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Single-point characterization of spectral amplitude and phase changes due to nonlinear propagation

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A frequency-domain representation of the Burgers equation reveals that the cross-spectrum between the pressure and pressure-squared waveforms can be used to calculate nonlinear frequency-domain effects of finite-amplitude sound propagation. The normalized version of the quadspectrum, Q/S, was introduced by Morfey and Howell and has since been used to point to the nonlinear transfer of energy between frequencies, in particular gaining use in the domain of high-amplitude jet noise propagation. However, one question that remained was that of the interpretation: The physical meaning of the amplitude of Q/S was unclear. Recent analytical work has recast Q/S and the normalized version of the cospectrum, C/S, as a way to estimate sound pressure level and phase changes due to nonlinearity with a single-point measurement. This paper uses various measurements within a plane-wave tube to verify the physical significance of the amplitude and phase changes predicted by Q/S and C/S. Experiments involving sinusoids and band-passed Gaussian noise at various amplitudes show the validity of the single-point measurement to measure the strength of nonlinear effects in both amplitude and phase.

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1. INTRODUCTION

In the ensemble-averaged, frequency-domain version of the Burgers equation, the cross-spectrum of the waveform with the square of the waveform becomes important in determining nonlinear effects.[1] The quadspectrum, or imaginary part of the cross-spectrum, is often referred to in the normalized form as Q/S or the Morfey-Howell indicator. This indicator, defined as

$$\frac{Q}{S} = \frac{Q_{pp^2}}{p_{\mathrm{rms}}\, S_{pp}} = \frac{\mathrm{Im}\{E[\mathcal{F}^*\{p(t)\}\mathcal{F}\{p^2(t)\}]\}}{p_{\mathrm{rms}} S_{pp}}, \quad (1)$$

has been used within nonlinear acoustics, and in particular within the jet[2,3] and rocket noise[4] communities, to describe a transfer of energy between frequencies due to nonlinear propagation. The normalized cospectrum, C/S, is defined as[5]

$$\frac{C}{S} = \frac{C_{pp^2}}{p_{\mathrm{rms}}\, S_{pp}} = \frac{\mathrm{Re}\{E[\mathcal{F}^*\{p(t)\}\mathcal{F}\{p^2(t)\}]\}}{p_{\mathrm{rms}} S_{pp}}, \quad (2)$$

While C/S has not been widely used, Q/S has been used to point to nonlinear effects in the frequency domain but has often suffered from a lack of quantitative interpretation, with the amplitude in particular proving difficult to decipher. However, recent work has recast the frequency-domain version of the Burgers equation to show the physical meaning of Q/S in an understandable form.

By taking the real part of the frequency-domain version of the Burgers equation, the change in level as a function of frequency can be calculated for a plane wave as a sum of two terms representing absorption and nonlinearity. For a plane wave, this formulation of the frequency-domain Burgers equation can be written as[6]

$$\frac{\partial L_p}{\partial x} = -10 \log(e) \left(2\alpha + k\beta \mathrm{M}_{\mathrm{rms}} \frac{Q}{S}\right). \quad (3)$$

These two terms, referred to hereafter as ν_α and ν_N, represent the change in level with distance due to absorption and nonlinearity, calculated as a function of frequency. The term α represents the real part of the absorption coefficient, while k represents the wavenumber. The variable β is the coefficient of nonlinearity (1.2 in air), and $\mathrm{M}_{\mathrm{rms}}$ is the rms acoustic Mach number, $p_{\mathrm{rms}}/p_{\mathrm{atm}}$. This formulation has been shown to accurately depict the rate of change in level for simple sinusoidal cases[6] and, with an additional term describing geometric spreading, has seen use within jet noise as well.[7]

