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Acoustic intensity of narrowband sources using the phase and amplitude gradient estimator method

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The phase and amplitude gradient estimator (PAGE) method [D.C. Thomas et al., J. Acoust. Soc. Am. 137, 3366-3376 (2015)] has proven successful in improving the accuracy of measured energy quantities over the traditional p-p method in several applications. One advantage of the PAGE method is the use of phase unwrapping, which can provide increased measurement bandwidth. However, narrowband sources often do not have coherent phase information over a sufficient bandwidth for a phase unwrapping algorithm to unwrap properly. Still, the PAGE method yields correct acoustic intensity measurements for frequencies up to the spatial Nyquist frequency for both broadband and narrowband sources. This is an improved bandwidth over the traditional method. For narrowband sources above the spatial Nyquist frequency, special conditions or processing are necessary for the PAGE method to provide accurate acoustic intensity. With sufficient bandwidth and a coherence of at least 0.1 at the spatial Nyquist frequency, a relatively narrowband source above the spatial Nyquist frequency can be unwrapped accurately. Also, with an assumption of a propagating field, and therefore linear phase, the extrapolated PAGE method uses the phase of a tone below the spatial Nyquist frequency to extrapolate phase above the spatial Nyquist frequency.

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1. INTRODUCTION

A. ACOUSTIC INTENSITY

Energy-based methods in acoustics can provide informative ways of analyzing acoustic fields. The main energy-based quantities are acoustic intensity, acoustic energy density, and specific acoustic impedance. The focus in this work is on acoustic intensity. Acoustic intensity is a vector, so it can provide not only magnitude information, but also provide a direction. For active intensity, this direction can aid in identifying propagation directions. Direction of propagation can help characterize a source by identifying which regions of the source are radiating more dominantly.[1] Intensity also can be used to find the sound power of a source.[2-5] Several methods for these sound power calculations as well as other applications of intensity have become published standards.[6-13] In addition, intensity can be useful in nearfield acoustical holography, which is a way to use pressure and/or particle velocity measurements at one location to visualize the field at another location.[14-19]

Complex acoustic intensity can be expressed in the frequency domain as:

$$\boldsymbol{I}_{\mathrm{c}} = \frac{1}{2} p \boldsymbol{u}^*, \qquad (1)$$

where p refers to pressure, $\boldsymbol{u}$ refers to particle velocity, vector quantities are in bold, and the * denotes the complex conjugate. The active intensity, expressed as $\boldsymbol{I}$, is the real part of $\boldsymbol{I}_{\mathrm{c}}$; the reactive intensity is expressed as $\boldsymbol{J}$ and is the imaginary part of $\boldsymbol{I}_{\mathrm{c}}$. Particle velocity can be directly measured using a particle velocity probe such as the Microflown[20-22], but such probes can be very sensitive to air flow in the acoustic field. Alternatively, when Euler's equation is used to relate particle velocity to the gradient of pressure, we can rewrite the complex acoustic intensity as

$$\boldsymbol{I}_{\mathrm{c}} = -j \frac{1}{\rho_0 \omega} p \nabla p^*. \qquad (2)$$

Acoustic intensity can be measured in several ways based on Eq. (2). One of the most prevalent ways is referred to in the literature as the p-p method, in which a probe with multiple microphones is used to estimate the gradient of pressure by using the change in the real and imaginary parts of pressure divided by the microphone spacing.[23-25] This p-p method is hereafter referred to as the traditional method.

The traditional method has several limitations of varying degree. One significant limitation is that estimating the gradient as the change in the real and imaginary parts of pressure over a distance between microphones is only a good estimation when the microphone spacing is small relative to a wavelength. This causes an underestimation of particle velocity when the microphone spacing begins to be sufficiently large relative to a wavelength. This is sometimes quantified by normalizing by the spatial Nyquist frequency, f_N, which is defined as the frequency at which half a wavelength is equal to the spacing between the microphones. At much lower frequencies, any inherent phase mismatch can cause significant errors because when the microphone spacing is very small relative to a wavelength, the actual phase difference being measured is small, and therefore the sensor phase mismatch becomes a relatively larger error. Between these two main types of errors, there is only a fairly limited bandwidth over which the traditional method can be adequately used. These and other errors have been discussed at length in the literature,[26-33] and many have tried to overcome the errors using varying experimental placement or processing.[34-37]

B. THE PAGE METHOD

To overcome some of the problems of the traditional method, the phase and amplitude gradient estimator (PAGE) method has been developed.[38,39] Instead of using formulations which split the complex pressure into real and imaginary parts, as is done in the traditional method, the formulations for the PAGE method represent the complex pressure as magnitude and phase, based on expressions from Mann et. al.[40] and Mann and Tichy.[41,42] The expressions for active and reactive intensity in the PAGE method are:

$$\boldsymbol{I} = \frac{1}{2\rho_0 \omega} P^2 \nabla \phi \qquad (3)$$

and

$$\boldsymbol{J} = -\frac{1}{2\rho_0 \omega} P \nabla P. \tag{4}$$

Here, P represents the pressure magnitude and ϕ represents the pressure phase. These expressions are advantageous, especially in propagating fields, because the magnitude and phase of pressure vary less spatially than the real and imaginary parts of pressure, which allows for a more accurate estimation of the particle velocity across a wider range of frequencies. However, although the PAGE method can be accurate to higher frequencies, the spatial Nyquist frequency can still be a limiting factor on the bandwidth for which PAGE calculations can be accurate.

i. Phase unwrapping

Under certain conditions, phase unwrapping can be used in the PAGE method to obtain meaningful particle velocity estimates above the spatial Nyquist frequency. The phase of a transfer function between microphones, an important element in obtaining the particle velocity estimate in the PAGE method, starts to be erroneous above the spatial Nyquist frequency due to a phenomenon known as phase wrapping. In this phenomenon, the phase difference between microphones reaches $-\pi$ or π at the spatial Nyquist frequency and multiples of it, at which points the phase makes a 2π jump. These jumps make the phase inaccurate above that frequency. With phase unwrapping,[43] all of the jumps are removed to obtain a continuous phase relationship, as demonstrated in Figure 1. Thus, phase unwrapping can provide correct particle velocity estimation and therefore accurate calculations of acoustic intensity above the spatial Nyquist frequency.

