



Lie algebraic approach to a nonstationary atom-cavity system

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In this work, we study the generation of photons inside an ideal cavity with resonantly oscillating boundaries in the presence of a two-level atom. We make use of Lie algebraic techniques to obtain an approximate time-evolution operator and not only evaluate the resonant and dispersive regimes but also explore different regions of parameters. We have found a very good agreement between our approximate results and those obtained by numerical means. © 2018 Optical Society of America

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1. INTRODUCTION

Creation of photons from vacuum fluctuations is one of the many fascinating effects in quantum theory. In a seminal paper dealing with the quantum theory of linearly polarized light propagating in a one-dimensional cavity bounded by two ideal, infinite, parallel, plane mirrors that move with arbitrary trajectories, Moore [1] predicted the creation of real photons generated from vacuum due to nonadiabatic variations in the boundary conditions of the field. This effect is now known as the dynamical Casimir effect (DCE); see Refs. [2–4] for recent reviews on the status of the DCE. For the generation of the effect, it is necessary to rapidly modulate the boundary conditions of the electromagnetic field with velocities close to the speed of light, which, for a physical mirror, may not be experimentally feasible. In order to circumvent these difficulties, experiments with analogous systems such as superconducting circuits consisting of coplanar transmission line resonators with tunable electrical length have been performed. In these experiments, the rate of change of the electrical length can be done very fast by modulating the inductance of a superconducting quantum interference device (SQUID) at high frequencies [5,6]. A different proposal, based on a trapped-ion implementation, has been made recently [7].

Interestingly, it was shown in Refs. [8,9] that one might expect a significant rate of photon generation inside ideal cavities with resonantly oscillating boundaries. The simplest model describing this effect takes into account a single resonant cavity mode whose frequency is rapidly modulated [10,11]. A quite different scenario can take place when featuring a secondary

system (a detector) inside a nonstationary cavity where the DCE is manifested. Along this line of research, the problem of the back action of different detectors on the rate of photon generation has been considered describing the detector as a two-level atom (or several atoms) [12,13] or by means of a harmonic oscillator tuned in resonance with the selected field mode [14,15]. More recent results have shown that DCE can also manifest if one allows the Zeeman splitting of the qubit or the atom-field coupling to be time-dependent [16–22].

Here, we are in line with the aforementioned studies of tackling the problem of exploring the effect of adding a secondary system viewed as a two-level atom on the evolution of the field in a nonstationary cavity. To do this, we put forward an alternative approach based on Lie algebraic techniques, since the constituent operators of the proposed unperturbed Hamiltonian model, the starting point of our treatment, turns out to generate a closed Lie algebra. This fact enables us to express the corresponding evolution operator of the whole system as a product of exponentials according to the well-known Wei–Norman theorem, as will be outlined in Section 2. Our algebraic procedure, therefore, allows us to derive closed-form semianalytical expressions for exploring, in Section 3, some quantities of physical interest to the problem at hand, such as the average value of Casimir photons, the variance of the field quadratures, and the evolution of the field on phase space in terms of its Q function; the extent to which our approximate solutions are applicable is also stated by comparing them with the corresponding entirely numerical results. And finally, in Section 4, some conclusions are given.

2. MODEL AND ITS APPROXIMATE SOLUTION

Consider the case of a closed cavity with a moving wall executing a periodic motion and a two-level atom inside it. The simplest Hamiltonian describing this system can be written as (in units of \hbar)

$$\hat{H} = \omega(t)\hat{n} + \chi(t)(\hat{a}^2 + \hat{a}^{\dagger 2}) + \frac{\Omega}{2}\hat{\sigma}_z + g(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-), \quad (1)$$

where \hat{n} , \hat{a} , and \hat{a}^\dagger are the usual number, annihilation, and creation operators, respectively, while $\hat{\sigma}_z, \hat{\sigma}_\pm$ refer to the Pauli matrices representing the two-level atom, Ω is the atomic level spacing, and g is the atom-field coupling constant with units of frequency. We have chosen $\chi(t) \equiv \frac{1}{4\omega(t)} \frac{d\omega(t)}{dt}$ and $\omega(t) = \omega_0[1 + \epsilon \sin(\eta t)]$, with $|\epsilon| \ll 1$ being the modulation amplitude and η the frequency of the modulation. It is known that in the absence of the atom-field interaction, the mean number of photons grows exponentially if $\eta \simeq 2\omega_0$ and $\epsilon\omega_0 t \geq 1$ [8,9].

