Three-microphone probe bias errors for acoustic intensity and specific acoustic impedance

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Abstract: In acoustic intensity estimation, adding a microphone at the probe center removes errors associated with pressure averaging. Analytical bias errors are presented for a one-dimensional, three-microphone probe for active intensity, reactive intensity, and specific acoustic impedance in a monopole field. Traditional estimation is compared with the Phase and Amplitude Gradient Estimator (PAGE) method; the PAGE method shows an increased bandwidth for all three quantities. The two- and three-microphone methods are compared experimentally, showing reduced bias errors with three-microphone PAGE for active and reactive intensity, whereas using two microphones is preferred for specific acoustic impedance.

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Date Received: December 1, 2017 **Date Accepted:** January 17, 2018

1. Introduction

Acoustic intensity and specific acoustic impedance are vital quantities for characterization of fields and sources. The traditional method for estimation of acoustic intensity using multi-microphone probes was introduced in the 1970s.[1–3] Due to high-frequency bias errors in the finite-sum and finite-difference formulations, this method has a limited bandwidth determined by the microphone spacing. To overcome these bias errors, the Phase and Amplitude Gradient Estimator (PAGE) method has been developed.[4,5] This processing method uses the same multi-microphone probes as the traditional method, and the PAGE method has been shown experimentally to extend the intensity estimation bandwidth by at least an order of magnitude for broadband sources.[6–8] In addition to active intensity, the method can be used to obtain multi-microphone estimates of reactive intensity and free-field specific acoustic impedance.

Analytical work for the two-microphone traditional intensity estimation has been done by Fahy[3] and Thompson and Tree,[9] who report bias errors of the method in several ideal fields. Champoux and L'espérance[10] performed a similar analysis for a two-microphone specific acoustic impedance estimation in the free-field. Building off this work, the analytical bias errors of the PAGE method for acoustic intensity and specific acoustic impedance have been reported by Whiting *et al.*[5] for a two-microphone probe in several ideal fields.

In this work, we seek to further develop the analytical foundation of Whiting *et al.* by extending it for a three-microphone probe in one dimension. Additionally, we seek to validate the analytical bias errors for both two and three microphones by presenting experimental data taken in a field produced by a monopole-like source.

2. Methodology

With a three-microphone probe [depicted in Fig. 1(a)], both the traditional and PAGE methods use the center microphone to obtain the complex pressure p, removing the need to estimate center pressure by averaging. This method has been previously employed in the literature for the traditional method, albeit rarely.[11,12] The outer two microphones are used for estimation of the pressure gradient, from which particle velocity is estimated using Euler's equation, $\boldsymbol{u} = (j/\rho_0\omega)\nabla p$. Here boldface represents a vector quantity, p is the frequency-dependent complex pressure, ρ_0 is the air density, and ω is the angular frequency. The estimates of active intensity and reactive intensity are $\boldsymbol{I} = \frac{1}{2}\mathrm{Re}\{p\boldsymbol{u}^*\}$ and $\boldsymbol{J} = \frac{1}{2}\mathrm{Im}\{p\boldsymbol{u}^*\}$, respectively, with * indicating complex

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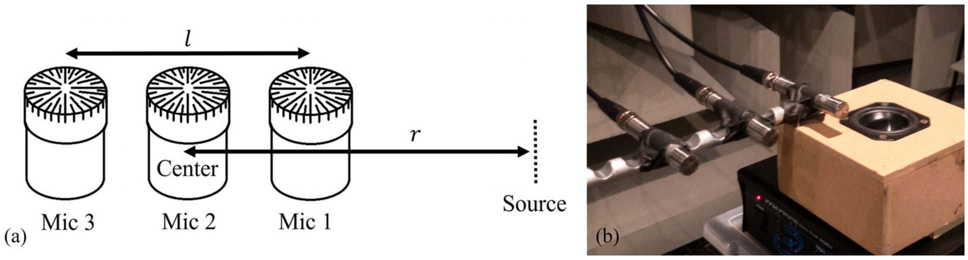

Fig. 1. (Color online) (a) Schematic of a one-dimensional intensity probe consisting of three microphones. The probe axis points toward the source, such that the sound first passes microphone 1. The distance between the microphones is $l/2$. (b) Layout of the experiment. The three-microphone probe is shown in its closest position to the source and is moved away from the source by a scanning system. The speaker approximates a monopole source over the frequency range analyzed.

conjugate. The estimate of specific acoustic impedance is $z = p/u_e$, with u_e indicating the particle velocity in the direction the specific acoustic impedance is to be measured.

