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## Acoustics `17 Boston



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### Signal Processing in Acoustics: Paper 4pSPb5

## Coherence-based phase unwrapping for broadband acoustic signals

**Mylan R. Cook, Kent L. Gee, Scott D. Sommerfeldt and Tracianne B. Neilsen**

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A coherence-based method for unwrapping the relative phase between microphones is investigated. For broadband signals, this method has the potential to lead to more accurate intensity vector estimations using the Phase Amplitude and Gradient Estimator (PAGE) method [D. C. Thomas et al., *J. Acoust. Soc. Am.* 137, 3366-3376 (2015)] Simple unwrapping methods function by detecting phase jumps above a threshold value, which works well for frequencies associated with high signal coherence. However, since unwrapping for these methods is triggered by only one previous frequency data point, frequency ranges of low coherence often contain unwrapping errors. By including coherence in a phase unwrapping algorithm, these errors can be avoided. Ranges of relatively low coherence are given less weight in phase unwrapping and are checked for unwrapping errors. For broadband signals with continuous relative phase, using both the coherence and multiple data points to unwrap, frequencies associated with low coherence result in fewer unwrapping errors. Phase values for jet noise data with low coherence ( $<0.1$ ) have been successfully unwrapped using this method, and have resulted in more reliable PAGE intensity estimates. This paper also investigates unwrapping in interference nulls produced by coherent, radiating sources.



## 1. INTRODUCTION

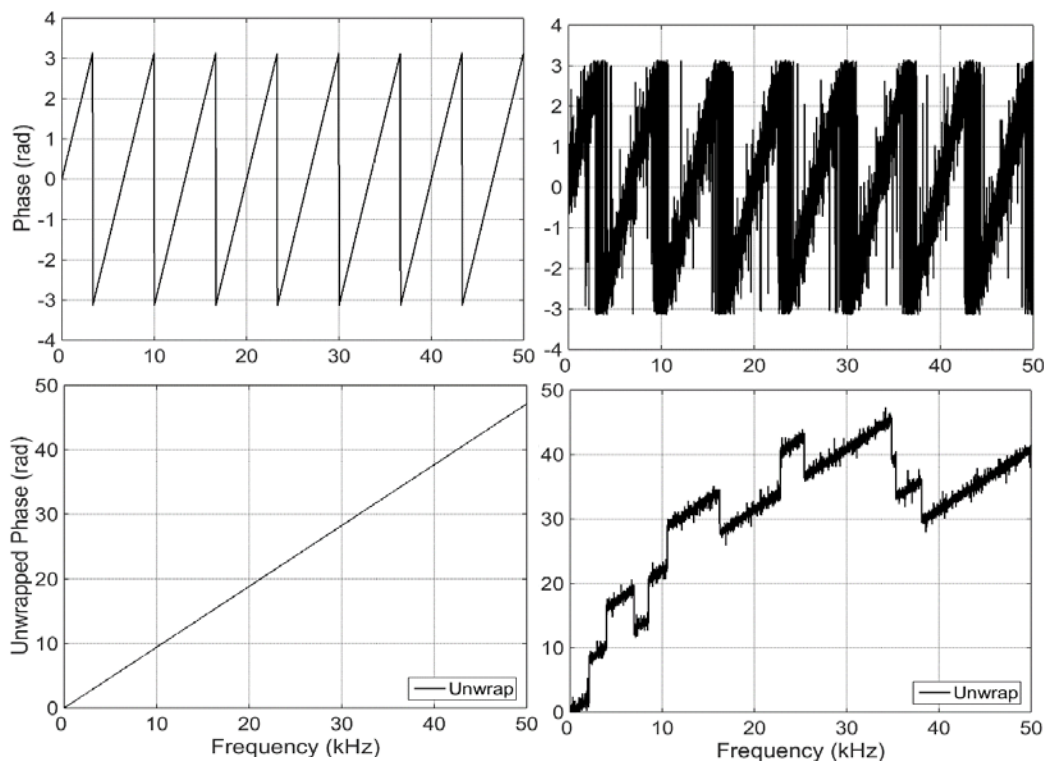
Complex-valued functions are often separated into real and imaginary values. For functions that demonstrate periodicity, a separation into magnitude and phase values is often more useful. As a simple example, consider a unit phasor rotating in the complex plane as a function of frequency  $e^{j\omega t}$ . The real part is represented by a cosine wave, and the imaginary part by a sine wave. The magnitude is constant, and the phase is piecewise linear, an example of which can be seen in the upper left plot of Figure 1. These phase values are known as the *wrapped phase*, because they are limited to an interval of  $2\pi$  radians. The values are aliased or wrapped, giving only the relative phase angle. The *absolute phase* gives the total angle—including complete cycles of the phasor as a function of frequency—instead of the current angle, which in some situations is necessary. The wrapped phase values can be shifted by  $2\pi$  radian intervals, a process known as unwrapping, to obtain a continuous absolute phase relation, as seen in the lower left plot of Figure 1.

Phase unwrapping is not always so straightforward, most especially when dealing with noisy data. Unwrapping is a common problem in such fields as signal processing, image processing, and optics<sup>1,2,3</sup>. Many acoustic variables are complex valued in frequency space—obtainable by using a Fourier transform with time domain data. Phase values at high frequencies are often aliased, making unwrapping useful for applications in areas such as beamforming, holography, and sound source localization<sup>4,5</sup>.