Since $|\epsilon| \ll 1$, we take $\omega(t) \simeq \omega_0$ and set the unperturbed Hamiltonian as

$$\hat{H}_0 = \omega_0\hat{n} + \frac{\Omega}{2}\hat{\sigma}_z + \chi(t)(\hat{a}^2 + \hat{a}^{\dagger 2}), \quad (2)$$

with $\chi(t) = (\epsilon\omega_0/2) \cos(2\omega_0 t)$, and the atom-field interaction Hamiltonian is given by

$$\hat{V} = g(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-). \quad (3)$$

By virtue of the fact that the constituent field operators of H_0 , namely, $\{\hat{n}, \hat{a}^{\dagger 2}, \hat{a}^2\}$, form the basis of an $su(1,1)$ Lie algebra, it is well known that the time-evolution operator corresponding to such a Hamiltonian can be cast in terms of a product of exponentials of the aforesaid operators [23]:

$$\hat{U}_0 = e^{-\frac{\Omega}{2}t\hat{\sigma}_z} e^{\gamma_1\hat{n}} e^{\gamma_2\hat{a}^{\dagger 2}} e^{\gamma_3\hat{a}^2} e^{\gamma_4}, \quad (4)$$

with complex, time-dependent functions $\gamma_i(t)$ to be determined. Substitution of Eq. (4) into Schrödinger's equation yields the set of ordinary differential equations:

$$\dot{\gamma}_1 = -i\omega_0 - 4ie^{2\gamma_1}\chi(t)\gamma_2, \quad (5)$$

$$\dot{\gamma}_2 = (-ie^{-2\gamma_1} + 4ie^{2\gamma_1}\gamma_2^2)\chi(t), \quad (6)$$

$$\dot{\gamma}_3 = -ie^{2\gamma_1}\chi(t), \quad (7)$$

$$\dot{\gamma}_4 = -2ie^{2\gamma_1}\chi(t)\gamma_2, \quad (8)$$

with the initial conditions $\gamma_i(t_0) = 0$, so that $U_0(t_0, t_0) = 1$. The maximum value for the function $\chi(t)$ is $\epsilon\omega_0/2$ and $|\epsilon| \ll 1$, so that at the beginning of the evolution $\gamma_1 \simeq -i\omega_0(t - t_0)$, and the functions γ_2, γ_3 , and γ_4 have small initial derivatives.

The time-evolution operator of the whole system in the interaction picture generated by \hat{U}_0 satisfies

$$i\partial_t \hat{U}_I = \hat{H}_I \hat{U}_I, \quad (9)$$

with $\hat{H}_I = \hat{U}_0^\dagger \hat{V} \hat{U}_0$ being the interaction picture Hamiltonian, and $\hat{U}_I(t_0, t_0) = 1$. This representation entails applying a Bogoliubov transformation to the annihilation and creation operators, namely,

$$\hat{U}_0^\dagger \hat{a} \hat{U}_0 = t_1 \hat{a} + t_2 \hat{a}^\dagger,$$

$$\hat{U}_0^\dagger \hat{a}^\dagger \hat{U}_0 = t_3 \hat{a} + t_4 \hat{a}^\dagger,$$

where $t_1^* = t_4$ and $t_2^* = t_3$, since the transformation is unitary. The transformation coefficients are given in terms of the γ_i terms as

$$t_1 = e^{\gamma_1} - 4e^{\gamma_1}\gamma_2\gamma_3, \quad (10)$$

$$t_2 = 2e^{\gamma_1}\gamma_2, \quad (11)$$

$$t_3 = -2e^{-\gamma_1}\gamma_3, \quad (12)$$

$$t_4 = e^{-\gamma_1}. \quad (13)$$

Thus, the interaction picture Hamiltonian takes the form

$$\hat{H}_I = g[(t_1 + t_3)\hat{a} + (t_2 + t_4)\hat{a}^\dagger](\hat{\sigma}_+ e^{i\Omega t} + \hat{\sigma}_- e^{-i\Omega t}). \quad (14)$$