The PAGE method differs from the traditional method by treating the complex pressure as an amplitude and phase, $p = Pe^{-j\phi}$. With this formulation, the PAGE formulas for active intensity, reactive intensity, and specific acoustic impedance are[5]

$$\boldsymbol{I}^{\mathrm{PAGE}} = \frac{P^2\widehat{\nabla\phi}}{2\rho_0\omega} = \frac{\bar{P}^2\widehat{\nabla\phi}}{\rho_0\omega}, \tag{1}$$

$$\boldsymbol{J}^{\mathrm{PAGE}} = -\frac{P\,\widehat{\nabla P}}{2\rho_0\omega} = -\frac{\bar{P}\,\widehat{\nabla P}}{\rho_0\omega}, \tag{2}$$

$$z^{\mathrm{PAGE}} = \frac{P\rho_0\omega}{\left[P\widehat{\nabla\phi} + j\widehat{\nabla P}\right]\cdot\hat{\boldsymbol{e}}} = \frac{P^2}{2\boldsymbol{I}_c^*\cdot\hat{\boldsymbol{e}}} = \frac{\bar{P}^2}{\boldsymbol{I}_c^*\cdot\hat{\boldsymbol{e}}}, \tag{3}$$

where an overhat indicates an estimated quantity, $\bar{P}$ is the ensemble-averaged root-mean-square pressure amplitude at frequency ω, $\boldsymbol{I}_c$ is the complex intensity calculated as $\boldsymbol{I}_c = \boldsymbol{I} + j\boldsymbol{J}$, and $\hat{\boldsymbol{e}}$ is the direction that specific acoustic impedance is to be measured in.

In practice, $\widehat{\nabla\phi}$ is obtained via the argument of pairwise transfer functions and is therefore wrapped to be within $-\pi$ and π, which makes $\boldsymbol{I}^{\mathrm{PAGE}}$ inaccurate past the spatial Nyquist frequency, where $kl = \pi$. However, for a broadband source and with sufficient coherence between the microphones, the phase difference can be unwrapped and $\boldsymbol{I}^{\mathrm{PAGE}}$ can be accurate for $kl > \pi$. The remainder of this letter assumes that unwrapping is possible for the collected data.[5,7,8,13]

This letter reports the three-microphone bias errors of both the traditional and PAGE methods in an analytical monopole field. Additionally, any phase mismatch present in the microphones can cause low-frequency estimation errors. These can be reduced by using phase-matched microphones, performing a switching technique to calibrate phase, or by increasing the microphone separation distance. The traditional method requires that all three microphones be well phase-matched. However, the PAGE method uses the center microphone only for pressure amplitude, eliminating the need for phase matching of the third microphone and making a PAGE-based three-microphone probe more cost effective.

To validate these analytical errors, and to compare the experimental performance between the two- and three-microphone probes, bias errors were measured using a small loudspeaker approximating a monopole, as shown in Fig. 1(b). Three microphones were attached to a scanning system in an anechoic chamber, with the axis of the probe in line with the source and a probe length of $l = 12$ cm. Using the scanning system to move the probe, broadband noise was recorded at multiple values of r that ranged from 10 cm to 5 m.

3. Monopole field

In order to understand performance of the PAGE method using three microphones, the bias errors are presented in this section for estimation of active intensity, reactive intensity, and specific acoustic impedance in an ideal monopole field.

The analytical bias errors depend on both the probe size relative to a wavelength, kl, and the distance from the source relative to a wavelength, kr. Here k is the acoustic wavenumber, l is the distance between outer microphones (expressed by Whiting *et al.*[5] as d for the two-microphone case), and r is the distance from the source to the probe center. It is useful to define a ratio β where $\beta = kl/kr$. As β approaches a

Table 1. The estimated-to-analytical error ratios for traditional and PAGE estimation of active intensity, for both two- and three-microphone probes.

