2 to estimate the complex pressure and its gradient, rather than estimating pressure amplitude and phase gradients as in the PAGE method. The performance of the higher-order traditional method for estimating reactive intensity in the same standing wave field is shown in Figure 3(b). The two-microphone traditional method result is identical to the two-microphone PAGE calculation because both calculations depend on a difference of autospectra.[7] However, the higher-order traditional method outperforms higher-order PAGE for reactive intensity in a standing wave field, with less than 0.2 dB error for $kd < 1.54$ for the six microphone probe, which is closer to the spatial Nyquist limit of $kd = \pi$ than the higher-order PAGE estimate. By using the higher-order traditional method with six microphones, the frequency range of accurate reactive intensity estimation in a standing wave field is nearly three times that of using two microphones.

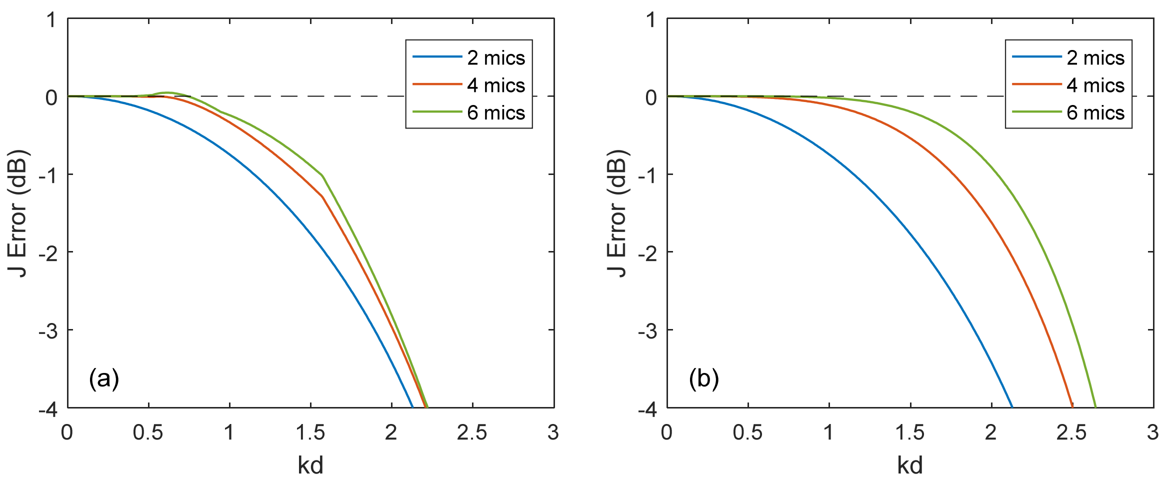

Figure 3. Reactive intensity error in a simulated standing wave field using (a) higher-order PAGE and (b) higher-order traditional. The probe center is halfway between a pressure node and a pressure antinode.

5. TWO-DIMENSIONAL PROBE

The higher-order method can be extended to two and three-dimensional probes. As an example, Fig. 4 shows a two-dimensional probe used in several applications.[6,11,12] This probe consists of microphones in an equilateral triangle with a fourth microphone at the centroid.

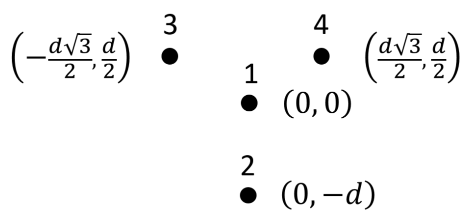

Figure 4. A two-dimensional intensity probe. Each microphone is labeled with a microphone number and its (x, y) coordinates, where d is the distance from each outer microphone to the center microphone.

For this probe, intensity estimation differs between the least-squares and higher-order PAGE methods. In both methods, pressure is obtained directly by the center microphone. Both methods estimate the x-component of the gradients using a difference of microphones 4 and 3. The methods differ in estimating the y-component of the gradients. The least-squares estimate of the y-component of $\nabla\phi$ is

$$\frac{\partial\phi}{\partial y} = \frac{1}{d}\left(-\frac{2}{3}\phi_2 + \frac{1}{3}\phi_3 + \frac{1}{3}\phi_4\right). \tag{18}$$

Again, in practice each value of ϕ is the unwrapped phase of a transfer function relative to one of the microphones. The higher-order estimate is

$$\frac{\partial\phi}{\partial y} = \frac{1}{d}\left(-\phi_1 - \frac{1}{3}\phi_2 + \frac{2}{3}\phi_3 + \frac{2}{3}\phi_4\right). \tag{19}$$

The higher-order PAGE estimate shown in Eq. (19) uses all four microphones instead of three, providing a higher-order of accuracy. Additionally, the higher-order method centers the y component of the gradient correctly at the origin. The least-squares method centers the estimate at $(0, -d/4)$, which means that the pressure and the particle velocity estimate are not collocated, causing error in the intensity estimate. The usefulness of higher-order PAGE in extending the bandwidth needs to be experimentally verified.

6. CONCLUSIONS

The method described in this paper can be used to obtain higher-order intensity estimates from probes in one, two, and three-dimensions, given a sufficient number of microphones. Both the traditional and the PAGE methods can benefit, in certain cases, from the higher-order estimates presented in this paper. The higher-order processing can be utilized with currently existing probes, as well as guide the creation of new probe designs.

The higher-order method can improve accuracy of active intensity estimation, however, the efficacy of higher-order estimation is of particular interest in reactive sound fields. In a monopole case, estimation of active intensity using four microphones and the higher-order PAGE method is within 0.2 dB error for distances closer to the monopole than using two microphones with the least-squares PAGE method. For reactive intensity in a standing wave field, using six microphones with the higher-order PAGE method is more accurate, but using six microphones with the higher-order traditional method extends the frequency range over which error is less than 0.2 dB even further, by nearly three times the two-microphone result. Future work may include investigation of more complicated analytical fields, experimental validation of bias errors, and considerations of additional probe designs.

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