The Hamiltonian \hat{H}_I is Hermitian. If we call $z_1 = (t_1 + t_3)$ and $z_2 = (t_2 + t_4)$, we have $z_1 = z_2^*$ and $|z_1|e^{i\phi_1} = |z_2|e^{-i\phi_2}$, so $\phi_1 = -\phi_2$. From Eq. (5), we know that at the early stages of the evolution $\gamma_1 \simeq -i\omega_0 t$, then $\phi_1 \simeq -i\omega_0 t$ and $\phi_2 \simeq i\omega_0 t$.

Taking the products in the equation above, one obtains terms that conserve the total number of excitations $\hat{a}\hat{\sigma}_+$, $\hat{a}^\dagger\hat{\sigma}_-$, and terms that do not conserve it, $\hat{a}\hat{\sigma}_-$, $\hat{a}^\dagger\hat{\sigma}_+$. The term $\hat{a}\hat{\sigma}_-$ corresponds to a transition from the upper level $|e\rangle$ to the lower level $|g\rangle$ and the annihilation of a photon; the second describes the reverse process. The terms that conserve the total number of excitations have an initial phase $\pm i(\Omega - \omega_0)t$, while those that do not conserve it have an initial phase $\pm i(\Omega + \omega_0)t$. The terms that oscillate rapidly will make a smaller contribution than those that oscillate slowly; then in what follows we will keep only the terms that conserve the total number of excitations (an approximation that will be grounded on comparing its predictive accuracy with the corresponding numerical results based upon the whole Hamiltonian) to get a Jaynes–Cummings-type interaction of the form [24]

$$\tilde{H}_I \approx g[\hat{a}\hat{\sigma}_+(t_1 + t_3)e^{i\Omega t} + \hat{a}^\dagger\hat{\sigma}_-(t_2 + t_4)e^{-i\Omega t}]. \quad (15)$$

Now we find it convenient to define the operators [25–28]:

$$\hat{b} = \frac{\hat{a}\sigma_+}{\sqrt{M}}, \quad \hat{b}^\dagger = \frac{\hat{a}^\dagger\sigma_-}{\sqrt{M}}, \quad (16)$$

with $M = n + \frac{1}{2}(1 + \sigma_z)$ being the total number of excitations in the corresponding ladder, so that the interaction Hamiltonian [Eq. (15)] can be recast as

$$\tilde{H}_I \approx \sqrt{M}g[(t_1 + t_3)\hat{b}e^{i\Omega t} + (t_2 + t_4)\hat{b}^\dagger e^{-i\Omega t}]. \quad (17)$$

The number of excitations for a given ladder is a constant, since the operators \hat{b} and \hat{b}^\dagger generate transitions between states of the same ladder (fixed $M = n + 1$). The action of such operators upon the states $|e, n\rangle$ and $|g, n + 1\rangle$ is given by

$$\begin{aligned} \hat{b}|e, n\rangle &= 0, & \hat{b}|g, n + 1\rangle &= |e, n\rangle, \\ \hat{b}^\dagger|e, n\rangle &= |g, n + 1\rangle, & \hat{b}^\dagger|g, n + 1\rangle &= 0, \end{aligned}$$

from which one can deduce the commutation relations,

$$[\hat{b}, \hat{b}^\dagger] = \hat{\sigma}_z, \quad [\hat{\sigma}_z, \hat{b}] = 2\hat{b}, \quad [\hat{\sigma}_z, \hat{b}^\dagger] = -2\hat{b}^\dagger.$$

Again, the Lie algebra generated by the set of operators $\{\hat{b}, \hat{b}^\dagger, \hat{\sigma}_z\}$ enables us to apply the Wei–Norman theorem and to write the time-evolution operator for the interaction part in a product form as

$$\hat{U}_I = e^{\beta_z \hat{\sigma}_z} e^{\beta_+ \hat{b}^\dagger} e^{\beta_- \hat{b}}. \quad (18)$$