Phase unwrapping uses a *transfer function* which, as the name indicates, gives the transformation of the complex-valued pressure recorded at one microphone location relative to that of a second microphone. The complex-valued pressure quantity  $\tilde{p}$  can instead be split into a magnitude  $P$  and a phase  $\phi$ <sup>6</sup>,

$$\tilde{p}(\omega) = P(\omega)e^{j\phi(\omega)} \quad (1)$$

where  $\phi$  gives the relative shift in waveforms of the same frequency between  $-\pi$  and  $\pi$  radians as measured by the two microphones. For a plane wave propagating in line with both microphones, the phase changes



**Figure 1.** Examples of numerical wrapped phase values (top) and the resulting unwrapped phase using MATLAB's function<sup>8</sup> "unwrap" (bottom). The noiseless case (left) is unwrapped perfectly, while the noisy case (right) contains obvious unwrapping errors.

linearly with increasing frequency. When the frequency is such that the microphones are separated by half of a wavelength, known as the *spatial Nyquist frequency*, the phase can thereafter wrap and become aliased.

Unwrapped phase values can be useful in a variety of situations. In particular, the Phase and Amplitude Gradient Estimator (PAGE) method<sup>7</sup> uses microphone pressure differences and the gradient of the transfer function's phase to obtain active acoustic intensity  $\hat{\mathbf{I}}_a$  estimates:

$$\hat{\mathbf{I}}_a(\omega) = \frac{1}{\omega \rho_0} P^2 \nabla \phi. \quad (2)$$

In order to properly obtain the gradient of the phase, represented by  $\nabla \phi$  in Eq. (2), phase values must be unwrapped properly. Using the traditional method, the microphone spacing limits the usable bandwidth of results. By using phase unwrapping, the PAGE method can find accurate acoustic intensity values well beyond the spatial Nyquist frequency<sup>8</sup>. Results of this process are in Section 4.

## 2. BACKGROUND

Phase unwrapping can be a difficult challenge in signal processing. There is not necessarily a clearly correct answer in every situation. Even when the trend can be seen visually, unwrapping algorithms often struggle. Even the most appropriate unwrapping can result in erratic jumps, such as when the phase exhibits multiple shifts of approximately  $\pi$  radians in a narrow frequency range. Phase values that are linear in nature, such as plane waves, are simpler to unwrap than rapidly-varying phase values. Li and Levinson<sup>9</sup> show that for linear phase, a high signal-to-noise ratio in the low frequencies—where the phase is not aliased—leads to the greatest chance of success. At 0 Hz the phase value is necessarily zero. Each frequency bin with its phase value is a data point  $(f_k, \phi_k)$ . The goal for unwrapping is to join these points in such a manner as to produce a continuous phase trend. The points are unwrapped by shifting points by any integer multiple of  $2\pi$  radians:

$$\bar{\phi}_k = \phi_k + 2\pi n, n = \{0, \pm 1, \pm 2, \dots\}. \quad (3)$$

In Eq. (3),  $\phi_k$  is the wrapped phase (between  $-\pi$  and  $\pi$  radians) and  $\bar{\phi}_k$  is the unwrapped phase, which is not restricted to a certain range. There are a number of different methods for performing unwrapping, each with its own benefits and limitations.

### A. SIMPLE UNWRAPPING METHOD

Common unwrapping methods, such as MATLAB's *unwrap* function, are conceptually very simple<sup>10</sup>. The unwrapping is performed point-by-point in order of increasing frequency, and relies only upon the single previous data point. The difference between data points is what triggers unwrapping. A cutoff value  $\gamma_{cut}$  is chosen—typically  $\pi$  radians since the wrapped phase is contained in a  $2\pi$  radian interval. Whenever the difference exceeds the cutoff value, all the following data points are shifted by  $2\pi$  radians:

$$\bar{\phi}_{k+1} = \phi_{k+1} + 2\pi n_{k+1}, n_{k+1} = \begin{cases} n_k + 1 & \text{if } \phi_{k+1} - \phi_k < -\gamma_{cut} \\ n_k - 1 & \text{if } \phi_{k+1} - \phi_k > \gamma_{cut} \\ n_k & \text{otherwise} \end{cases}, n_1 = 0. \quad (4)$$

This ensures that the largest possible phase jump between adjacent points is  $\pi$  radians. This works very well in many circumstances, such as for linearly varying phase values and data with high signal-to-noise ratios; however, many problems can arise. Erroneous phase jumps are often a result of uncorrelated noise between the microphone pair. The algorithm shifts values incorrectly, even when the phase trend is clearly visible to the human eye. An example of this is visible in the right plots of Figure 1.

## B. LEAST-SQUARES METHOD

Unfortunately, phase values do not always vary linearly, and signal-to-noise ratios are not always high. Cusack et al.<sup>11</sup> showed that for two-dimensional phase unwrapping, a modified nearest-neighbor algorithm can mitigate problems caused by noise. Huntley<sup>12</sup> also showed that smoothing improves unwrapping. For one-dimensional phase unwrapping, it is therefore reasonable to use a smoothing technique such as the least-squares method.