Phase unwrapping, and therefore energy quantity calculation above the Nyquist frequency, consistently works only under specific conditions. The first requirement is that the phase vary linearly as frequency increases, or at least is locally linear. This requires a field that is at least mostly propagating. Also, the signal must have sufficient frequency information for an unwrapping algorithm to resolve how to properly remove phase jumps and obtain an accurate continuous phase relationship. For example, phase unwrapping works well for broadband noise because there is phase information at every frequency.[44-48]

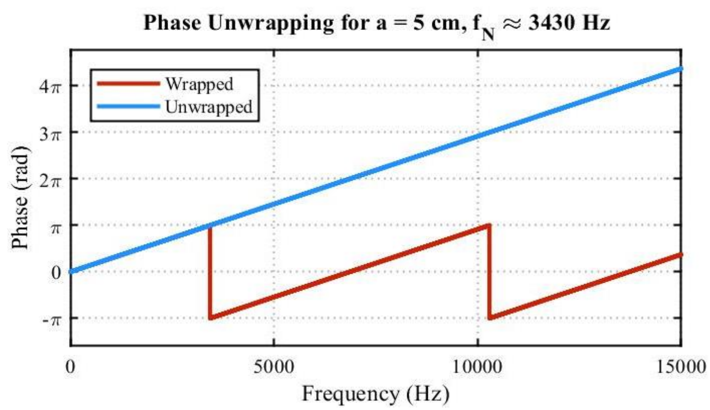

Figure 1: An illustration of phase unwrapping for two microphones with 5 cm spacing and a spatial Nyquist frequency of 3430 Hz.

C. THE NARROWBAND PROBLEM

Although phase unwrapping and the PAGE method have been shown to work well for broadband sources, problems may arise when the source is not broadband. In narrowband signals, such as a sawtooth wave, there is only coherent phase information at very specific frequencies corresponding to the peaks in the sawtooth wave. Therefore, phase unwrapping is unable to piece together a correct phase for the portions of the signal above the spatial Nyquist frequency because it does not have phase information at enough frequencies. The limits of the PAGE method for narrowband sources are demonstrated through the exploration of three signals: sine waves, combinations of tones, and bandlimited white noise.

2. SINE WAVES

A. EXPERIMENT

The main purpose of this experiment is to verify that the PAGE method successfully estimates sine waves at frequencies up to the spatial Nyquist frequency and to observe if the spatial Nyquist frequency changes based on the rotation angle of the probe relative to the source.

A two-dimensional, five-microphone probe with five GRAS phase-matched microphones was used for all experiments. The probe has two orthogonal pairs with one microphone in the middle, as can be observed in Figure 2. The probe diameter was 4 in (0.102 m) and therefore had an approximate spatial Nyquist frequency of $f_N \approx 1688$ Hz. In all cases in this paper, the microphone probe was mounted on a turntable, allowing for different angles of incidence from the speaker to the probe. The rotation angle of the probe was considered to be 0° when the source was on the same line as microphones 1, 2, and 3, and microphone 2 was the closest to the source (see Figure 2). Also, where possible the source or sources were kept at approximately the same height as the top of the microphones, and all sources were at least 2 m away so that the sound field would behave locally like a plane wave at the probe, where applicable.

With the speaker at different angles of incidence to the probe, the microphone spacing across which the plane wave propagates is expected to change, resulting in a different spatial Nyquist frequency. For example, if the probe is at 45°, the microphone spacing would be effectively smaller, as seen by the wave. It was expected that the effective spatial Nyquist frequency would increase as

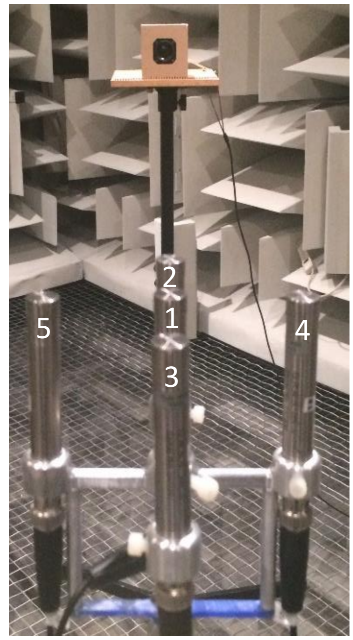

Figure 2: Setup with speaker on a stand and the five-microphone probe.

$$f_{N,\mathrm{eff},23} = \left|\frac{f_{N,0°}}{\cos(\theta_{\mathrm{rotation}})}\right| \quad \textit{(5)}$$

and

$$f_{N,\mathrm{eff},45} = \left|\frac{f_{N,0°}}{\sin(\theta_{\mathrm{rotation}})}\right|, \quad \textit{(6)}$$

where $f_{N,\mathrm{eff},ij}$ is the effective spatial Nyquist frequency for microphones i and j, $f_{N,0°}$ is the spatial Nyquist frequency for a 0° angle of rotation, and $\theta_{\mathrm{rotation}}$ is the angle of rotation. Due to the combination of the two orthogonal pairs on the same probe, the overall effective spatial Nyquist frequency of the probe, or $f_{N,\mathrm{eff}}$, is the lower frequency of $f_{N,\mathrm{eff},23}$ and $f_{N,\mathrm{eff},45}$. This results in the highest possible effective spatial Nyquist frequency occurring at $\theta_{\mathrm{rotation}} = 45°$, where there would be an effective microphone spacing of $2\sqrt{2}$ in (0.072 m). The resulting spatial Nyquist frequency for the whole probe would be $f_{N,\mathrm{eff}} \approx 2387$ Hz, which would be a significant improvement over the $f_{N,0°}$ of about 1688 Hz.