Substitution of this evolution operator into Schrödinger's equation [Eq. (9)] allows us to arrive at the following set of coupled differential equations for the complex, time-dependent functions $\beta_z(t)$ and $\beta_\pm(t)$:

$$\dot{\beta}_z = -ie^{i\Omega t - 2\beta_z} g \sqrt{M} (t_1 + t_3) \beta_+, \quad (19)$$

$$\dot{\beta}_+ = -ig \sqrt{M} (e^{-i\Omega t + 2\beta_z} (t_2 + t_4) + e^{i\Omega t - 2\beta_z} (t_1 + t_3) \beta_+^2), \quad (20)$$

$$\dot{\beta}_- = -ie^{i\Omega t - 2\beta_z} g \sqrt{M} (t_1 + t_3). \quad (21)$$

The solution to these equations must be obtained by numerical means. Nonetheless, having established the evolution operator of the whole system, $\hat{U} = \hat{U}_0 \hat{U}_I$, the algebraic scheme enables us to readily proceed to the calculation of any physical observable.

Let us consider first the case when the atom is in its ground state and the field has no photons, that is, $|\Psi(t_0)\rangle = |g, 0\rangle$, the total number of excitations is $M = 0$, and application of the evolution operator \hat{U}_I upon the initial state does not modify it; that is, $\hat{U}_I(t, t_0)|g, 0\rangle = |g, 0\rangle$. This is due to the fact that \hat{H}_I has only terms that conserve M . The state at time t is then given by $|\Psi(t)\rangle = U_0(t, t_0)|g, 0\rangle$, and the atomic state will not change during this evolution. There is however, generation of photons due to the Hamiltonian H_0 .

Consider now the case when the state of the system at time t_0 is $|\Psi(t_0)\rangle = \alpha|e, n\rangle + \beta|g, n+1\rangle$, where $|e\rangle$, $|g\rangle$ refer to the excited and ground atomic states, respectively, and $|n\rangle$ corresponds to a field with n photons. Since the state is normalized, we require $|\alpha|^2 + |\beta|^2 = 1$. At time t , the system has evolved into $|\Psi(t)\rangle = \hat{U}_0 \hat{U}_I |\Psi(t_0)\rangle$. Once obtained in its explicit form, one can evaluate almost straightforwardly the average value of a given observable $\hat{O}(t)$:

$$\langle \hat{O}(t) \rangle = \langle \Psi(t) | \hat{O} | \Psi(t) \rangle = \langle \Psi(t_0) | \hat{U}_I^\dagger \hat{O} \hat{U}_0 \hat{U}_I | \Psi(t_0) \rangle. \quad (22)$$

Consider, for instance, the case where the operator \hat{O} is the number operator. Application of \hat{U}_I to the initial state $|\Psi(t_0)\rangle$ yields

$$|\Psi_I(t)\rangle = e^{\beta_z} (\alpha \beta_-) |e, n\rangle + e^{-\beta_z} (\alpha \beta_+ + \beta(1 + \beta_+ \beta_-)) |g, n+1\rangle, \quad (23)$$

$$= C_{e,n} |e, n\rangle + C_{g,n+1} |g, n+1\rangle, \quad (24)$$

and the interaction picture representation of the number operator is

$$\hat{n}_I(t) = U_0^\dagger \hat{a}^\dagger U_0 U_0^\dagger \hat{a} U_0 = (t_3 \hat{a} + t_4 \hat{a}^\dagger)(t_1 \hat{a} + t_2 \hat{a}^\dagger),$$

which may be written as

$$\hat{n}_I(t) = (1 - 8\gamma_2 \gamma_3) \hat{n} + 2\gamma_2 \hat{a}^{\dagger 2} + 2\gamma_3 (4\gamma_2 \gamma_3 - 1) \hat{a}^2 - 4\gamma_2 \gamma_3, \\ = g_{11} \hat{n} + g_{20} \hat{a}^{\dagger 2} + g_{02} \hat{a}^2 + g_{00}. \quad (25)$$