A least-squares method can prevent many of the unwrapping errors to which the simple unwrapping method is susceptible. Single points with erratic phase values do not trigger an erroneous unwrapping. An additional parameter is necessary in this case: the number of data points  $N$  to use for the least-squares fit. To unwrap the point  $(f_k, \phi_k)$ , the least-squares method uses the  $N$  previously unwrapped frequency data points  $\{(f_i, \bar{\phi}_i)\}_{i=k-N}^{k-1}$  to obtain the slope  $m_k$  and offset  $b_k$  of the fitted line by way of the least-squares equation:

$$\mathbf{A}_k^T \mathbf{A}_k \mathbf{x}_k = \mathbf{A}_k^T \boldsymbol{\Phi}_k \quad \text{where} \quad \mathbf{A}_k = \begin{bmatrix} f_{k-N} & 1 \\ f_{k-N+1} & 1 \\ f_{k-N+2} & 1 \\ \vdots & \vdots \\ f_{k-1} & 1 \end{bmatrix}, \quad \boldsymbol{\Phi}_k = \begin{bmatrix} \bar{\phi}_{k-N} \\ \bar{\phi}_{k-N+1} \\ \bar{\phi}_{k-N+2} \\ \vdots \\ \bar{\phi}_{k-1} \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} m_k \\ b_k \end{bmatrix} \quad (5)$$

The predicted unwrapped phase value  $\tilde{\phi}_k$  for frequency  $f_k$  is then  $\tilde{\phi}_k = m_k f_k + b_k$ . The unwrapped phase value  $\bar{\phi}_k$  is found by shifting  $\phi_k$  by  $2\pi$  intervals to be as close to  $\tilde{\phi}_k$  as possible, i.e.  $n$  is chosen such that  $|\tilde{\phi}_k - \bar{\phi}_k| = |(m_k f_k + b_k) - (\phi_k + 2\pi n)| < \pi$ . This is likewise performed for each point in order of increasing frequency, where  $\phi = 0$  at  $f = 0$ .

The least-squares method can prevent erroneous jumps in certain situations. In frequency ranges of excessive noise, where many phase values are erratic, this method can still give a poorly unwrapped phase. Though the phase itself is expected to be inaccurate in these ranges, unwrapping errors can also shift the phase values for all higher frequencies, hence the need for a better phase unwrapping algorithm.

## 3. COHERENCE-BASED APPROACH

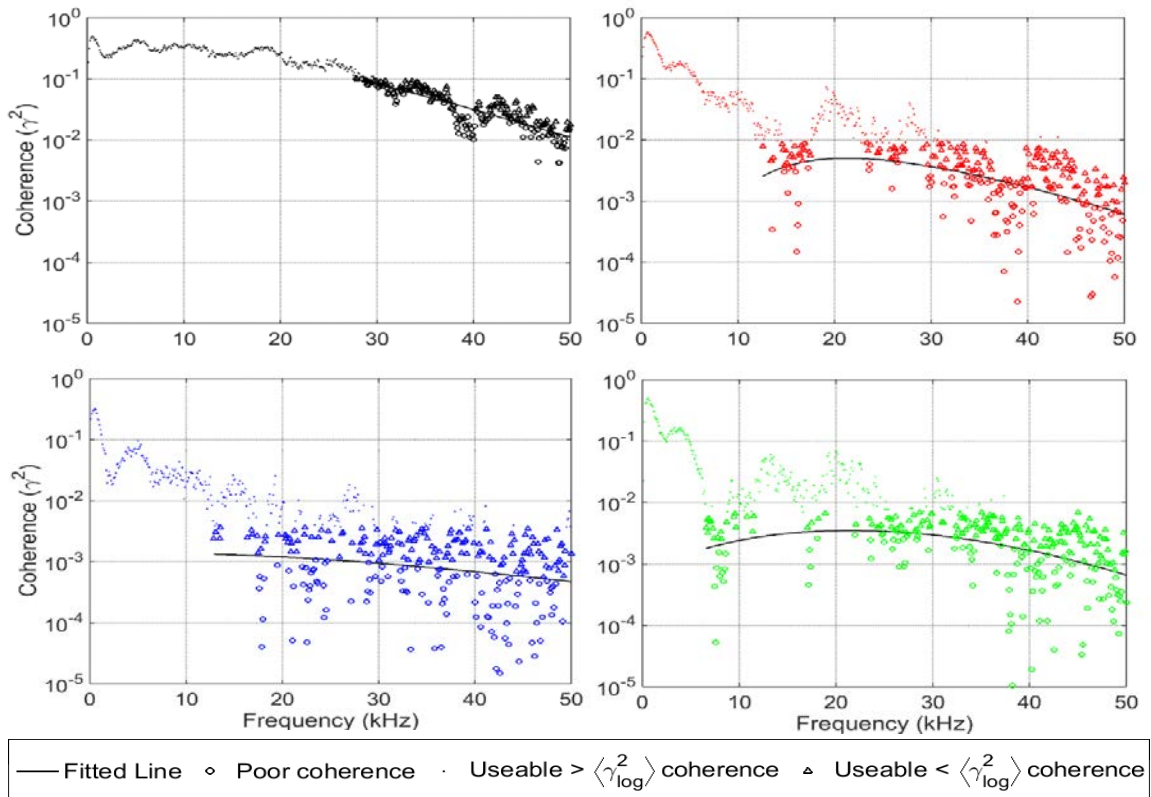
Using a coherence-based approach, many unwrapping errors can be avoided, because inaccurate unwrapping usually occurs in frequencies of poor coherence. The algorithm described here shares many similarities with the least-squares approach. The main difference is, naturally, the use of the coherence in order to accomplish unwrapping. Coherence  $\gamma_{ij}^2(f_k)$  is a frequency-domain measure of the similarity of the signals received by microphones  $i$  and  $j$ , with values between zero and one defined as:

$$\gamma_k^2 = \gamma_{ij}^2(f_k) = \frac{|G_{ij}(f_k)|^2}{G_{ii}(f_k)G_{jj}(f_k)} \quad (6)$$

The autospectrum of microphone  $i$  is represented as  $G_{ii}$ , while  $G_{ij}$  gives the crossspectrum of microphones  $i$  and  $j$ . Coherence is often shown on a logarithmic scale and is more useful than linear coherence when applied in this unwrapping algorithm due to the fitting explained below.

### A. COHERENCE CLASSIFICATION

In order to use coherence to prevent erroneous unwrapping, which often occurs in ranges of poor coherence, frequency data points must be given a coherence classification or measure. A basic classification is a division into two groups, one of usable coherence and the other of poor coherence. There are many possible ways to make this distinction, for example by picking a coherence threshold value. This is useful in some situations, though the method used here takes a different approach. It is done in the following manner:



**Figure 2.** Classification of usable coherence and poor coherence for four different microphone pairs. The data for these coherence values come from the jet noise data described in section 4 B. The unwrapped phases for these data are seen in Figure 4, with corresponding coloring.

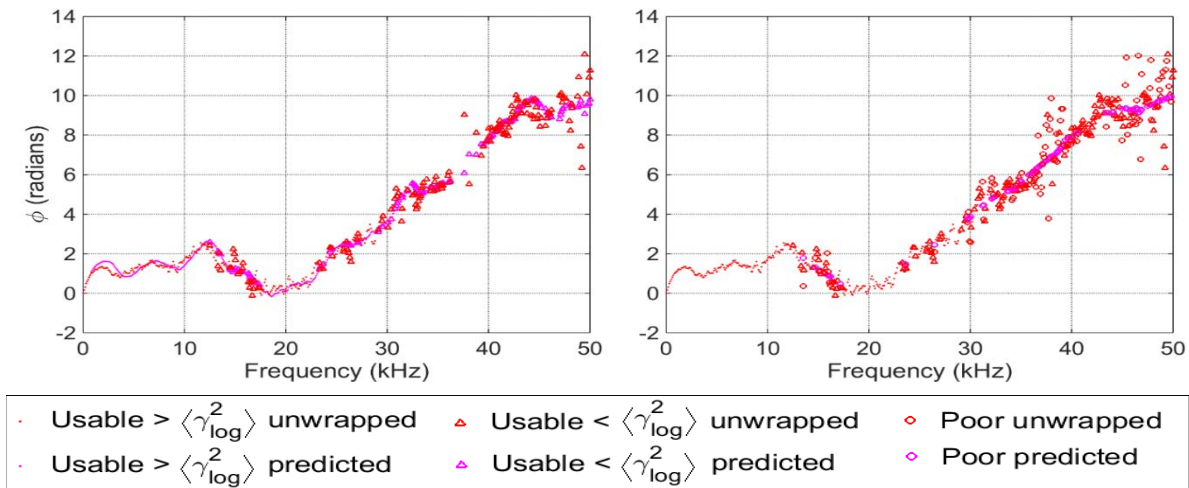
- The average logarithmic coherence is computed as a threshold value  $\langle \gamma_{\log}^2 \rangle = \frac{1}{S} \sum_{k=1}^S \log_{10} \gamma_k^2$ , and all points above this threshold are classified as having usable coherence. Other threshold values can be useful depending on the application.
- A curve is fit to the points below the threshold, using a double exponential model  $c_1 e^{c_2 f_k} + c_3 e^{c_4 f_k}$  where  $c_i$  is some constant. Other fitting models may be used, though the double exponential is versatile enough to fit many different coherence trends.
- Points above the fitted line are classified as having usable coherence, and those below the line as having poor coherence.

This classification ensures that not too many points are marked as poorly coherent. It also ensures that there will not be long frequency ranges with only points of poor coherence. The dips in coherence are found relatively well using this method. For a visual example of fitting to data, see Figure 2.

## B. UNWRAPPING METHOD

After the data points have been classified by their coherence values, the unwrapping is performed using the least-squares approach. The points with usable coherence are first unwrapped independently of those of poor coherence, using the  $N$  usable points lower in frequency. Phase values are shifted in  $2\pi$  intervals so as to be placed as close as possible to the least-squares prediction. The points with poor coherence are not used for unwrapping these points. This ensures that the ranges of poor coherence do not affect the overall phase trend. An example is pictured in the left plot of Figure 3.