Quantity	**3-microphone**	**2-microphone**
$\frac{\boldsymbol{I}^{\mathrm{TRAD}}}{\boldsymbol{I}}$	$\frac{1}{1-\beta^2/4}\mathrm{sinc}(kl/2)$	$\frac{1}{1-\beta^2/4}\mathrm{sinc}(kl)$
$\frac{\boldsymbol{I}^{\mathrm{PAGE}}}{\boldsymbol{I}}$	1	$\left(\frac{1}{1-\beta^2/4}\right)^2$

minimum value of 0, the probe is far from the source relative to the microphone spacing, and the field becomes planar. The maximum value for β is 2, where an outer microphone overlaps the source location. In this near-field case, the sound field has significant curvature and is highly reactive, whereas in the far field, the field has nearly constant amplitude and is primarily active.

The spatially-dependent complex pressure in an ideal monopole field can be expressed as $p = Ae^{-jkr}/r$, where A is the amplitude. The analytical radial active intensity is $\boldsymbol{I} = A^2/2\rho_0 c r^2$, where c is the sound speed. Table 1 reports ratios of estimated-to-analytical active intensity for the two methods, for both two- and three-microphone probes. These error ratios are derived by evaluating the traditional and PAGE expressions for estimation in a monopole field and dividing by the analytical quantity. The two-microphone ratios were reported preciously by Whiting *et al.*[5]

The three-microphone traditional method error level, $L_{\varepsilon,I} = 10\log_{10}(|\boldsymbol{I}^{\mathrm{TRAD}}/\boldsymbol{I}|)$, is shown in Fig. 2(a). This color plot (as well as the other color plots in this letter) shows the error magnitude in dB versus both kl and kr. A black diagonal line shows where $\beta = 2$, where an outer microphone overlaps the source. Lines of constant β run parallel to this line, and β is smallest toward the top left corner of the plot, where the field becomes more planar. For the traditional method, there is a significant error in active intensity estimation close to the source, and estimation error is greater than 5% for $\beta > 0.44$. For a probe length of 12 cm as in Fig. 1(b), this corresponds to a distance from the source to the probe center of 0.11 m. Additionally, there is error as kl increases, with more than 5% error for $kl > 1.1$, which corresponds to a frequency limit of 500 Hz for $l = 12$ cm. This is twice the bandwidth of the two-microphone case of $kl > 0.55$[5] since the center pressure is measured and has no estimation error.

The three-microphone PAGE method results in zero bias error for active intensity in a monopole field, plotted in Fig. 2(b). Unlike two-microphone PAGE which has