So, based upon Eq. (22), the average value of the number operator is

$$\langle \Psi_I(t) | \hat{n}_I(t) | \Psi_I(t) \rangle = g_{11} [n |C_{e,n}|^2 + (n+1) |C_{g,n+1}|^2] + g_{00}, \quad (26)$$

where we have made use of the relation $|C_{e,n}|^2 + |C_{g,n+1}|^2 = 1$ that follows from the normalization condition $\langle \Psi_I(t) | \Psi_I(t) \rangle = 1$. We shall apply this prescription in the following section to explore the effectiveness of our semianalytical approach, comparing our results with the entirely numerical solution for the whole Hamiltonian [Eq. (1)]. As a remark, we should mention that the algebraic approach described above can also be suitable for the study of initial multiphoton Gaussian states of the electromagnetic mode, such as coherent and squeezed states.

3. SEMIANALYTICAL AND NUMERICAL RESULTS

Let us now focus on reviewing to what extent the semianalytical approach outlined above can be used to explore the effect of the atom on features of physical relevance concerning the dynamics of the cavity field, viz., the expectation value of the number of Casimir photons, the variances of its quadratures, and its evolution in phase space based upon the Q function.

Since we are primarily interested in the aforesaid features when the state of the system at the initial time is $|\Psi(t_0)\rangle = |e, 0\rangle$, it is found that the average number of Casimir photons is given by

$$\langle \Psi_I(t) | \hat{n}_I(t) | \Psi_I(t) \rangle = g_{11} |\beta_+|^2 e^{-2\Re\{\beta_z\}} + g_{00}, \quad (27)$$

with $\Re\{\cdot\}$ denoting the real part of the corresponding time-dependent complex function. The influence of the atom-field interaction on photon creation is encapsulated in the time-dependent functions β_+ and β_z . The outcome of Eq. (27) is depicted in the top panel of Fig. 1 for parameters $\Omega = 0.2\omega_0$, $\epsilon = 0.02$, and $g = 0.05\omega_0$, chosen in a way such that the atom-field interplay falls into the dispersive regime in which $\omega_0 \gg \Omega$ and $|\Omega - \omega_0| \gg g$; unless otherwise specified, we shall make use of red and black lines to label the numerical and approximate results, respectively, in subsequent descriptions. So, on the basis of the numerical result, we see that our approach works very well within the time interval displayed in the plot. Although a slightly oscillatory conduct is exhibited at the very beginning of the evolution, the archetypal exponential growth of such a quantity rules its overall profile. On the other hand, as far as the atom is concerned, the probability of finding it in its excited state turns out to be given by the expression

$$P_e(t) = \frac{e^{2\Re\{\gamma_4 + \beta_z\}}}{\sqrt{1 - 4|\gamma_2|^2 e^{4\Re\{\gamma_1\}}}}, \quad (28)$$

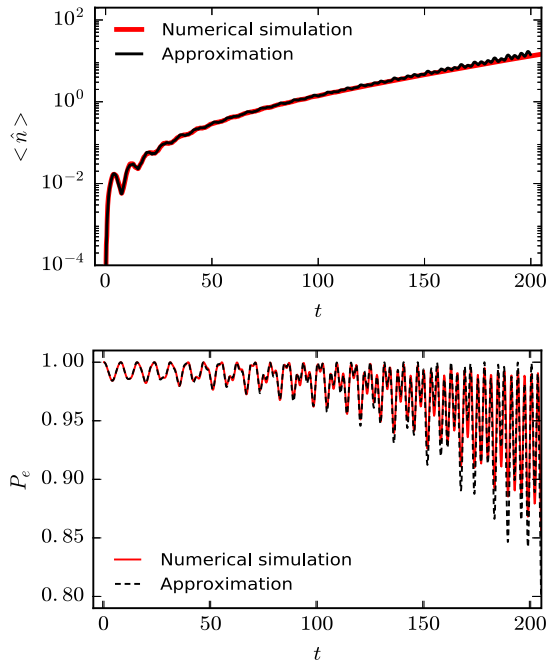


Fig. 1. (Top) Average value of the number operator and (bottom) the probability of finding the atom in its excited state as functions of time (in units of $\tau_0 = 1/\omega_0$). The initial state is taken to be $\rho_{AF}(0) = |e, 0\rangle\langle e, 0|$. Parameter set (dispersive regime): $\{\Omega, \eta, g\} = \{0.2, 2.0, 0.05\}\omega_0$, $\epsilon = 0.02$.