In order to unwrap the points of poor coherence, the  $N$  closest points, including both points of lower and higher frequencies, with usable coherence are used in the least-squares approach. An example is pictured in the right plot of Figure 3.

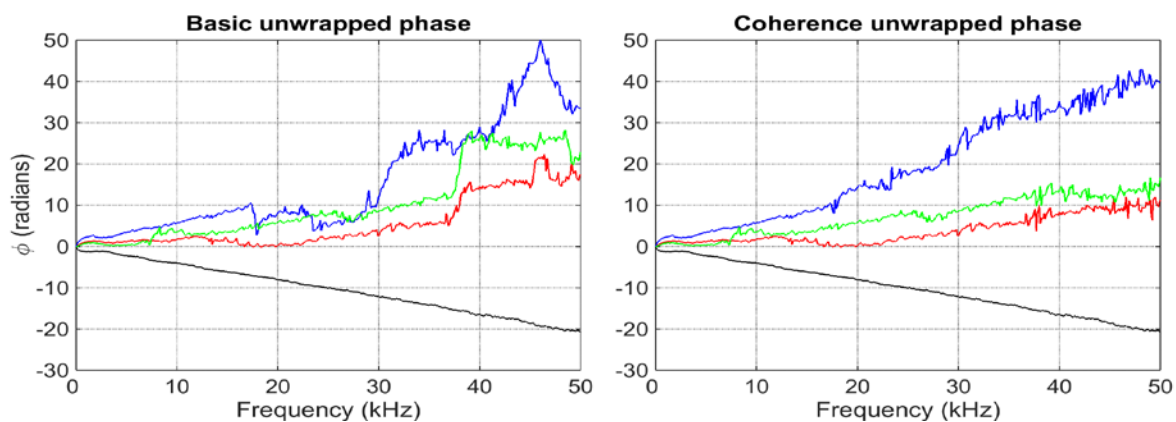


**Figure 3.** The points of usable coherence are unwrapped first using least-squares (left), with  $N=30$  for this case. Then the points of poor coherence are unwrapped to fit the trend (right). As could be expected, the points with poor coherence do not fit as well as the points with usable coherence, just as those above the threshold fit better than those below. These data correspond to the red coherence values seen in Figure 2, and the red line in Figure 4.

The main disadvantage of this approach can be seen when phase values are approximately  $\pi$  radians away from the predicted values. The closest match may be above or below, and this can lead to a jagged-looking unwrapped phase, such as the 38 kHz range in Figure 4. However, an erroneous phase value at  $f = f_k$  does not cause erroneous unwrapping that shifts the phase for  $f > f_k$  as it does using the simple unwrapping method. The phase can be unwrapped *across* the ranges of poor frequency, not necessarily *in* the ranges of poor frequency. This is what is necessary to find the proper phase gradient. The results using this unwrapping method are seen in the right plot of Figure 4.

### C. ALTERNATIVE WEIGHTING METHOD

A variation can be made to this method by using weighted least-squares in place of regular least-squares. Each point  $(f_k, \phi_k)$  is given a weighting  $w_k$ . A simple weighting takes a scaled logarithmic coherence value as the weight. Additionally, points nearer in frequency may be given a larger weighting.



**Figure 4.** The results of basic unwrapping (left) compared to coherence-based unwrapping (right) for four different microphone pairs using jet noise data. The coherence for each set of data is seen in Figure 2 with corresponding colors. The obviously erroneous phase jumps seen using basic unwrapping have been removed by using coherence unwrapping.

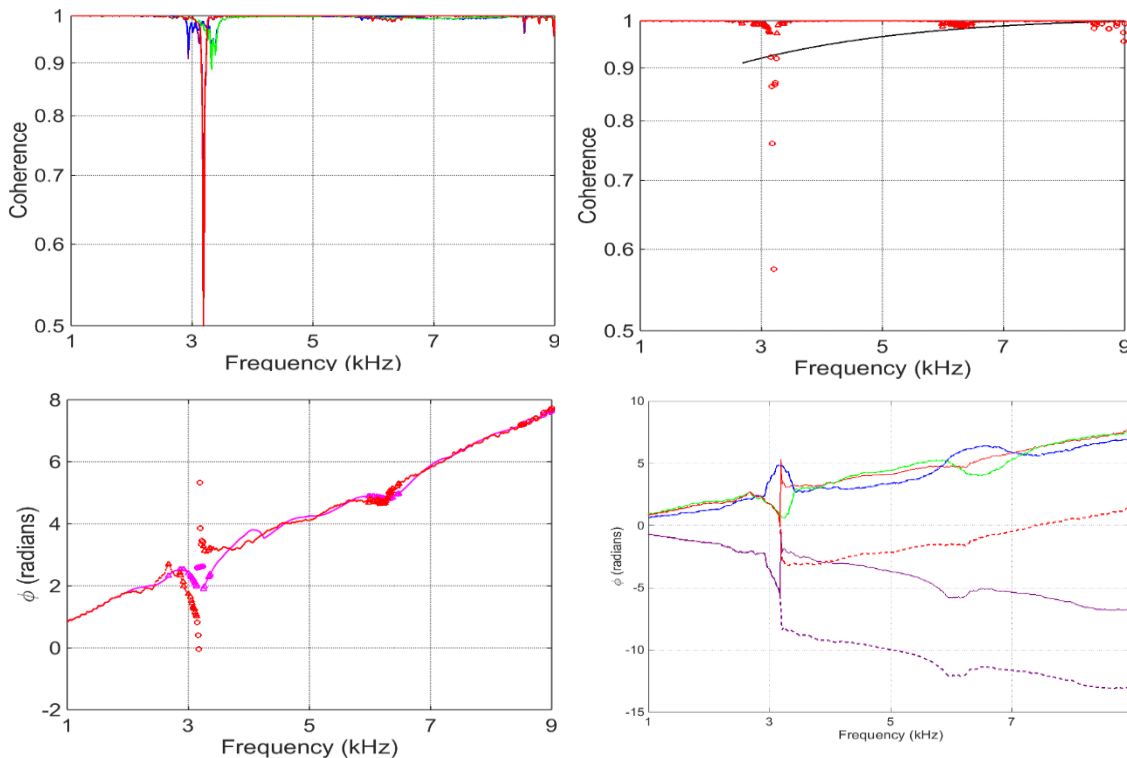
Many different weightings are possible. The weighted least-squares equation is given in Eq. (7), using the same definitions as in Eq. (5):