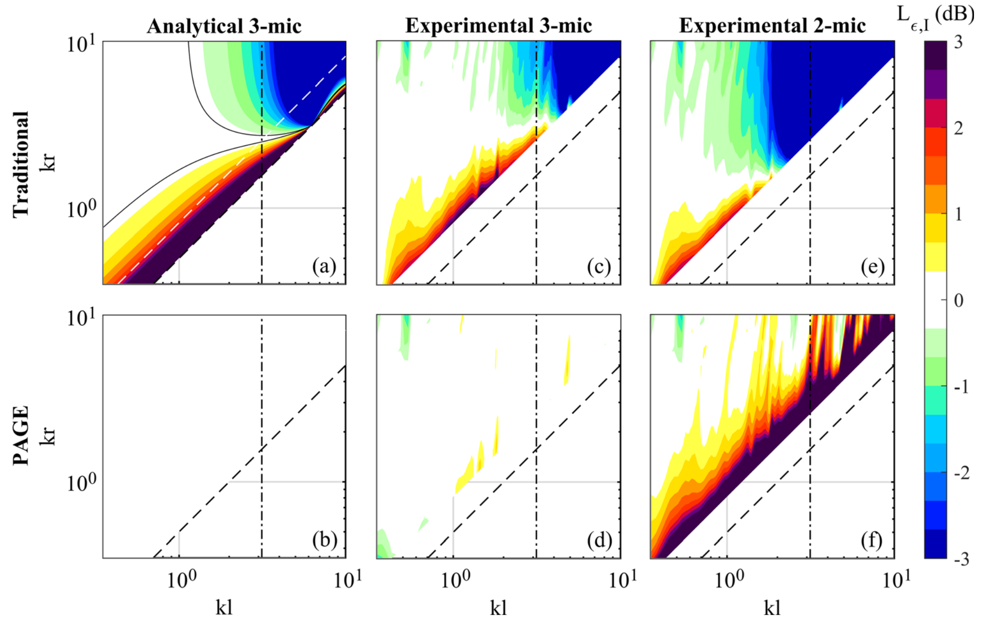

Fig. 2. (Color online) Bias errors in estimates of the magnitude of active intensity for a monopole field as a function of kl and kr: three-microphone, analytical (a) $\boldsymbol{I}^{\mathrm{TRAD}}$ and (b) unwrapped $\boldsymbol{I}^{\mathrm{PAGE}}$; three-microphone, experimental (c) $\boldsymbol{I}^{\mathrm{TRAD}}$ and (d) unwrapped $\boldsymbol{I}^{\mathrm{PAGE}}$; and two-microphone, experimental (e) $\boldsymbol{I}^{\mathrm{TRAD}}$ and (f) unwrapped $\boldsymbol{I}^{\mathrm{PAGE}}$. The vertical dashed line is the spatial Nyquist limit. To the left of this line, wrapped and unwrapped PAGE give the same results. The black diagonal dashed lines follow $r = l/2$. In the analytical plots, the solid black lines trace the limit of 5% error, and the white diagonal dashed line follows the closest distance to the source achievable in the experiment, for reference.

Table 2. The estimated to analytical error ratios for traditional and PAGE estimation of reactive intensity, for both two- and three-microphone probes.

Quantity	3-microphone	2-microphone
$\frac{\boldsymbol{J}^{\mathrm{TRAD}}}{\boldsymbol{J}}$	$\frac{1}{1-\beta^2/4}\cos(kl/2)$	$\left(\frac{1}{1-\beta^2/4}\right)^2$
$\frac{\boldsymbol{J}^{\mathrm{PAGE}}}{\boldsymbol{J}}$	$\frac{1}{1-\beta^2/4}$	$\left(\frac{1}{1-\beta^2/4}\right)^2$

near-field error,[5] three-microphone PAGE is accurate no matter the microphone spacing or distance to the source. Therefore, of the methods considered here, three-microphone PAGE is the most accurate for active intensity estimation in a monopole field. Also, since the PAGE method uses the additional center microphone only to measure pressure amplitude, it only needs to be amplitude-calibrated and not phase calibrated.

The experimental three-microphone bias errors are shown in Figs. 2(c) and 2(d), and Figs. 2(e) and 2(f) show the two-microphone bias errors using the outer microphones, to be compared with the analytical bias errors in Fig. 6 of Ref. 5. The error plotted is the ratio between the experimental intensity estimate and the expected intensity calculated from the measured center pressure, on a log scale as in Figs. 2(a) and 2(b). The expected intensity in a monopole field is $\boldsymbol{I} = P^2/2\rho_0 c$. For both probes, the traditional method shows near-field error and high-frequency error as expected. By using three-microphone PAGE, both the distance requirement and the upper-frequency limit vanish. This is an improvement over two-microphone PAGE, which has significant near-field error.

Another quantity of interest is reactive intensity, which in a monopole field is $\boldsymbol{J} = A^2/2\rho_0 ckr^3$. Unlike the two-microphone case where the traditional and PAGE estimations of reactive intensity are equivalent,[5] the three-microphone estimates are different for the two methods. The estimated to analytical error ratios are reported in Table 2, calculated from the expressions for traditional and PAGE estimation of reactive intensity.

The analytical error ratio, $L_{\varepsilon,J} = 10\log_{10}(|\boldsymbol{J}^{\mathrm{TRAD}}/\boldsymbol{J}|)$, is plotted in Fig. 3(a). Similar to active intensity, traditional reactive intensity estimates have errors both in the near-field and for large kl. For small values of kl, the three-microphone probe has less than a 5% error for $\beta < 0.44$, which is better than the two-microphone probe with the constraint $\beta < 0.31$.[5] However, the three-microphone probe also requires $kl < 0.64$ due to errors in the cross-spectral terms.