If instead of taking the real part of the Burgers equation, the imaginary is selected, the real part of the normalized cross-spectrum, or normalized cospectrum C/S, can be seen to represent a phase change due to nonlinear propagation. As the Burgers equation relates to the retarded time waveform, this phase change is not due to propagation, but due to parts of the waveform traveling faster or slower relative to the rest of the waveform, which change the phase of the complex Fourier transform. The derivation, performed by Ohm *et al.*,[5] results in

$$\frac{\partial \theta}{\partial x} = -\delta + \frac{\beta k M_{\mathrm{rms}}}{2} \frac{C}{S}. \quad (4)$$

Similar to the terminology used to describe Eq. (3), the terms in Eq. (4) will be referred to as ξ_δ and ξ_N. Linear dispersion is accounted for by δ (the imaginary component of the complex absorption coefficient), which is usually a small effect in air, but can be significant in a plane-wave-tube environment. In contrast, ξ_N describes a nonlinear phase change due to steepening in the time domain. For an initially sinusoidal signal, phase of harmonics is constant and C/S would be expected to be zero. However, in noise portions of the waveform will have greater speed relative to the rest of the waveform, resulting in phase changes.

Together, Q/S and C/S can function as single-point indicators of nonlinear effects on both level and phase changes due to nonlinearity. Though both equations have received some validation in the form of numerical simulations,[6,5,8] this paper presents the first test of their accuracy in assessing changes due to nonlinearity in the experimental setting of a plane wave tube experiment. After the experimental setup is described, changes in level and phase are calculated using the equations above from a center microphone and compared with measured changes from pairs of microphones located at different distances from the center microphone. This comparison shows that, in the form used here, Q/S and C/S are useful single-point measures of the spatial rate of change of level and phase caused by nonlinear propagation.

2. EXPERIMENTAL SETUP

To verify the validity of using Q/S and C/S as single-point indicators of nonlinear behavior, the spectral evolution was measured within a plane-wave tube. The PVC tube had a 2 in. diameter and was made of two separate 6.1 m sections, with an anechoic termination placed at the end of the second section. Holes were drilled so that 1/8" G.R.A.S. 40DD microphones could be mounted flush with the inner wall of the PVC. The tube was excited with a compression driver playing Gaussian noise band-passed from 500-1500 Hz. Microphones were placed at distances of 1.85, 2.25, 2.35, 2.45, and 2.85 m from the driver, so that estimates of the spatial derivative in level at a distance change of $\Delta x = 1.0$ m and $\Delta x = 0.2$ m can be compared against predictions from Q/S and C/S using the center microphone at 2.35 m.

3. RESULTS

A. SPECTRA

Spectral changes with distance are shown in Fig. 1. Though the noise played through the source driver was initially band-passed from 500-1500 Hz, at a distance of 1.85 m (red) significant high-frequency energy is present as nonlinear propagation has steepened the waveform and pushed energy into higher frequencies. As the waveform propagates further downstream to 2.85 m from the source (green) the high-frequency energy has increased to greater levels, and a combination of nonlinear effects and linear absorption have led to a slight decrease in level in the peak frequency region from 500-1500 Hz. However, because propagation takes place in a plane-wave tube environment, overall sound pressure level (OASPL) remains nearly constant throughout the measurement, dropping slightly from 148.5 dB to 148.1 dB. Some small spectral effects, in particular at 2-3 kHz at 2.85 m, can be seen as a result of imperfect junctions in between sections of the PVC pipe which cause reflections and interference at some microphones.