$$\mathbf{A}_k^T \mathbf{W}_k \mathbf{A}_k \mathbf{x}_k = \mathbf{A}_k^T \mathbf{W}_k \boldsymbol{\Phi}_k \quad \text{where} \quad \mathbf{W}_k = \begin{bmatrix} w_{k-N} & 0 & 0 & \dots & 0 \\ 0 & w_{k-N+1} & 0 & \dots & 0 \\ 0 & 0 & w_{k-N+2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & w_{k-1} \end{bmatrix} \quad (7)$$

The same unwrapping procedure described in the previous section is followed in this variation. The points with poor coherence are not used to unwrap the points of usable coherence. Results are very similar in most cases, but not necessarily identical, especially within frequency ranges of poor coherence.

## 4. EXPERIMENTAL RESULTS

In addition to numerical data, two different data sets have been investigated in great detail with this phase unwrapping algorithm, namely anechoic chamber measurements of a dipole-like radiation field and jet noise. Active acoustic intensity results for each using the coherence-based phase unwrapping algorithm are compared to that using the MATLAB *unwrap* function, using the Phase and Amplitude Gradient Estimator (PAGE) Method<sup>7,13</sup>. As explained previously, this method uses the gradient of the phase and therefore needs accurately unwrapped phase values to produce accurate active acoustic intensity vectors above the spatial Nyquist frequency. The acoustic intensity direction can be greatly impacted by incorrectly unwrapped phase values.



**Figure 5.** For comparison, the results of the unwrapping algorithm for the dipole case are shown. The red lines in each correspond to one another. The dashed lines (bottom right) show the simple unwrapping results, and the solid lines the coherence-based results. See previous figures for detailed explanations.



## A. ANECHOIC EXPERIMENT

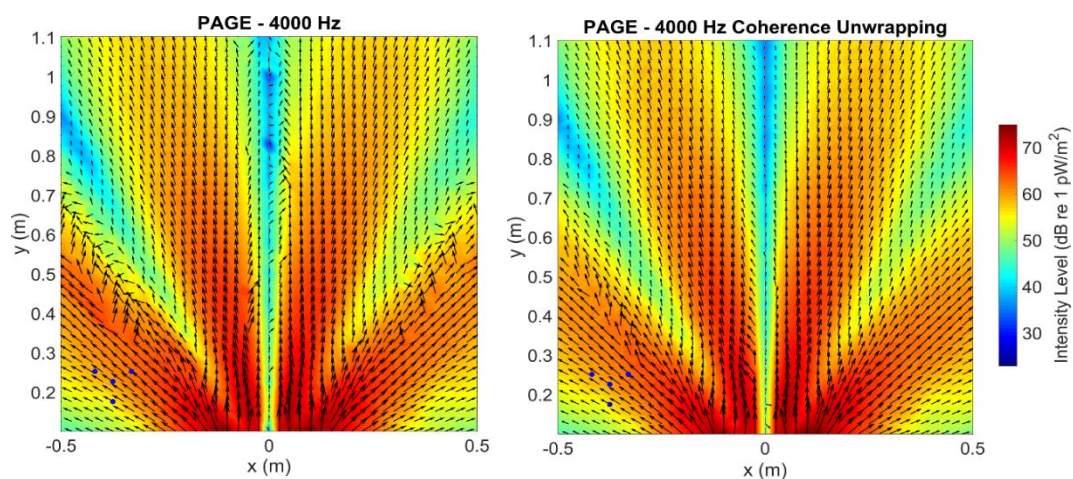
### i. Experimental Setup

Measurements were made in the anechoic chamber at BYU by D. K. Torrie<sup>8</sup> in order to test the efficacy of the PAGE method. A two-dimensional probe consisting of three microphones in an equilateral triangle arrangement around a center microphone was used for the receiver. The microphone spacing is 2 inches. The source consisted of the middle two elements (or one of the middle elements for the monopole case) of a loudspeaker array consisting of four 6.3 cm loudspeakers spaced 17.78 cm apart<sup>8</sup>. For most frequencies, the coherence is very high, exceeding 0.99. However, due to the lobe patterns of a dipole at low frequencies and more complex interference patterns at higher frequencies, the coherence drops markedly at specific frequencies and locations for which one microphone is located in an interference null. Coherence and phase values for microphone pairs with the probe at a single location are shown in Figure 5.