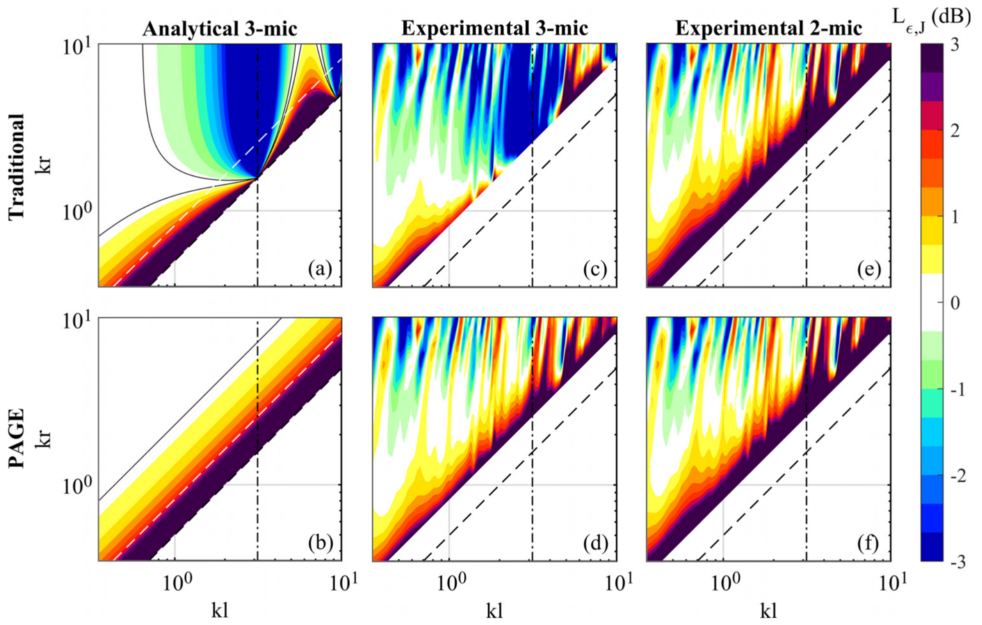

Fig. 3. (Color online) Similar to Fig. 2, except for the reactive intensity, $\boldsymbol{J}$.

Table 3. The estimated to analytical error ratios for traditional and PAGE estimation of specific acoustic impedance, for both two- and three-microphone probes.

Quantity	3-microphone	2-microphone
$\frac{z^{\mathrm{TRAD}}}{z}$	$\frac{\beta(1-\beta^2/4)(1+jkr)}{\beta\cos(kl/2)+j2\sin(kl/2)}$	$\beta\frac{(1+jkr)[2\cos(kd/2)+j\beta\sin(kd/2)]}{2[\beta\cos(kd/2)+j2\sin(kd/2)]}$
$\frac{z^{\mathrm{PAGE}}}{z}$	$\frac{kr-j}{kr-j(1-\beta^2/4)}$	1

The analytical PAGE error level is plotted in Fig. 3(b). Three-microphone PAGE outperforms two-microphone PAGE for all values of kl, with less than 5% error for $\beta < 0.44$ as opposed to $\beta < 0.31$.[5] Also, three-microphone PAGE outperforms three-microphone traditional at high values of kl.

Experimental data for reactive intensity are plotted in Figs. 3(c)–3(f), showing error from the expected reactive intensity of $\boldsymbol{J} = P^2/2\rho_0 ckr$. The three-microphone data are compared with the analytical results shown in Figs. 3(a) and 3(b). The near-field behavior matches the analytical errors, and the traditional method shows the correct trend of increasing error as kl approaches π. The two-microphone data also show correct trends, matching the analytical results shown in Fig. 6 of Ref. 5. However, for all cases, large estimation errors occur at far distances as the field becomes more planar (active). For example, J is an order of magnitude smaller than I at the plot limit of $kr = 10$. For these small values, the estimation accuracy is limited by the signal-to-noise ratio and scattering.

