Plane-wave tube validation of bandwidth extension for energy-based quantities using pressure gradient methods

Daxton Hawks, Tracianne B. Neilsen, Kent L. Gee, and Scott D. Sommerfeldt

Citation: Proc. Mtgs. Acoust. **30**, 055019 (2017); doi: 10.1121/2.0001044
View online: https://doi.org/10.1121/2.0001044
View Table of Contents: https://asa.scitation.org/toc/pma/30/1
Published by the Acoustical Society of America

Volume 30 http://acousticalsociety.org/

Acoustics `17 Boston

173rd Meeting of Acoustical Society of America and 8th Forum Acusticum
Boston, Massachusetts
25-29 June 2017

Signal Processing in Acoustics: Paper 2aSPb2

Plane-wave tube validation of bandwidth extension for energy-based quantities using pressure gradient methods

Daxton Hawks
Brigham Young University-Idaho, Rexburg, IDAHO; daxtonhawks5@gmail.com

Tracianne B. Neilsen
Department of Physics and Astronomy, Brigham Young University, Provo, UTAH, 84602; tbn@byu.edu; tbnbyu@gmail.com

Kent L. Gee and Scott D. Sommerfeldt
Brigham Young University, Provo, UTAH, 84602; kentgee@byu.edu; scott sommerfeldt@byu.edu

Energy-based quantities, such as acoustic vector intensity, kinetic energy density, and specific acoustic impedance, rely on the acoustic particle velocity. The particle velocity is often approximated via Euler's equation using the gradient of the complex pressure across closely spaced microphones, which is traditionally found using the cross-spectral density. In contrast, the Phase and Amplitude Gradient Estimator (PAGE) method [Thomas et al., J. Acoust. Soc. Am., 137, 3366-3376 (2015)] relies on gradients of pressure magnitude and phase. For a broadband source, the PAGE method allows for the phase to be unwrapped, which extends the usable bandwidth of the particle velocity calculations well above the spatial Nyquist frequency. The benefits of the PAGE method are demonstrated in plane wave tube experiments in which two microphones are spaced 10-360 cm apart. The traditional processing method underestimates active acoustic intensity and kinetic energy density well below the spatial Nyquist frequency. The PAGE method, however, extends the reliable bandwidth of active acoustic intensity, kinetic energy density, and specific acoustic impedance, to the spatial Nyquist frequency, and above when phase unwrapping can be applied. This increased high frequency reliability allows for microphones to be spaced farther apart, which then increases the low frequency reliability as well.

Published by the Acoustical Society of America

 https://doi.org/10.1121/2.0001044

1. INTRODUCTION

Plane-wave tube experiments were conducted to validate the ability of the the Phase and Amplitude Gradient Estimator (PAGE) method [1] (PAGE) method to extend the bandwidth of energy-based quantities using a two-microphone probe. The PAGE method was originally developed to maximize the useable bandwidth of active, acoustic vector intensity measurements in rocket noise source characterization in outdoor settings.[2] The PAGE method has also been applied to measure acoustic intensity near military aircraft[3] and laboratory-scale jets.[4]

The standard p-p method for active acoustic intensity, which will be referred to as the traditional method, involves ensemble-averaging of finite sums and differences of the complex pressure between pairs of microphones to estimate the acoustic particle velocity.[5] The traditional method has been used in both engineering and research applications for source modeling and localization.[6] As microphone spacing becomes comparable to wavelengths, spatial aliasing causes bias errors in the estimated pressure and particle velocity required for the intensity calculation.[7] This limitation[8] has been studied,[9] the appropriate bandwidth has been defined as a function of microphone spacing and included in many standards.[10,11] The PAGE method uses the amplitude and phase components of the complex pressures, which typically vary more linearly across the microphone pair than the real and imaginary parts.[1] Bias error studies have recently been completed for the PAGE processing using two[12] and three[13] microphone probes. Both the theory and initial implementations of the PAGE method indicate robustness that extends the usable bandwidth of active intensity calculations.