### ii. Results

The coherence-based approach can deal with unwrapping errors in frequency ranges that exhibit poor coherence. Figure 6 shows a spatial map of acoustic intensity vectors for the given frequency. The unwrapping is done across frequency for each position individually. When unwrapping errors have occurred at lower frequencies the vectors appear incorrect.

Something important to note is that the intensity vectors within frequency nulls are not necessarily improved. This, however, is not the goal; instead, the vectors should be valid for frequencies *above* which a frequency null has swept across the probe location. We are concerned with unwrapping *across* frequencies that exhibit poor coherence (when the vector is in a null), rather than unwrapping *in* the frequency ranges of poor coherence. For the spatial map, the erroneous vectors in the areas with high intensity are the result of unwrapping errors at lower frequencies, when this position was in a null. By using coherence unwrapping, many of these errors are avoided.



**Figure 6.** A comparison of using simple unwrapping (left) and coherence-based unwrapping (right) to calculate active acoustic intensity using the PAGE method. Many of the erroneous vectors have been markedly improved. The blue dots represent the probe microphones for the selected location.

## B. JET NOISE EXPERIMENT

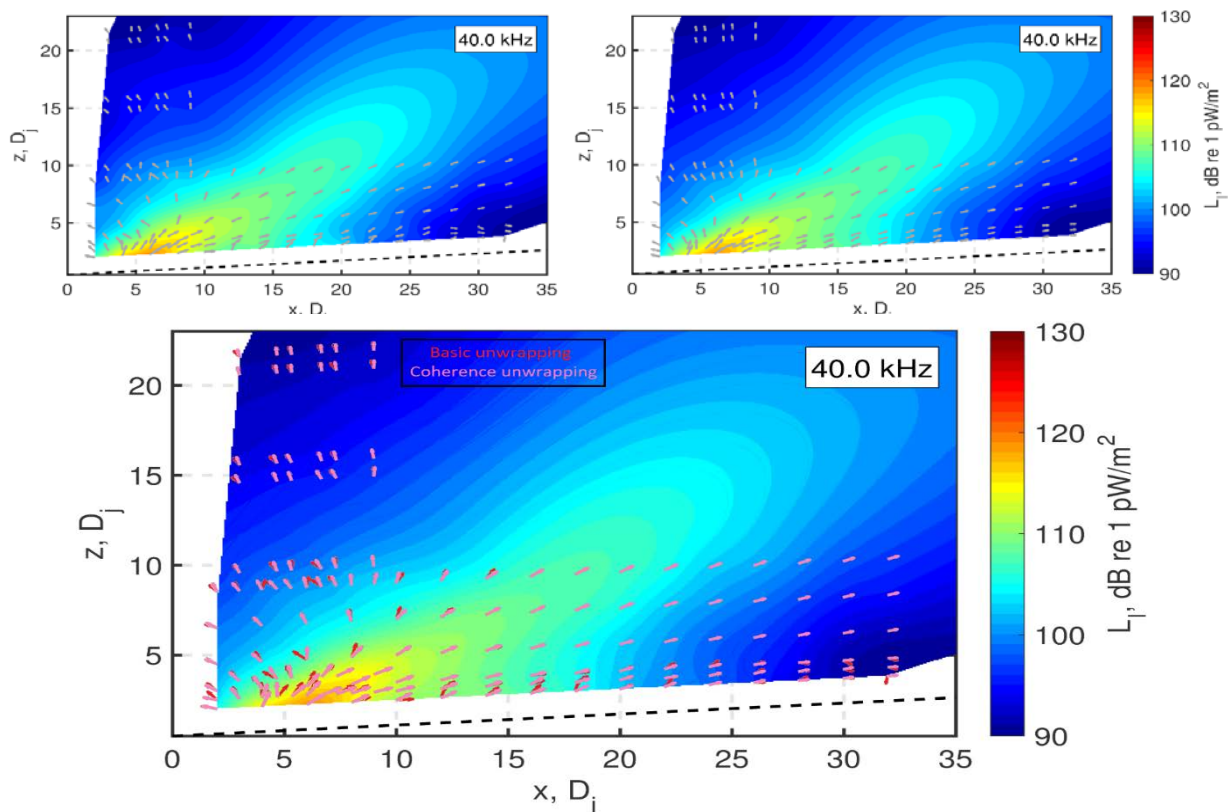
### i. Experimental Setup

Acoustical measurements were made at a jet facility at the Hypersonic High-enthalpy Wind Tunnel at Kashiwa Campus of the University of Tokyo. An unheated jet was ideally expanded through a 20-mm diameter converging-diverging nozzle for a design Mach number of 1.8. Although the facility is not anechoic, nearby reflecting surfaces were wrapped in fiberglass to limit reflections<sup>14</sup>. The same microphone probe configuration described in the dipole experiment was used to obtain measurements. The data used to describe the unwrapping method come from this experiment.