B. RESULTS

As expected, the PAGE method calculates the intensity magnitude and direction correctly below the spatial Nyquist frequency. For example, at 1600 Hz, the PAGE estimates of intensity magnitude and direction are shown in Figure 3 and Figure 4. The intensity direction error in degrees is calculated for each rotation angle as

$$I_{\mathrm{dir\ error}} = \frac{180}{\pi}\cos^{-1}\left(\frac{\boldsymbol{I}_{\mathrm{calc}} \cdot \boldsymbol{I}_{\mathrm{bench}}}{\left|\left|\boldsymbol{I}_{\mathrm{calc}}\right|\right|\left|\left|\boldsymbol{I}_{\mathrm{bench}}\right|\right|}\right), \quad \textit{(7)}$$

where $\boldsymbol{I}_{\mathrm{calc}}$ is the calculated two-dimensional vector, and $\boldsymbol{I}_{\mathrm{bench}}$ is the vector two-dimensional benchmark intensity obtained based on the rotation angle and a magnitude of $\frac{p_{rms}^2}{\rho_0 c}$ from the pressure at the center microphone. The improvement of the PAGE calculation over the traditional calculation can be seen in Figure 4 for intensity magnitude at a rotation angle of 0° and for intensity direction error over all of the rotation angles in Figure 5.

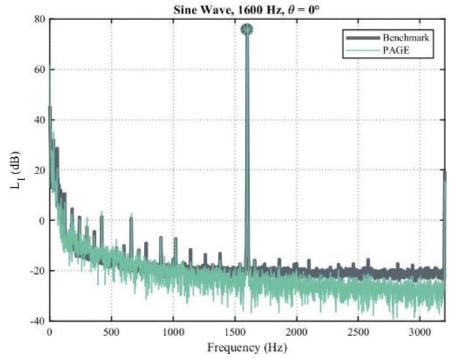

Figure 3: Comparison of intensity level, L_I, for a benchmark of $I = \frac{p_{rms}^2}{\rho_0 c}$ and the PAGE calculation for a probe with a 0° angle of rotation for a 1600 Hz sine wave in an anechoic chamber. Markers on each curve are at the frequency of the sine wave.

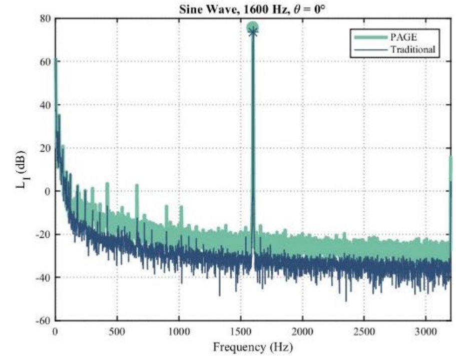

Figure 4: Comparison of intensity level, L_I, calculated from the PAGE and traditional methods of active intensity for a probe with a 0° angle of rotation for a 1600 Hz sine wave in an anechoic chamber. Markers on each curve are at the frequency of the sine wave.

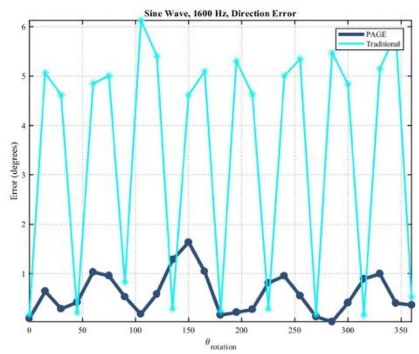

Figure 5: Comparison of the error in intensity direction for the PAGE and traditional methods as a function of rotation angle for a 1600 Hz sine wave in an anechoic chamber.

The upper limit of $f_{N,\text{eff}}$ was tested by seeing how high in frequency the sine wave magnitude and direction could be correctly calculated using the PAGE method. The theoretical maximum possible $f_{N,\text{eff}}$ is about 2387 Hz based on a rotation angle of 45° in Eq. (5) and Eq. (6). Based on this limit, a sine wave at 2300 Hz was tested. As can be seen in Figure 6, the PAGE method underestimates the intensity level at 2300 Hz for a rotation angle of 45°. Due to this, a lower frequency of 2100 Hz was tested for the same angle of rotation. The results of this test are in Figure 7 and it can be seen that the PAGE method accurately calculates the intensity level for this frequency. The comparison with the traditional method is observable in Figure 8 and Figure 9. The improvement for magnitude in Figure 8 is not as significant as might be expected because the effective microphone spacing is also smaller for the traditional method. In Figure 9, it is shown that some of the smallest errors at 2100 Hz are at 45°, 135°, etc., as expected. It is important to note that the measured $f_{N,\text{eff}}$ is at least 2100 Hz, with the possibility of it being between 2100 Hz and 2300 Hz.