While the PAGE method has greatly increased the usable bandwidth of intensity calculations in various applications,[2-4] a controlled plane-wave experiment is presented herein to validate the performance of the PAGE method, not just for acoustic intensity, but for other energy quantities as well. This work investigates the performance of the PAGE method in a plane-wave tube environment by comparing it to the results from the traditional method and analytical expressions. Calculations using both the traditional method and the PAGE method for active intensity, reactive intensity, potential energy, kinetic energy, and impedance are set forth. Results from this plane-wave tube experiment show how the PAGE method can extend the bandwidth of active intensity vector, kinetic energy density, and specific acoustic impedance.

2. BACKGROUND

Energy-based acoustical quantities can be used to characterize a sound field. These energy-based spectral quantities include acoustic intensity, acoustic energy density, and specific acoustic impedance. These quantities are obtained from the complex acoustic pressure, p, and particle velocity vector, $\boldsymbol{u}$, as a function of position r and angular frequency ω. While there are transducers that measure $\boldsymbol{u}$ directly, it is commonly estimated using the pressure gradient between closely-spaced microphones, The traditional method for estimating this gradient computes the difference between the real and imaginary parts of the complex spectra, whereas the PAGE method uses differences in the magnitude and phase. Both of these methods are described in this section.

A. DEFINTIONS

Definitions of the energy-based quantities are given prior to describing the processing methods. The complex pressure from microphone m can be written in two ways:

$$p_m = \mathrm{Re}(p_m) + j\,\mathrm{Im}(p_m) = P_m e^{-j\Phi_m},$$

where P and Φ are the magnitude and phase of the complex pressure. Sound pressure level (L_p) is calculated using the root-mean-square of the pressure, p_{rms}: $L_p = 20\log_{10}\left(\frac{p_{m,\mathrm{rms}}}{p_{\mathrm{ref}}}\right)$, where $p_{\mathrm{ref}} = 20\ \mu Pa$. The acoustic particle velocity can be expressed as

$$\boldsymbol{u} = \frac{j}{\omega\rho_0}\nabla p,$$

where ρ_0 is the ambient density. This expression is a representation of Newton's second law for a time-harmonic process. In practice, time-averaged values are used for particle velocity. Acoustic intensity is defined as the amount of sound power per unit area. In the frequency domain, the complex vector intensity is

$$\boldsymbol{I} = \frac{1}{2}\, p\boldsymbol{u}^*,$$

where * indicates complex conjugate. This complex acoustic intensity has real and imaginary parts that correspond to active and reactive intensity, $\boldsymbol{I}_a$ and $\boldsymbol{I}_r$ respectively. The sound intensity level (L_I) is $\mathrm{L_I} = 10\log_{10}\left(\frac{|I|}{I_0}\right)$, with $I_0 = 10^{-12}$ W/m^2. The potential energy density is

$$E_p = \frac{|p|^2}{4\rho_0 c},$$

where c is the ambient sound speed, and the kinetic energy density is

$$E_k = \frac{\rho_0}{4}|\boldsymbol{u}|^2.$$

Lastly, acoustic impedance is defined as a ratio of the complex pressure and the particle velocity. The specific acoustic impedance in the direction l is related to the component of acoustic particle velocity in that direction, u_l:

$$z_l = \frac{p}{u_l}.$$

B. PLANE-WAVE CASE

As an initial investigation of the performance of the PAGE method to calculate acoustic energy quantities, a plane-wave tube is used. A plane wave is defined such that there is constant phase and amplitude on surfaces perpendicular to the direction of travel. Investigating a plane-wave environment is ideal for the scope of this work for several reasons. Plane waves can describe the propagation of waves far from the sound source. In addition, general wave fields can be described as a superposition of plane waves with various amplitudes, frequencies, and phases. Expressions for the energy-based quantities for the case of a plane wave are now given.