which is calculated from the trace operation $\text{Tr}\{\hat{\rho}_A|e\rangle\langle e|\}$, with $\hat{\rho}_A = \text{Tr}_F\{\hat{\rho}_{AF}\}$ being the reduced density operator of the atom (Tr_F means tracing over the degree of freedom of the field) and $\rho_{AF} = |\Psi(t)\rangle\langle\Psi(t)|$ the density operator of the composite system. This quantity is displayed in the bottom panel of Fig. 1. One can see that its spiky behavior in the early stages of the system's evolution is in agreement with the corresponding numerical result; this conduct is also in accord with the fact that the state of the atom, being out of resonance with the field, remains almost unchanged around the state it started up ($P_e(t) \approx 1$).

Neglecting the terms $\hat{a}\hat{\sigma}_-$ and $\hat{a}^\dagger\hat{\sigma}_+$ from the Hamiltonian given in Eq. (1), we obtained a Jaynes–Cummings-like Hamiltonian whose time-evolution operator we constructed in an exact form. The differences found between the purely numerical computation using the full Hamiltonian, and the results obtained with our semianalytical method are due only to the terms we have neglected. In the case far from resonance, we have found that the effect of these terms is negligible. However, when the interaction between the atom and the field is in resonance ($\Omega = \omega_0$), we can see that for long enough times, their effect is no longer negligible, as can be seen in Fig. 2, where the photon production is plotted as a function of time for the set of system parameters indicated in the figure caption. In this regime, one can see, again, a preponderant exponential growth in photon production, whereas our approximation starts displaying, as time elapses, a conspicuous oscillatory profile around the value given by the entirely numerical solution using the full Hamiltonian.

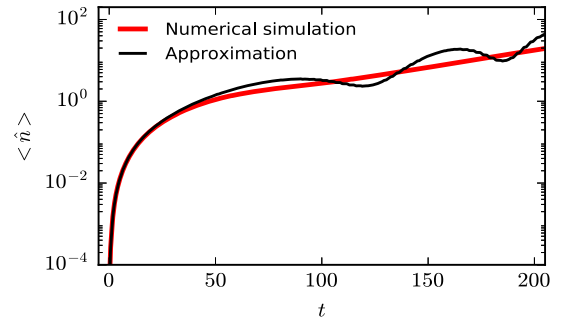


Fig. 2. Average value of the number operator as a function of time (in units of $\tau_0 = 1/\omega_0$). The initial state is taken to be $\rho_{AF}(0) = |e, 0\rangle\langle e, 0|$. Parameter setting (resonant regime): $\{\Omega, \eta, g\} = \{1.0, 2.0, 0.02\}\omega_0$, $\epsilon = 0.02$.

At this stage, it is worth mentioning that our semianalytical approach goes beyond the algebraic treatment reported in Ref. [12], where the coupling strength of the cavity field with the atom is regarded as a small perturbation parameter in comparison with the modulation amplitude, i.e., $g \ll \epsilon\omega_0$. Indeed, besides being applicable in this limit, the present approach encompasses both dispersive and resonant regimes and allows the aforesaid parameters to have values of the same order of magnitude [as those used for Fig. 2].