Possible reasons for the experimental result to not quite reach the theoretical maximum could be directional alignment errors in the original setup, possible separation of the microphones from the centerline of propagation, or a sound speed different from the nominal 343 m/s used in calculations. Directional alignment errors in the setup were always kept to less than 3°, but a 3° error could decrease the theoretical maximum down to about $f_{N,\text{eff}} \approx 2271$ Hz. In this orientation, microphones 2 and 5 are the closest to the source (see Figure 2), but would be a maximum of an approximate 1° angle from the source. This 1° difference could make a difference of no more than 41 Hz on the spatial Nyquist frequency depending on the angle of rotation. If this error is combined with a 3° alignment error, the spatial Nyquist frequency maximum drops to approximately 2237 Hz. Differences in sound speed would only make a difference of about 7 Hz to the spatial Nyquist frequency per 1 m/s. The explanations presented are only possible reasons and one or all of them could be contributing factors to the experimental results not matching the theoretical maximum.

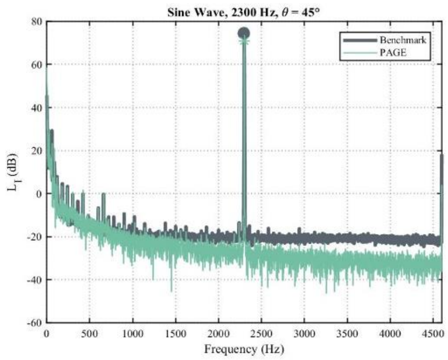

Figure 6: Comparison of PAGE and benchmark L_I for a 2300 Hz sine wave at 45°. Similar to Fig. 3.

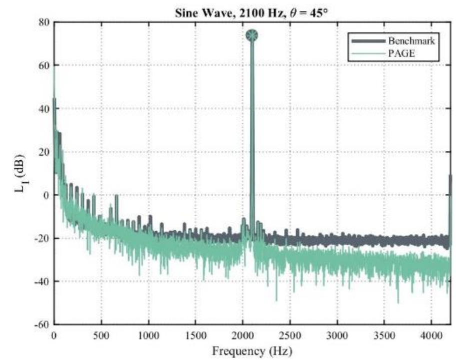

Figure 7: Comparison of PAGE and benchmark L_I for a 2100 Hz sine wave at 45°. Similar to Fig. 3.

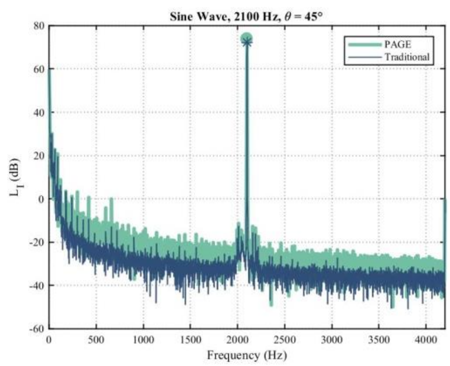

Figure 8: Comparison of PAGE and traditional L_I for a 2100 Hz sine wave at 45°. Similar to Fig. 4.

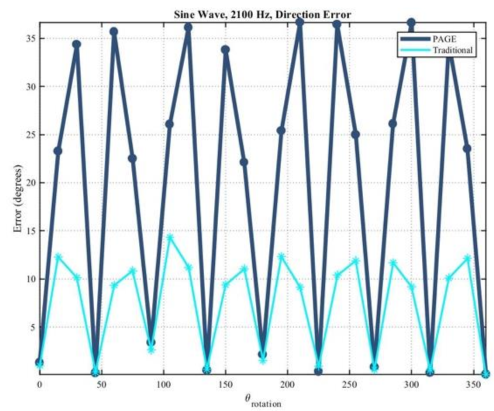

Figure 9: Comparison of PAGE and traditional intensity direction errors for a 2100 Hz sine wave. Similar to Fig. 5

In Figure 9, the smallest intensity direction errors occur at 0° and 45° and their counterparts (90°, 135°, etc.) for any frequency above $f_{N,0°}$. The relatively low errors at 0°, 90°, etc. despite their lower spatial Nyquist frequencies is due to the fact that the phase difference across one pair of microphones at that angle is very close to zero, which makes it easy for the direction to be calculated correctly even when the magnitude is not. It is also worth noting from Figure 9 that at angles where the frequency is only a few hundred Hz above the effective spatial Nyquist frequency for that rotation angle, the traditional method has a noticeably smaller direction error than the PAGE method does.

It is important to note that all of the calculations for the sine wave case were done without unwrapping. If unwrapping was attempted, significant errors occurred. In our processing, it was hardcoded that no unwrapping could occur until close to $f_{N,0°}$ to prevent erroneous unwrapping before it made sense for it to occur. However, since the spatial Nyquist frequency changes with angle of incidence, having a fixed frequency where unwrapping can begin to occur leads to erroneous wrapping results. Further details regarding this phenomenon can be found in Ref. 49.

3. MULTIPLE TONES

A. SAWTOOTH WAVES

i. Experiment

Experiments with sawtooth waves were conducted as a way to test a narrowband source containing several separate tones. It was expected that the PAGE method would work up to the effective spatial Nyquist frequency and that for a sawtooth wave, an extrapolated PAGE method could be used to obtain correct intensity magnitude and direction above the spatial Nyquist frequency. The experimental setup was the same as was used for sine waves, and can be seen in Figure 2. For reasons similar to those cited in the sine wave case, unwrapping was turned off for the normal PAGE processing.

ii. Results

As expected, the intensity of any peaks below the spatial Nyquist frequency was calculated correctly using the PAGE method. A 250 Hz sawtooth wave case demonstrates this effectively because it shows many peaks both above and below the spatial Nyquist frequency. The plot for the 250 Hz sawtooth wave intensity level for the PAGE method at a rotation angle of 45° can be seen in Figure 10. Notice that for this rotation angle, the PAGE method matches the benchmark well up through the peak at 2000 Hz. After that, the PAGE method starts to underestimate the benchmark. This further confirms our findings in the previous sections and narrows the upper range of the effective spatial Nyquist frequency for this spacing to be between 2100 Hz and 2250 Hz.