The final quantity explored here is specific acoustic impedance, which in a monopole field is $z = \rho_0 ckr/(kr - j)$. Table 3 reports the estimated to analytical error ratios, calculated from the expressions for traditional and PAGE estimation of specific acoustic impedance.

Figure 4(a) shows the traditional error level, $L_{\varepsilon,z} = 20\log_{10}(|z^{\mathrm{TRAD}}/z|)$, Fig. 4(b) shows the PAGE error level, and Figs. 4(c)–4(f) show the experimental data, to be compared with analytical results shown in Figs. 4(a) and 4(b) and in Fig. 7 of Ref. 5. The experimental data shows error from the analytical value, which depends on r. The analytical three-microphone traditional method shows an improved high-frequency limit of $kl < 1.08$ over the two-microphone traditional with $kl < 0.77$. On the other hand, the three-microphone PAGE method has no high-frequency limit. However, both methods have a near-field limit of $\beta > 0.44$, as opposed to the two-microphone case with no near-field limit for either method. Because two-microphone PAGE has zero bias error, it is preferred over three-microphone PAGE for specific acoustic impedance. The experimental data in Fig. 4(f) match these trends.

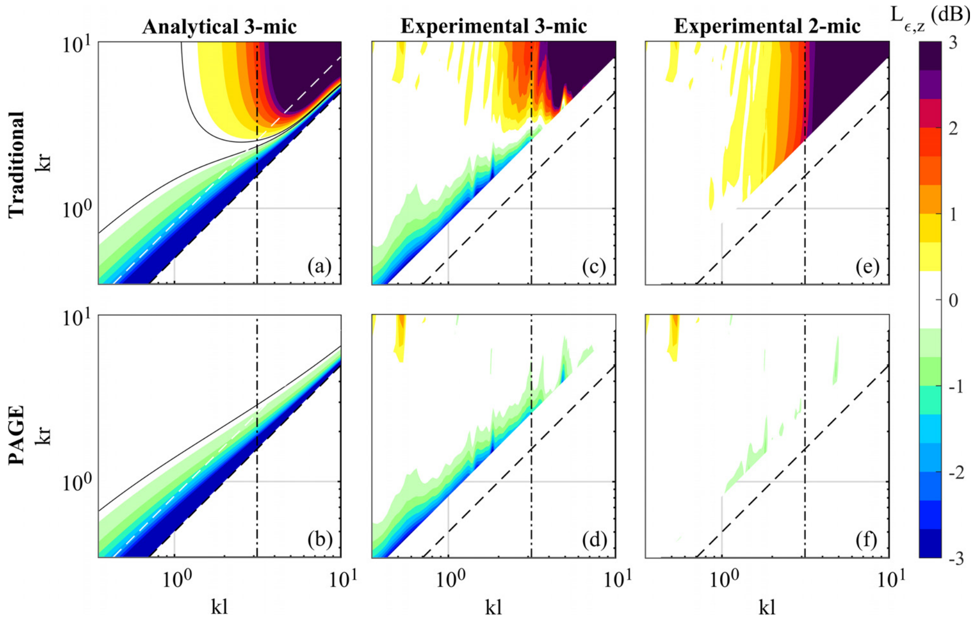

Fig. 4. (Color online) Similar to Fig. 2, except for the specific acoustic impedance, z.

4. Conclusion

In this letter, the theoretical foundation of the PAGE method has been extended by presenting monopole bias errors for a three-microphone probe for three quantities: active intensity, reactive intensity, and specific acoustic impedance. To validate these bias errors, and to make an experimental comparison with the two-microphone bias errors reported by Whiting *et al.*,[5] bias errors were obtained using a small loudspeaker to approximate a monopole.