Before proceeding to explore the evolution of the field on phase space, let us compute the variances of the field quadratures, labeled as $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ and $\hat{P} = (\hat{a} - \hat{a}^\dagger)/\sqrt{2}i$, that can be determined from Eq. (22) by using the expressions $\langle\Psi_I(t)|\hat{a}_I^\dagger|\Psi_I(t)\rangle = (\beta\beta_+^*\alpha^* - 2\beta^*\beta_+\alpha\gamma_3)e^{-2\Re\{\beta_z\} - \gamma_1}$ and $\langle\Psi_I(t)|\hat{a}_I^{\dagger 2}|\Psi_I(t)\rangle = -2\gamma_3e^{-2\gamma_1}(2|\alpha\beta_+|^2e^{-2\Re\{\beta_z\}} + 1)$, together with Eq. (27). For the same set of parameters and initial conditions as in Fig. 2, the outcome of the variance $\langle(\Delta\hat{X})^2\rangle = \langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2$ as a function of time is displayed in the top panel of Fig. 3, along with the corresponding numerical result for the sake of comparison. Again, the short time behavior of the approximate result matches that of the converged numerical one, as seen in the inset of the figure, revealing a small degree of squeezing (i.e., $\langle(\Delta\hat{X})^2\rangle < 1/2$, see the blue dashed line in the figure as a reference) at certain values of time. Discrepancies between the approximate and the numerical results become noticeable for times larger than $\omega_0 t \simeq 40$, where, unlike the numerical outcome, our approximation still predicts squeezing in both quadratures around the interval $60 < \omega_0 t < 80$ and a more pronounced dispersion at certain periods of time. A similar behavior is observed for the dispersion in the conjugate quadrature \hat{P} , as seen in the bottom panel.

In order to make a link between the previous results and the evolution of the field on phase space let us consider the Husimi Q function, which is defined as the diagonal matrix element of the field density operator $\hat{\rho}_F = \text{Tr}_A\{\rho_{AF}\}$ (the trace is now over the atomic variables) between standard coherent states, i.e.,

$$Q(z) = \frac{1}{\pi} \langle z|\hat{\rho}_F|z\rangle, \quad (29)$$

where $|z\rangle = \exp(-|z|^2/2)\sum_k z^k/\sqrt{k!}|k\rangle$. For the particular case we have been focused on ($\Psi(t_0) = |e, 0\rangle$), substitution

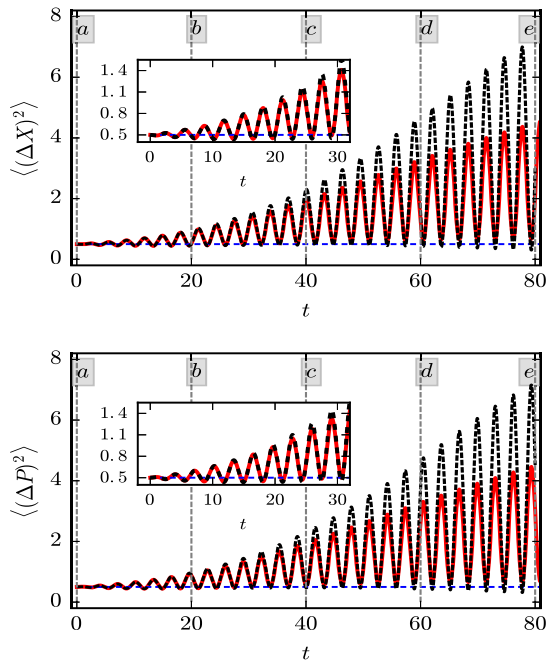


Fig. 3. (Top) Plot of the variance of the field quadratures $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ and (bottom) $\hat{P} = (\hat{a} - \hat{a}^\dagger)/\sqrt{2}i$ as functions of time for the same set of parameters as in Fig. 2. The inset focuses on the shortened time interval, $0 < \omega_0 t < 30$, within which a good agreement between the numerical (red line) and semianalytical (black dashed line) results can be observed.

of $\hat{\rho}_F$ into this expression allows us to arrive at the sought result:

$$Q(z) = \frac{1}{\pi} e^{-|z|^2 + 2\Re\{\gamma_4 + z^{*2}\gamma_2 e^{2\gamma_1}\}} (e^{2\Re\{\beta_z\}} + |z\beta_+|^2 e^{2\Re\{\gamma_1 - \beta_z\}}). \tag{30}$$

This distribution function is portrayed as a density plot in the upper row of Fig. 4 (the lower row corresponds to the numerical result) at the time instants $\omega_0 t = 0, 20, 40, 60,$ and $80,$