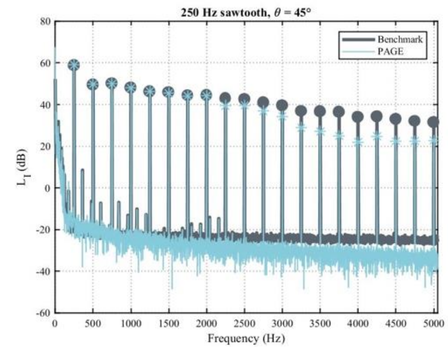

Figure 10: Comparison of PAGE and benchmark L_I for a 250 Hz sawtooth wave at 45°. Similar to Fig. 3.

The extrapolated PAGE method requires a propagating field and for at least one peak of the sawtooth to be below the effective spatial Nyquist frequency. For the two-dimensional probe, the method of extrapolating the phase based on the phase of the first peak was done for each microphone pair. The extrapolation is performed by assuming linear phase from 0 Hz through the frequency of the fundamental, and then extrapolating that linear phase through all higher harmonics. Due to the symmetry of the probe, the only pairs that end up contributing to the answer are microphones 2 and 3 and microphones 4 and 5 (see Figure 2). The intensity magnitude results of the PAGE method without extrapolation and with extrapolation are shown in Figure 11 and Figure 12, respectively, for a 1000 Hz sawtooth wave at a rotation angle of 45°. Note that the frequency range of these plots extends up to 20 kHz. The regular PAGE method in Figure 11 only matches the benchmark for the peaks at 1 kHz and 2 kHz, as would be expected for a 45° rotation angle. Also notice in Figure 12 that despite the sawtooth wave not being an ideal sawtooth due to the imperfections of the speaker, the extrapolated PAGE matches the benchmark up to 20 kHz. Figure 13 shows the improvement of the extrapolated PAGE method over the traditional method for intensity magnitude. The comparison of the direction error that is calculated from Eq. (7) for the extrapolated PAGE method, the PAGE method, and the traditional method are compared as a function of rotation angle at 2 kHz in Figure 14 and at 20 kHz in Figure 15. For the comparison at 2 kHz in Figure 14, the PAGE and traditional methods predictably have their lowest error at rotation angles where 2 kHz is below the effective spatial Nyquist frequency, and the extrapolated PAGE method always has an error of less than 2°. For the comparison at 20 kHz in Figure 15, the extrapolated PAGE method still always has less than 2° of error but the other two methods have very large errors at all angles of rotation. Based on the estimated uncertainty in our setup, within about 3° was considered an acceptable tolerance for direction; the extrapolated PAGE method was always within this tolerance. The extrapolated PAGE method was able to successfully calculate intensity magnitude and direction at frequencies far above the spatial Nyquist frequency.

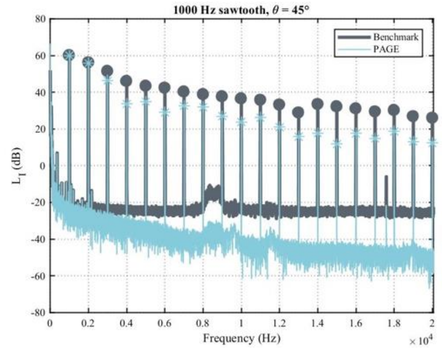

Figure 11**: Comparison of PAGE and benchmark *L_I* for a 1000 Hz sawtooth wave at 45°. Similar to Fig. 3.***

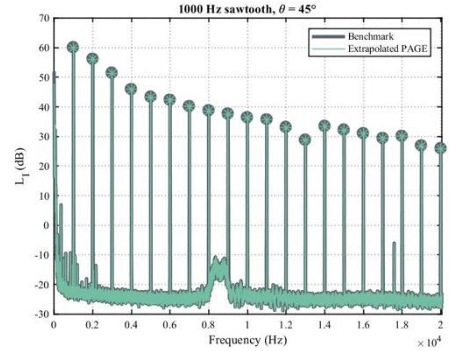

Figure 12: Comparison of extrapolated PAGE and benchmark L_I for a 1000 Hz sawtooth wave at 45°. Similar to Fig. 3.

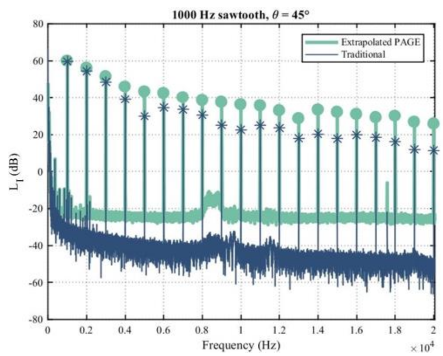

***Figure 13: Comparison of PAGE and traditional L_I for a 1000 Hz sawtooth wave at 45°. Similar to Fig. 4*.**

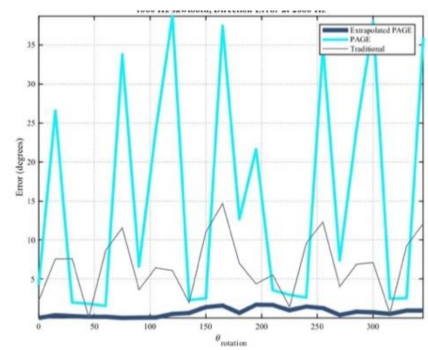

Figure 14: Comparison of PAGE and traditional intensity direction errors at 2000 Hz for a 1000 Hz sawtooth wave. Similar to Fig. 5.