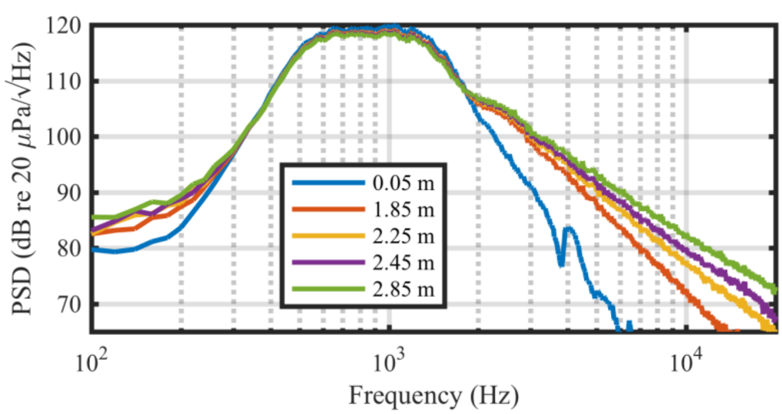

Figure 1. **Spectra at different distances along the plane wave tube.**

B. CHANGE IN LEVEL

The change in level predicted using Q/S in Eq. (3) can be verified by taking the difference in levels seen in Fig. 1 around the midpoint of 2.35 m from the noise source. The waveforms from this mid-point microphone are used to calculate the changes in Eqs. (3) and (4). To better compare against the theoretical predictions, the difference between spectra in Fig. 1 are divided by the distance to give a rate of change in terms of dB/m. This rate of change is shown in Fig. 2(a) for the two microphones located at 2.25 and 2.45 m, and in Fig. 2(b) for the microphones located at 1.85 and 2.85 m. Because of the spectral effects associated with reflections within the tube, some harmonic interference is seen in Fig. 2(a) for the experimental results. However, the predicted changes closely mirror the expected behavior if the harmonic interference is averaged out. The predicted changes of $\nu_N + \nu_\alpha$ show even greater agreement in Fig. 2(b), where the two microphones were separated by 1 m. The accurate estimation of change in spectral levels show the usefulness of Q/S as a single-point measure to predict changes in level due to nonlinearity, accurately estimating the change in level with less noise than a set of two microphones located 0.2 m apart.

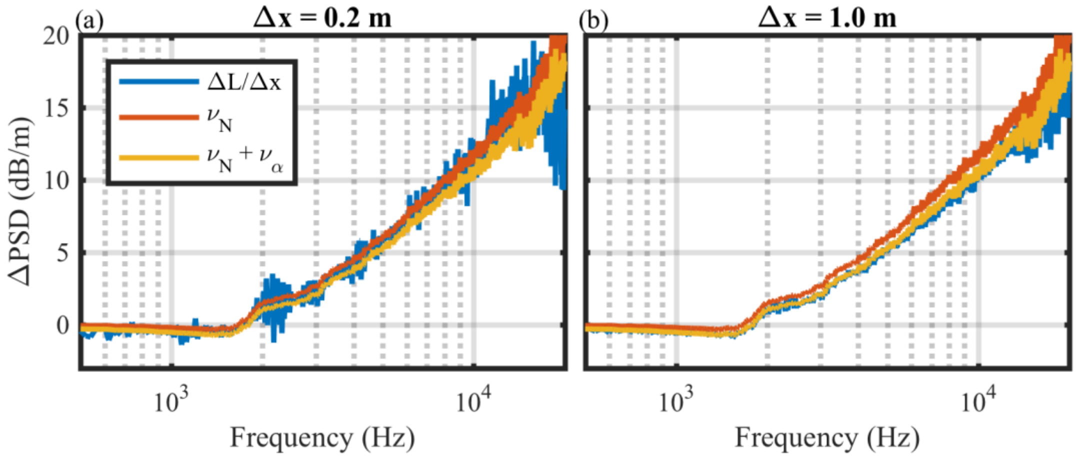

Figure 2. **Comparison of experimentally determined $\partial L/\partial x$ to ν_N and $\nu_N + \nu_\alpha$.**

C. CHANGE IN PHASE

While change in level is an important verification of Eq. (3), the change in phase presented in Eq. (4) presents another interesting test for the validity of *Q/S* and *C/S* as single-point indicators of nonlinear behavior. Similar to the changes in level shown in Fig. 2, the rate of change in phase across frequency is calculated between microphones and shown in Fig. 3. The change in phase is taken using the unwrapped phase of the cross-spectrum, with a linear phase shift added to correct for changes due to propagation distance since the Burgers equation predicts changes with respect to retarded time.