For simple acoustic sources, analytical expressions for the energy quantities can be found. For a plane wave, the acoustic pressure and particle velocity are

$$p_A = P_0 e^{-jkr},$$

and

$$u_A = \frac{p_A}{\rho_0 c} = \frac{P_0}{\rho_0 c} e^{-jkr},$$

where k is the wavenumber. For a propagating wave, the sound pressure magnitude should match the active intensity magnitude

$$|\boldsymbol{I}_{a_A}| = \frac{P^2}{2\rho_0 c},$$

and the active intensity vector should point in the direction of propagation down the tube. The potential and kinetic energy density for a plane wave can be calculated using

$$E_{p_A} = \frac{P^2}{4\rho_0 c^2},$$
$$E_{k_A} = \frac{P^2}{4\rho_0 c^2}.$$

Finally, because ideally there are no standing waves in the plane wave case, the specific acoustic impedance is approximately

$$z_A = \rho_0 c,$$

and the expected reactive intensity should be zero. These analytical expressions allow us to evaluate the performance of the traditional and PAGE method calculations for this broadband plane-wave tube experiment.

3. METHODS

A. PROCESSING

Both the traditional method and the PAGE method for estimating acoustic particle velocity involve using the complex pressure as measured by various microphones, and the differences between the two methods are discussed.

The p-p method for approximating the particle velocity $\boldsymbol{u}$ involves estimating a gradient using pressures measured by two or more microphones. For a two-mic case, the gradient is approximated by

$$\widehat{\nabla p} = \frac{\Delta p}{d},$$

where Δp is the difference of measured pressures between microphones, and d is the spacing between the two microphones. In practice, the traditional method, involves ensemble-averaging of finite differences of the complex pressure between pairs of microphones to estimate the gradient.[14] For cases using more than two microphones, the division by d is replaced by a matrix operation as described in Thomas *et al.*[1] This gradient estimate is then used in Euler's equation to estimate $\boldsymbol{u}$. This finite-difference approximation is bandlimited due to two effects related to d. First, the highest frequency at which a reliable estimate can be obtained is restricted by the spatial Nyquist frequency (f_{Nyq}): the highest frequency at which a wave can be sampled at least twice per wavelength, $f_{\mathrm{Nyq}} = c/2d$. For $f > f_{\mathrm{Nyq}}$, spatial aliasing occurs. Thus, to extend the estimate to higher frequencies, d should be smaller. However, a smaller d increases the lowest frequencies at which reliable estimates of $\boldsymbol{u}$ can be obtained. The low frequency limit occurs when the phase difference between the microphones due to spacing d is comparable to the inherent phase mismatch of the microphones; to obtain reliable estimates of $\boldsymbol{u}$ at lower frequencies, d should be larger. Because the recommendation for extending the bandwidth to higher and lower frequencies contradict each other, a new processing method is proposed.

The traditional method for estimating $\boldsymbol{u}$ uses the difference between the real and imaginary parts of the complex pressure from the microphones to obtain Δp. Examples of the bias errors for plane waves and other simple sources are shown in Whiting *et al.*[12] for a two-microphone probe and Lawrence *et al.*[13] for a three-microphone one-dimensional probe. The traditional method begins underestimating the level of $\boldsymbol{u}$ at frequencies below f_{Nyq} .

These bias error studies[12-13] compare the bias errors from the traditional method with those obtained using the Phase and Amplitude Gradient Estimator method (or PAGE method), which has been developed as a method for extending the bandwidth of calculations of energy-based quantities. Similar to the traditional method, the particle velocity is estimated by estimating a pressure gradient across microphones. However, instead of using differences in the real and imaginary parts of the complex pressure, the PAGE method uses the magnitude and phase. The pressure amplitude gradient in the direction of propagation is estimated as

$$\widehat{\nabla P} = \frac{\Delta P}{d},$$

and the phase gradient using

$$\widehat{\nabla \phi} = \frac{\Delta \Phi}{d},$$

where ΔP and $\Delta \Phi$ are the magnitude and phase differences between two microphones. The phase difference is calculated by the argument of the transfer function, which can be expressed by