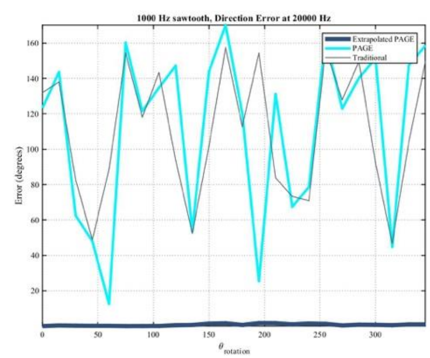

Figure 15: Comparison of extrapolated PAGE, PAGE and traditional intensity direction errors at 20000 Hz for a 1000 Hz sawtooth wave. Similar to Fig. 5.

B. TONES FROM SEPARATE SOURCES

i. Experiment

The purpose of this experiment was to see if the extrapolated PAGE method used for sawtooth waves could also be applied when the tone below the spatial Nyquist frequency came from a different source and potentially at a different angle than the tone above the spatial Nyquist frequency.

The setup for this experiment used two separate speakers. One speaker was on an arm connected to the turntable and therefore rotated with the probe, meaning it always had a 0° rotation angle. The other speaker was on a stand, slightly higher than the speaker on the arm to allow them to both be at a 0° rotation angle at the same time. This setup can be seen in Figure 16. The speaker on the arm, which is the lower one in Figure 16, was raised up on a piece of wood to decrease the effects of scattering off the arm. The blue piece of foam on the arm is also for the purpose of decreasing scattering. Both speakers are approximately 2 m away from the center of the probe in the horizontal direction.

One speaker broadcasted a sine wave at 1000 Hz and the other broadcasted a different, higher-frequency sine wave. The higher-frequency sine wave from the second speaker was chosen to be above the spatial Nyquist frequency for all rotation angles, and the 1000 Hz sine wave from the first speaker was always below the spatial Nyquist frequency. For reasons similar to those cited in the sine wave case, unwrapping was turned off for the normal PAGE processing.

Figure 16: A two-speaker setup in an anechoic chamber. The speaker on the arm rotates, so the angle of rotation is the same as the angle of separation between the speakers.

ii. Results

From normal PAGE processing, as expected, it was found that the normal PAGE method underestimates the intensity level for the higher frequency from the second speaker because it is above the spatial Nyquist frequency.

Similar to the sawtooth wave case, the extrapolated PAGE method was used in an effort to improve on the PAGE method. However, in this case the 1000 Hz tone from one speaker was used to extrapolate the phase of the transfer function in hopes of being able to correctly calculate the intensity for the tone above the spatial Nyquist frequency from the other speaker. The extrapolated phase was first utilized at a 0° rotation angle. For this rotation angle, it was expected that the extrapolated PAGE would work very well for both direction and angle because both sources are propagating from the same direction. From Figure 17, it can be seen that the extrapolated PAGE method was successful in matching the benchmark intensity magnitude even for the 2500 Hz tone that is above the spatial Nyquist frequency and from a different source. The improvement over the traditional method can be seen in Figure 18. The comparison of the error in direction for the extrapolated PAGE method, the PAGE method, and the traditional method at the 2500 Hz is in Figure 19, as calculated from Eq. (7). This graph will be discussed more in subsequent paragraphs, but at 0° it can be observed that the extrapolated PAGE method has no direction error.

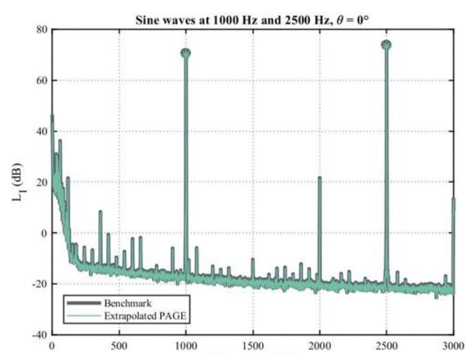

Figure 17: Comparison of extrapolated PAGE and benchmark L_I for two sine waves at 1000 Hz and 2500 Hz. Similar to Fig. 3.

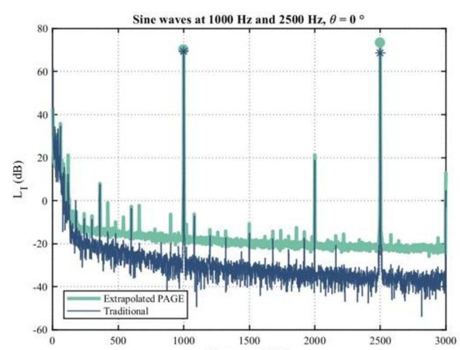

Figure 18: Comparison of extrapolated PAGE and traditional L_I for two sine waves at 1000 Hz and 2500 Hz. Similar to Fig. 4.

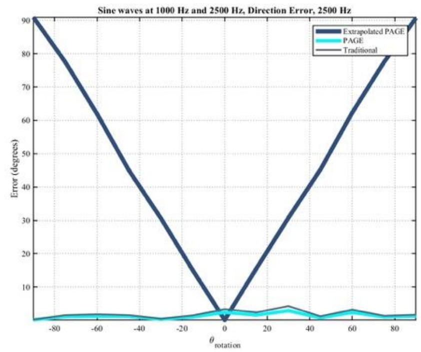

Figure 19: Comparison of extrapolated PAGE, PAGE and traditional intensity direction errors for two sine waves at 1000 Hz and 2500 Hz. Similar to Fig. 5.