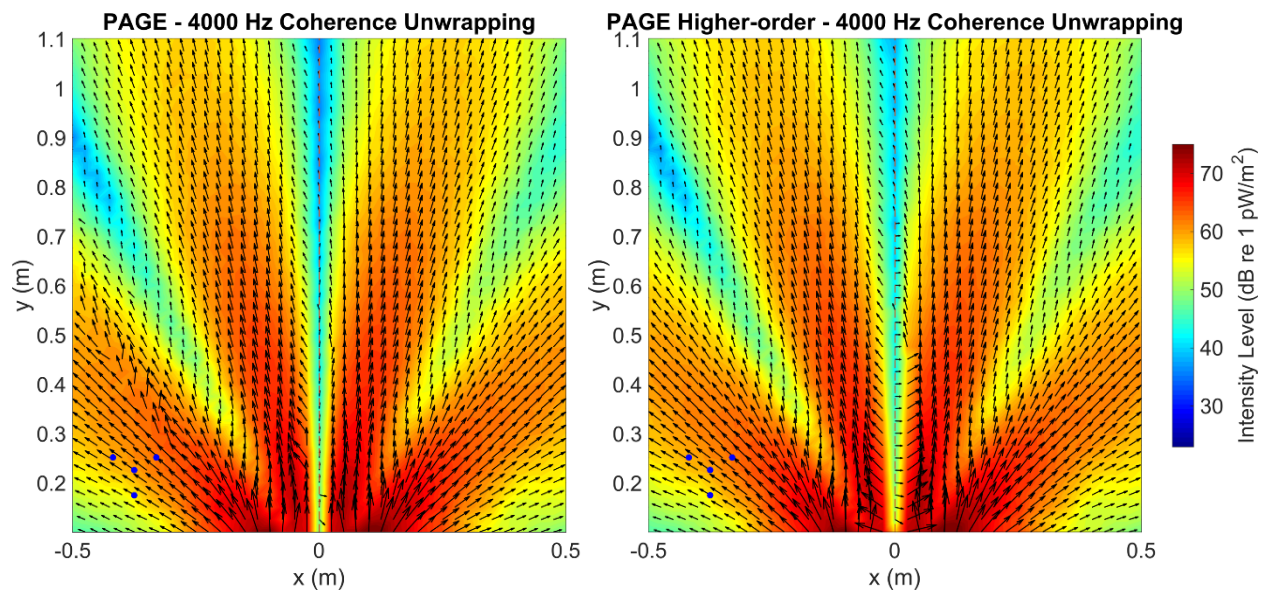
### ii. Results

Whereas the dipole experiment measurements exhibit excellent coherence, the jet noise experiment measurements exhibits poor coherence between probe microphone pairs, with typical values of less than 0.01. In spite of this extremely low coherence, the phase values still vary rather linearly with frequency. There are relative peaks and dips in coherence across the frequency range of interest. The coherence-fitting algorithm described above works well with this, catching the dips and appropriately classifying frequency ranges of poor coherence. The large phase jumps in these ranges result in a very poorly unwrapped phase when using the simple approach. The coherence-based approach, on the other hand, is not thrown off by these false jumps, and recovers remarkably well.

Figure 7 contains spatial maps for the acoustic intensity in the jet noise experiment. The upper figures show the results using regular (left) and coherence-based unwrapping (right). To compare the two, the plots have been superimposed (bottom) and the vectors have been colored. The results using the coherence-based approach vary more smoothly in space, as we would expect to happen physically.



**Figure 7.** Active acoustic intensity vector plots resulting from using simple unwrapping (top left) and coherence-based unwrapping (top right). To aid with visual comparison, these plots have been superimposed (bottom) with differently-colored vectors.



**Figure 8.** Active acoustic intensity vector plots using coherence unwrapping with the PAGE calculation (left) and coherence unwrapping with a higher-order PAGE calculation (right).

## 5. FUTRURE WORK

This phase unwrapping algorithm has been applied to situations other than active acoustic intensity, such as for beamforming, and has shown marked improvements<sup>4</sup>. Investigations into higher-order PAGE calculations for finding active acoustic intensity are currently ongoing<sup>15</sup>. Preliminary results of this method combined with coherence unwrapping using the anechoic chamber data show further improvements, and can be seen in Figure 8.

## 6. CONCLUSION

A coherence-based phase unwrapping algorithm can better determine absolute phase values than can simple unwrapping methods. Phase unwrapping is a problem that may not always have a viable solution. Some frequencies ranges contain so many jumps that one cannot be sure what the phase is supposed to be. In other situations, a phase trend can be picked out visually, but algorithms can produce results with many false jumps. There is not a one-case-fits-all solution.

In spite of these difficulties, it is possible to improve results by using a coherence-based approach. Phase unwrapping errors are often the result of trying to unwrap in ranges of relatively poor coherence. By giving ranges of poor coherence no weight (or less weight) in unwrapping, a more viable phase trend can be obtained. This in turn leads to less error in active acoustic intensity vectors using the PAGE method, which can increase the bandwidth to well beyond the spatial Nyquist frequency.

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