$$\Delta \Phi = \arg\left(\frac{p_1}{p_2}\right),$$

where p_1 and p_2 are pressure expressions as measured by two microphones. The PAGE method, by estimating the pressure amplitude and phase gradients separately, is able to overcome the frequency limitations of the traditional method and extend the usable bandwidth to $f = f_{\mathrm{Nyq}}$. This method is effective because the amplitude and phase vary more linearly over space than the real and imaginary parts of the complex pressure, and thus the estimate for acoustic intensity is more likely to be accurate. For $f > f_{\mathrm{Nyq}}$, errors are introduced in the calculation of $\Delta \Phi$ because of phase wrapping. The PAGE method has the potential to fix this problem by unwrapping the phase. Phase unwrapping works well for broadband noise sources. Details of the phase unwrapping are given in Cook *et al.*[15] The possibility of using background noise to apply the PAGE method to narrowband signals has been studied,[16] along with the effect of background noise.[17]

B. EXPERIMENT

The bandwidth extension capability of the PAGE method is demonstrated using a plane-wave tube experiment, shown in Figure 1. The plane wave tube is made of acrylic and has an anechoic termination. The tube's 10 cm inner diameter restricts the propagation to plane waves below 2kHz, as the first cross-modes have frequencies around 2.3 kHz. For this experiment, pressure waveform data were synchronously recorded on phase-matched G.R.A.S. 40AE 12.7mm microphones with a 24-bit National Instruments PXI-4462 card. The microphone diaphragms were flush with the inside of the tube and were placed with separation distances d (relative to the first microphone) of 10-360 cm. For reference, the maximum microphone separation distance used in most commercial intensity probes is only 10 cm, with an upper frequency limit (-2 dB error) of 1.2 kHz.

Figure 1. Photos of experimental setup. (Left) The DAQ is connected to the speaker system (on the left end of the plane wave tube), as well as the National Instruments Hardware on the shelf. The anechoic termination is visible on the far right of the image. (Right) Seven microphones spaced 10 cm apart, placed roughly in the middle of the tube. The middle microphone is not phase-matched, but the remaining six are phase-matched.

4. RESULTS

This section contains the results of using traditional and PAGE methods to estimate energy quantities from the plane wave tube experiment. Comparisons are made for different microphone spacings of the magnitude and direction of active intensities, kinetic and potential energy density, and impedance. Reactive intensity is the last quantity considered. For this two-microphone probe, these quantities are being estimated at the midpoint between the two microphones. The results using traditional and PAGE processing methods are compared to analytical values expected for a plane wave. Unless otherwise specified, the black lines indicate the analytical values expected for a plane wave, the blue line the traditional method calculations, and the red line the PAGE method.

A. ACTIVE INTENSITY

The first quantity compared is active intensity for a variety of microphone spacings, d. The magnitude and direction of the active intensity, I_a, are shown in Figure 2 and Figure 3, respectively, for $d = 10$, 40, 60, 120, 180, and 360 cm. In all cases, the traditional estimate of $|I_a|$ departs from the analytical value well below f_{Nyq}. The PAGE estimate of $|I_a|$ matches the analytical estimate to f_{Nyq} and then beyond, when phase unwrapping is applied, with one primary exception. The variations in the levels between 2300 and 3000 Hz are attributed to frequencies at which the first cross modes of the tube are excited by the broadband noise. Because of the coordinate system used, the correct direction of the active intensity vector is 90°, which is obtained by both processing methods until $f = f_{\mathrm{Nyq}}$; for higher frequencies, the PAGE method with phase unwrapping maintains the correct direction, while the traditional estimate changes between $\pm 90°$. These results show how the PAGE method can expand the bandwidth of active acoustic intensity vector estimates for broadband noise in a plane wave tube for different microphone spacings.