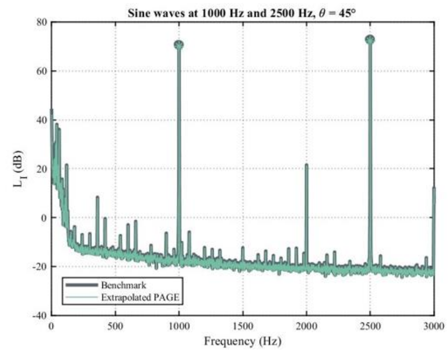

Figure 20: Comparison of extrapolated PAGE and benchmark L_I for two sine waves at 1000 Hz and 2500 Hz at 45°. Similar to Fig. 3.

As seen in Figure 19, the direction error for the extrapolated PAGE method is essentially the same as the rotation angle. In retrospect, this makes sense since the 1000 Hz tone is always at a rotation angle of 0°, and the phase of the 1000 Hz tone is what is used to extrapolate the phase of the 2500 Hz tone. Thus, the 2500 Hz tone always gets a direction from the extrapolated PAGE method of 0°. However, at every tested angle the extrapolated PAGE method did calculate the intensity level correctly for all higher frequencies tested. As an example, for a frequency from the second speaker of 2500 Hz, the intensity level results can be seen for extrapolated PAGE at 45° in Figure 20. This figure shows that the magnitude is correctly calculated using the extrapolated PAGE method even though the error in direction is 45°.

From the results in this section, it is seen that the extrapolated PAGE method for the two-speaker case explored here is only valuable when the angle of rotation is within the allowable direction error for a given measurement. Within this range, the intensity magnitude and direction are both calculated correctly by the extrapolated PAGE method, even if one tone is above the spatial Nyquist frequency. However, at greater angular differences, it may be just as valuable to just use one microphone to get the magnitude without frequency limitations because the direction obtained from this method is so inaccurate.

4. BANDLIMITED WHITE NOISE

A. EXPERIMENT

The purpose of these experiments was to discover if unwrapping could occur with bandlimited noise that is entirely above the spatial Nyquist frequency. The bandlimited white noise was obtained by applying low-pass and high-pass filters to broadband noise. The cutoffs of the filters are the frequencies used to specify the bands, which are denoted by Δf, and the filters being used are third-order Butterworth filters. It was expected that with sufficient bandwidth, even bands of noise at very high frequencies could be accurately unwrapped. It was also expected that larger bands would be required for proper unwrapping at higher frequencies. The experimental setup had the probe on the turntable and one speaker on a stand about 2 m away and at approximately the same height, as seen in Figure 2.

B. RESULTS

As with most experiments involving broadband noise, the amount of averaging in the processing makes a significant difference on the random error. The time data used for this experiment was 15 seconds long, collected at a 96 kHz sampling frequency. The averaging was done with a 50% overlap of blocks and a block size of 9600, resulting in about 300 averages. This helped overcome some problematic extraneous noise in the setup, such as the electrical noise of the turntable.

The higher the frequency band was above the spatial Nyquist frequency ($f_N \approx$ 1688 Hz), the higher Δf had to be for unwrapping to occur properly. A graph of the increasing amount of bandwidth needed for unwrapping to work properly as the center frequency of the band increased can be seen in Figure 21. Only bands of noise completely above the spatial Nyquist frequency were tested, but some of the coherent bandwidth could potentially extend below the spatial Nyquist frequency. Each case was only considered to "work" if phase unwrapping worked for all rotation angles. For the highest frequency band tested, a center frequency (f_c) of 18 kHz required a Δf of 12 kHz for unwrapping to occur properly. The smallest band had an f_c of 1800 Hz and only required a Δf of 100 Hz.

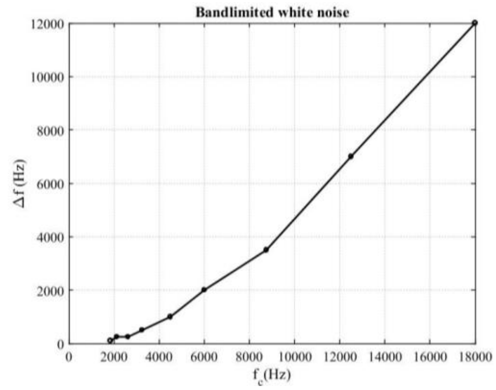

Figure 21: For bandlimited white noise, the center frequency of each band (f_c) is com-pared to the amount of bandwidth needed for unwrapping to work properly (Δf).

Regardless of the other parameters, all cases worked if the coherence for both microphone pairs was above 0.1 for the entire band from 1500-1800 Hz ($f_N \approx$ 1688 Hz). Some cases still worked with a lower coherence than 0.1 for that bandwidth, but less consistently. This suggests that only rarely can bands of noise which have an entire coherent bandwidth far above the spatial Nyquist frequency use phase unwrapping accurately. However, a coherence of 0.1 is still relatively low, and other methods such as coherence-based unwrapping[50] could be used to improve the results.

One example of a working case is with a f_c = 6000 Hz and a Δf = 2000 Hz. The comparison between the benchmark and the PAGE and traditional methods for intensity magnitude is shown in Figure 22, with the accompanying phase and coherence results in Figure 23. These results are for a 0° angle of rotation. Notice the slight increase in coherence around 1500 Hz in Figure 23, resulting in a coherence above 0.1 above 1500 Hz. The noisy intensity level and phase below the spatial Nyquist frequency can be attributed to the very low coherence there. The improvement over the traditional method is in Figure 24 for direction across all angles of rotation. The results for all other cases were very similar, with the improvement over the traditional method growing even more impressive with increasing center frequency.