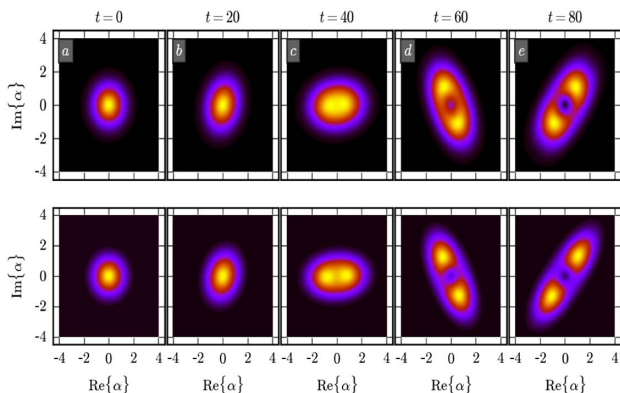


Fig. 4. Evolution of the Q function of the field density operator at time (in units of $1/\omega_0$) at instants $t = 0, 20, 40, 60,$ and 80 when the initial state of the composite system is such that $\rho_{AF}(0) = |e, 0\rangle\langle e, 0|$. The numerical and semianalytical results correspond, respectively, to the upper and lower rows. The set of system parameters is the same as in Fig. 2 (resonant regime).

which, in turn, are indicated by the dashed black vertical lines labeled as $a, b, c, d,$ and $e,$ respectively, in Fig. 3. At $\omega_0 t = 0,$ the state of the field starts as a minimum-uncertainty state. It evolves and at, say, $\omega_0 t = 20,$ it attains a somewhat more elongated form along the vertical axis whose dispersion, given by $\langle (\Delta \hat{P})^2 \rangle$ (see right panel of Fig. 3) is duly quantified to be slightly above the coherent-state value. We also see that the presence of the atom fosters the formation of what seems to be well localized two-phase components [see frames (d) and (e)], which in fact begin to be barely observable at $\omega_0 t = 40$ [frame (c)]. Although our approach assesses a wider spreading effect of the state’s phase-space distribution, reflected in the dispersion relations of Fig. 3, than the one observed for the numerical outcomes for $\omega_0 t > 40,$ it correctly predicts (qualitatively) the foregoing cat-like splitting behavior on short time scales. In Ref. [29], we calculated the Husimi function for a single nonstationary cavity mode immersed in a nonlinear Kerr medium, and we found a similar pattern, that is, the formation of distinguishable quantum superpositions arising from the vacuum state due to the presence of the nonlinear medium.

4. CONCLUDING REMARKS AND OUTLOOKS

In this work, we have developed an approximate method for the study of a closed cavity with a moving wall executing periodic motion with a two-level atom inside the cavity. We take as unperturbed Hamiltonian that of the atom, the field, and the part arising from the motion of the mirror. The interaction part of the Hamiltonian is that corresponding to the atom-field interaction, which is treated within the rotating wave approximation, and we thus have an interaction that preserves the total number of excitations. This approximation is valid whenever the atom-field coupling $g \ll \Omega.$ In order to test the validity of our method, we considered the same set of parameters as those used in Ref. [12], where analytical expressions were obtained in the dispersive and the resonant limits, and we found a good agreement with their results. We also considered cases far from the resonance and the dispersive regimes, and we made a numerical evaluation in order to test our approximate results. We want to stress the fact that in the numerical evaluation, we did not make use of the rotating wave approximation. One of the main effects due to the presence of the atom inside the cavity is an enhancement in the generation of Casimir photons.

Finally, it is worth underlining the possibility of adapting the present algebraic scheme to carry out the assessment of quantum fluctuations of physical interest that, as far as we know, has not been undertaken in this context, such as time-dependent spectrum of light [30], phase- and intensity-intensity correlation measurements [31], which involves the calculation of standard correlation functions, provided we restrict ourselves to the short-time behavior of the system. Another potential application would be to provide a protocol and/or scheme to the thermalization of the cavity through a random injection of atoms into it (see, e.g., Refs. [32,33]) like the micromaser scenario; this would open up the possibility of implementing quantum heat engines in the context of nonstationary cavities. A modest step toward tackling the problem of an empty nonstationary lossy cavity where the DCE in its bounded regime is manifested can be found in Ref. [34].

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