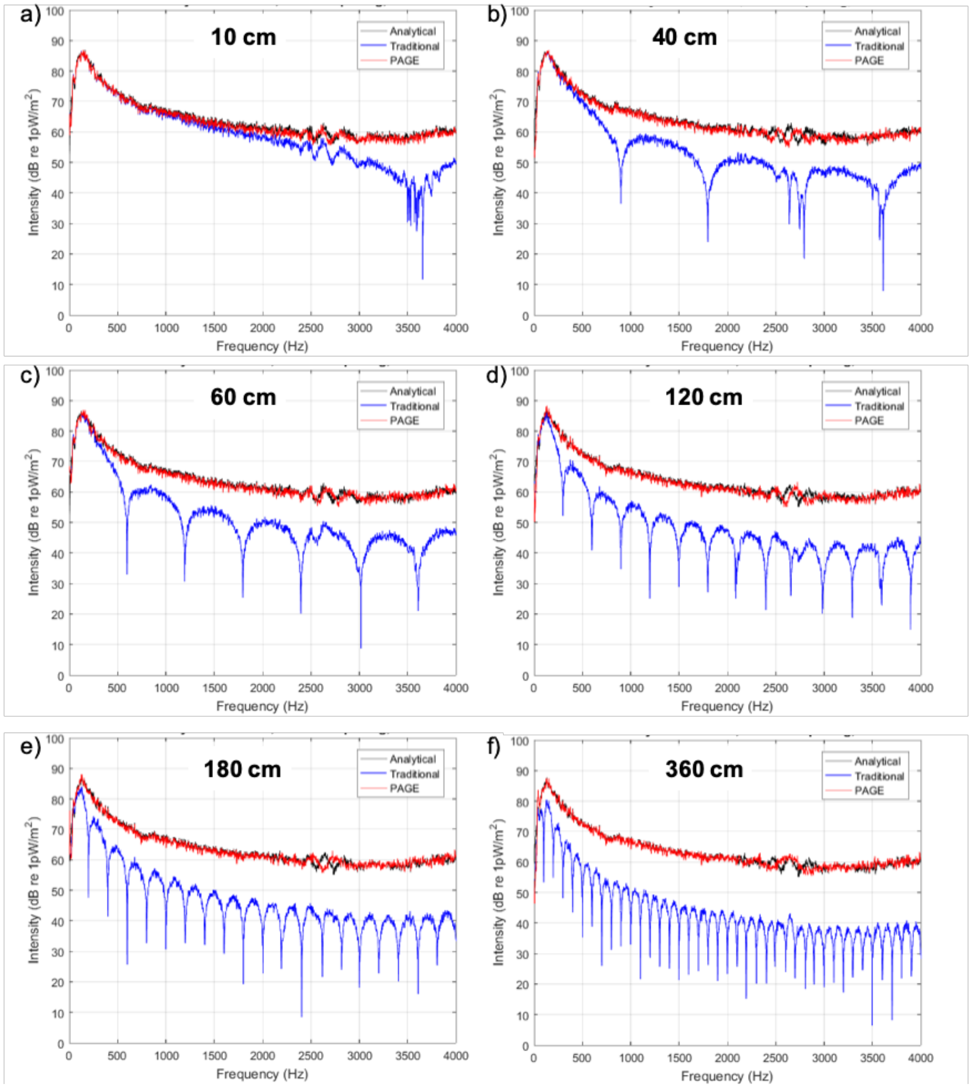

Figure 2 Magnitude of the active intensity using the analytical plane wave expression (black) compared to that estimated by the traditional method (blue), and the PAGE method (red) as microphone spacing, d, increases from 10 cm at the upper left to 360 cm in to the lower right. The traditional method result begins to underestimate the intensity below f_{Nyq} (the first downward spike on each figure). The PAGE method is consistent with the analytical calculations through except for the frequencies of the cross modes (2300-3000 Hz).