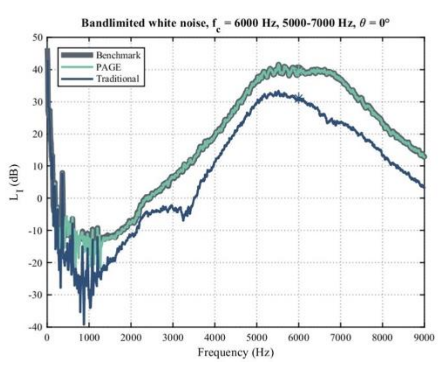

Figure 22: Comparison of PAGE, traditional, and benchmark L_I for bandlimited white noise from 5000-7000 Hz. A marker on each curve is at f_c.

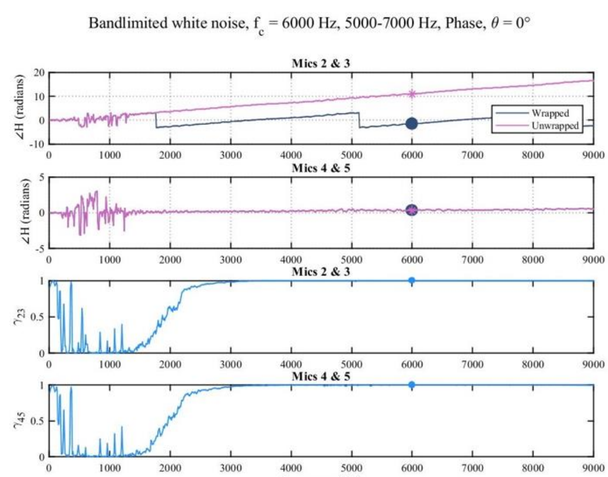

Figure 23: Comparison of the wrapped and unwrapped phase of the transfer function as well as the coherence for each microphone pair for filtered white noise from 5000-7000 Hz. A marker on each curve is at f_c.

5. CONCLUSIONS

In an anechoic chamber, experiments using sine waves, multiple tones, and bandlimited white noise were performed to test the narrowband limits of the PAGE method. For all cases, a probe with a spatial Nyquist frequency of about 1688 Hz for normal incidence was used.

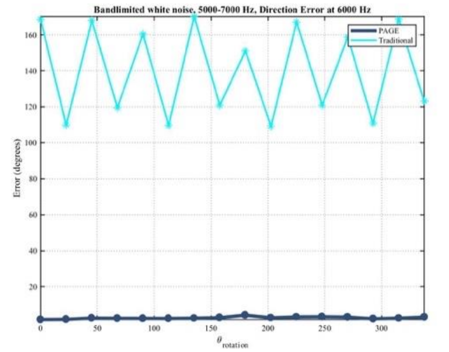

Figure 24: Comparison of PAGE and traditional intensity direction errors for bandlimited white noise from 5000-7000 Hz.

In the first experiment, it was confirmed that the intensity magnitude and direction of sine waves were calculated correctly using the PAGE method up to the spatial Nyquist frequency. As part of this testing, it was found that an effective spatial Nyquist frequency could be obtained from the effective microphone spacing at a rotation angle. This effective spatial Nyquist frequency is higher than the spatial Nyquist frequency based only on the maximum spacing of the microphones, or the microphone spacing for a 0° angle of rotation. It was found that the actual effective spatial Nyquist frequency was at least 2100 Hz but less than 2250 Hz, despite a theoretical value of 2387 Hz. Possible reasons for this error were discussed, including possible alignment errors in the experimental setup. All successful PAGE calculations were improvements over the traditional method.

The sawtooth wave results were as expected. First, they confirmed the effective spatial Nyquist frequency findings from the sine wave experiments. Then, an extrapolated PAGE method was used to accurately predict the magnitude and phase of the sawtooth wave up to 20 kHz. This is the entire range of human hearing and represents a bandwidth extension of over 10 times. This represents a significant improvement over both the normal PAGE method and the traditional method.

The extrapolated PAGE method was then applied to sound from two speakers, each playing different sine waves: one produced a tone above the spatial Nyquist frequency and one produced a tone below the spatial Nyquist frequency. When the two speakers broadcasted in the same direction, this method worked very well. However, due to how the method is implemented, the intensity direction always stayed the same as the tone below the spatial Nyquist frequency, meaning the error in degrees was essentially equal to the angle of separation between the speakers. The intensity magnitude was always correct using the extrapolated PAGE method. For intensity magnitude, the extrapolated PAGE method represents an improvement over the normal PAGE method and the traditional method. However, for intensity direction the extrapolated PAGE only does as well as the normal PAGE method and the traditional method for very small angles of separation between the speakers.

To measure the magnitude and direction of bandlimited white noise completely above the spatial Nyquist frequency, it was found that phase unwrapping can work properly for a bandwidth of noise at least mainly above the spatial Nyquist frequency, but it requires a greater bandwidth at higher frequencies. However, for phase unwrapping to work consistently, a coherence of at least 0.1 in the frequency range from 1500-1800 Hz ($f_N \approx$ 1688 Hz) was required. The results obtained represent a significant improvement over the traditional method.

Although the extrapolated PAGE method used was effective for some cases, several improvements could be made to the extrapolated PAGE method to make it more widely applicable. The extrapolated PAGE method required some phase information below the spatial Nyquist frequency, usually in the form of a tone or fundamental of the sawtooth. Also, the assumption of a propagating field was necessary for it to work. Further, all of the frequency content above the spatial Nyquist frequency needs to be coming from approximately the same direction as the source with the tone below the spatial Nyquist frequency, or else significant direction errors result. One possible improvement would be to take into account all phase information below the spatial Nyquist frequency. Another, more significant improvement would be to indirectly extrapolate by using the phase information below the spatial Nyquist frequency and assumption of linear phase to make the necessary number of "unwraps," or 2π jumps, instead of imposing an unwrapped phase based on one point. The advantage of this is it is more based on the real data being taken above the spatial Nyquist frequency, and would likely increase the separation angle over which it could be effective.

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