The analytical and experimental results support the following findings. First, the accuracy of active intensity estimates is significantly improved by adding a center microphone, removing error in center pressure estimation. The bandwidth of traditional method active intensity is twice that of using two microphones. Second, the three-microphone PAGE method has zero error in active intensity estimation up to the spatial Nyquist frequency, regardless of probe size or distance to the source. For broadband sources and with sufficient coherence between the microphones, the phase can be unwrapped, which can extend the bandwidth to be an order of magnitude greater than that of the traditional method.[6] Third, the PAGE method does not require the center microphone to be phase-matched with the other microphones, so three-microphone PAGE is the most accurate of the discussed methods with no loss of feasibility other than obtaining an amplitude-calibrated center microphone. Fourth, calculation of reactive intensity with three-microphone PAGE is improved over the two-microphone methods. Finally, the three-microphone PAGE method introduces error to specific acoustic impedance estimates, so this quantity is best estimated using a probe's outer two microphones. Future work may include consideration of multi-dimensional probes, and higher-order estimation of gradients.[14]

Acknowledgments

This work was supported by National Science Foundation Grant No. 1538550, "Developing New Methods for Obtaining Energy-based Acoustic Quantities."

References and links

[1] F. J. Fahy, "Measurement of acoustic intensity using the cross-spectral density of two microphone signals," J. Acoust. Soc. Am. **62**, 1057–1059 (1977).

[2] J. Y. Chung, "Cross-spectral method of measuring acoustic intensity without error caused by instrument phase mismatch," J. Acoust. Soc. Am. **64**, 1613–1616 (1978).

[3] F. J. Fahy, *Sound Intensity* (Spon, London, 1995), pp. 1–295.

[4] D. C. Thomas, B. Y. Christensen, and K. L. Gee, "Phase and amplitude gradient method for the estimation of acoustic vector quantities," J. Acoust. Soc. Am. **137**, 3366–3376 (2015).

[5] E. B. Whiting, J. S. Lawrence, K. L. Gee, T. B. Neilsen, and S. D. Sommerfeldt, "Bias error analysis for phase and amplitude gradient estimation of acoustic intensity and specific acoustic impedance," J. Acoust. Soc. Am. **142**, 2208–2218 (2017).

[6] K. L. Gee, T. B. Neilsen, S. D. Sommerfeldt, M. Akamine, and K. Okamoto, "Experimental validation of acoustic intensity bandwidth extension by phase unwrapping," J. Acoust. Soc. Am. **141**, EL357–EL362 (2017).

[7] K. L. Gee, E. B. Whiting, T. B. Neilsen, M. M. James, and A. R. Salton, "Development of a near-field intensity measurement capability for static rocket firings," Trans. Jpn. Soc. Aeronaut. Space Sci. **14**, PO_2_9–PO_2_15 (2016).

[8] K. L. Gee, M. Akamine, K. Okamoto, T. B. Neilsen, M. Cook, S. Tsutsumi, S. Teramoto, and T. Okuuki, "Characterization of supersonic laboratory-scale jet noise with vector acoustic intensity," in *23rd AIAA/CEAS Aeroacoustics Conference*, AIAA Paper 2017-3519 (2017).

[9] J. K. Thompson and D. R. Tree, "Finite-difference approximation errors in acoustic intensity measurements," J. Sound Vib. **75**, 229–238 (1981).

[10] Y. Champoux and A. L'espérance, "Numerical evaluation of errors associated with the measurement of acoustic impedance in a free field using two microphones and a spectrum analyzer," J. Acoust. Soc. Am. **84**, 30–38 (1988).

[11] B. S. Cazzolato and C. H. Hansen, "Errors arising from three-dimensional energy density sensing in one-dimensional sound fields," J. Sound Vib. **236**, 375–400 (2000).

[12] J.-C. Pascal and J.-F. Li, "A systematic method to obtain 3D finite-difference formulations for acoustic intensity and other energy quantities," J. Sound Vib. **310**, 1093–1111 (2008).

[13] M. R. Cook, K. L. Gee, S. D. Sommerfeldt, and T. B. Neilsen, "Coherence-based phase unwrapping for broadband acoustic signals," Proc. Mtgs. Acoust. **30**, 055005 (2017).

[14] J. S. Lawrence, K. L. Gee, T. B. Neilsen, and S. D. Sommerfeldt, "Higher-order estimation of active and reactive acoustic intensity," Proc. Mtgs. Acoust. **30**, 055004 (2017).