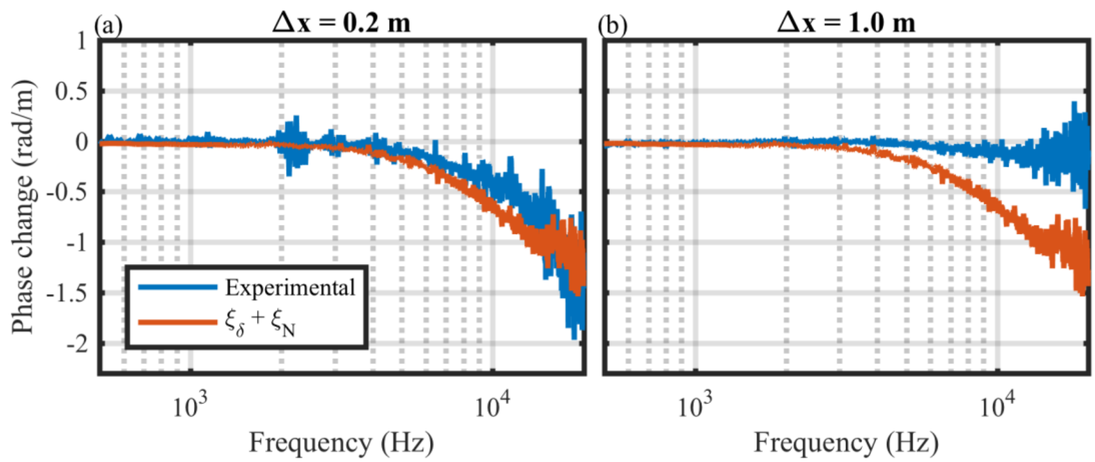

Figure 3. **Comparison of experimentally determined $\partial\theta/\partial x$ to $\xi_N + \xi_\delta$.**

At frequencies below 6-7 kHz the changes predicted using C/S align closely with those measured between microphones at 2.25 and 2.45 m, shown in Fig. 3(a). However, the experimentally measured phase change between the microphones at 1.85 and 2.85 m, shown in Fig. 3(b), does not agree with either the C/S prediction or the experimental change from Fig. 3(a). The reason for this is that the coherence (not shown here) drastically drops between these two microphones located 1 m apart. At 4 kHz, the coherence drops below 0.9, and at 10 kHz it is as a value of 0.7. Because the high-frequency noise is increasing significantly between these two microphone locations (10 dB/m at 10 kHz) the phase changes cannot be accurately estimated using the phase of the cross-spectrum, as much of the high-frequency information did not exist at the first measurement point. This also explains the differences seen in Fig. 3(a) above 6-7 kHz, as high-frequency coherence also begins to drop at this frequency for the closer microphone spacing. However, the agreement shown in Fig. 3(a) shows that changes in phase can be approximated using a single-point measurement, but comparison with numerical simulations may further strengthen its validity.

4. CONCLUSION

Recent analytical formulations of the frequency-domain, ensemble-averaged Burgers equation have been tested in an experimental setting to show their validity as single-point indicators of change due to nonlinear propagation. The use of Q/S and C/S is shown to agree well with experimentally determined rates of change. The ability to directly predict the rate of change in level at a point allows for a direct comparison of nonlinear and absorptive effects, while changes in phase may be more inherently tied to the waveform steepening process and give insight to where shock formation is occuring. In addition, the single-point nature of the indicators shows significant advantages over sets of microphones in this plane-wave tube experiment. Changes in level are better shown over a pair of microphones separated by a larger distance, while low coherence at farther separation distance limits the ability to predict a consistent phase change with the same set of microphones. The use of these single-point indicators can accurately estimate changes in both phase and level without concern for the competing causes of microphone separation distance.

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