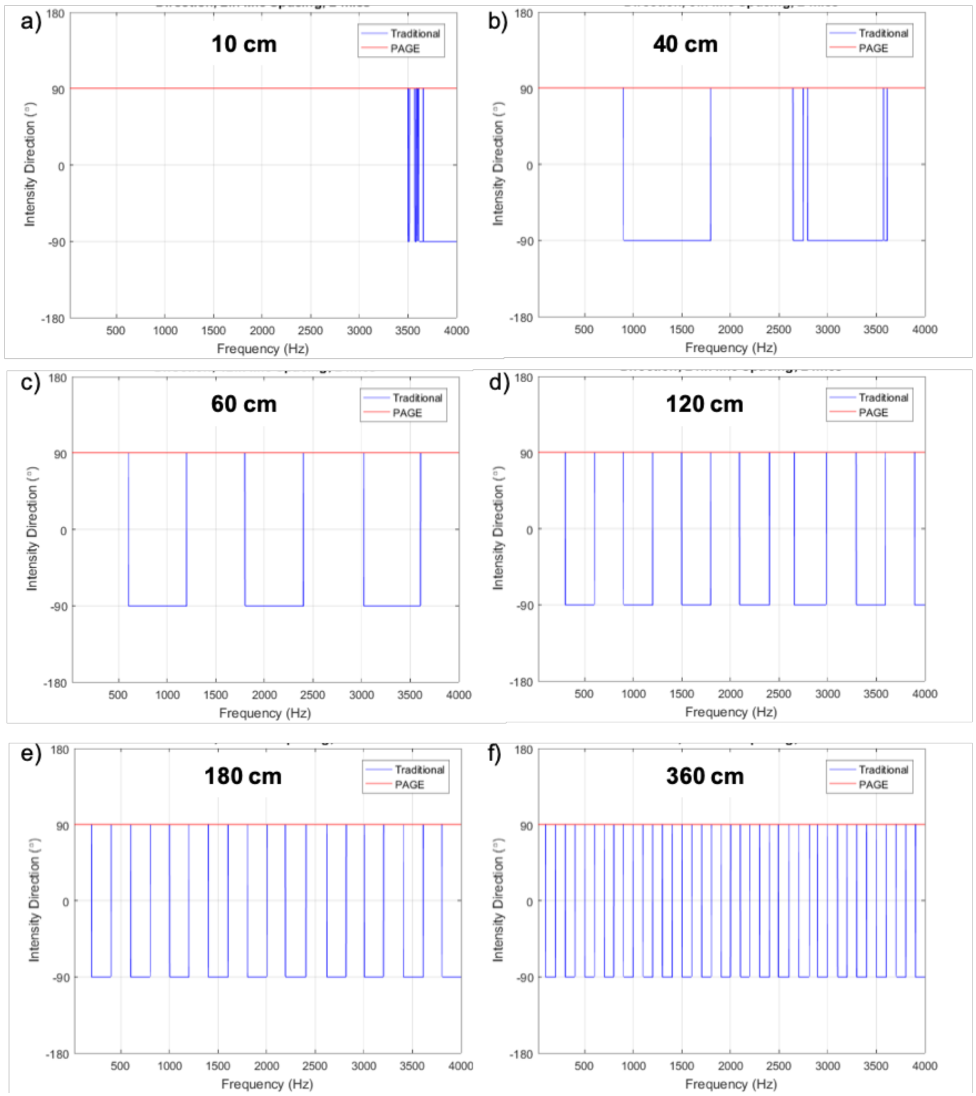

Figure 3. Direction of the active intensity estimated by the traditional method (blue), and the PAGE method (red) as microphone spacing, d, increases from 10 cm at the upper left to 360 cm in the lower right. For this plane wave tube experiment, the expected direction is down the tube—defined as 90°. Above f_{Nyq} the direction obtained with traditional processing alternately points in the opposite or same direction. The PAGE method, however, with unwrapping applied, calculates the direction as 90° for $f > f_{Nyq}$.

B. ENERGY DENSITY

The bandwidth of kinetic energy density calculations can also be extended using the PAGE method. The potential and kinetic energy density for the broadband noise in the plane wave tube are shown in Figure 4 for d =10, 60, and 360 cm. The calculated values from both the traditional and the PAGE methods for the potential energy are the same because they both rely on the average of the pressure between the two microphones. Because of the direct dependence on the center pressure, the estimate of potential energy density tends to improve when a microphone is included at the center of the probe. The estimates of kinetic energy density, which depend on the estimate of $\boldsymbol{u}$, exhibit the same features as $|I_a|$. The PAGE processing extends the bandwidth of reliable intensity estimates to $\mathrm{f} = \mathrm{f}_{\mathrm{Nyq}}$, and above with phase unwrapping.

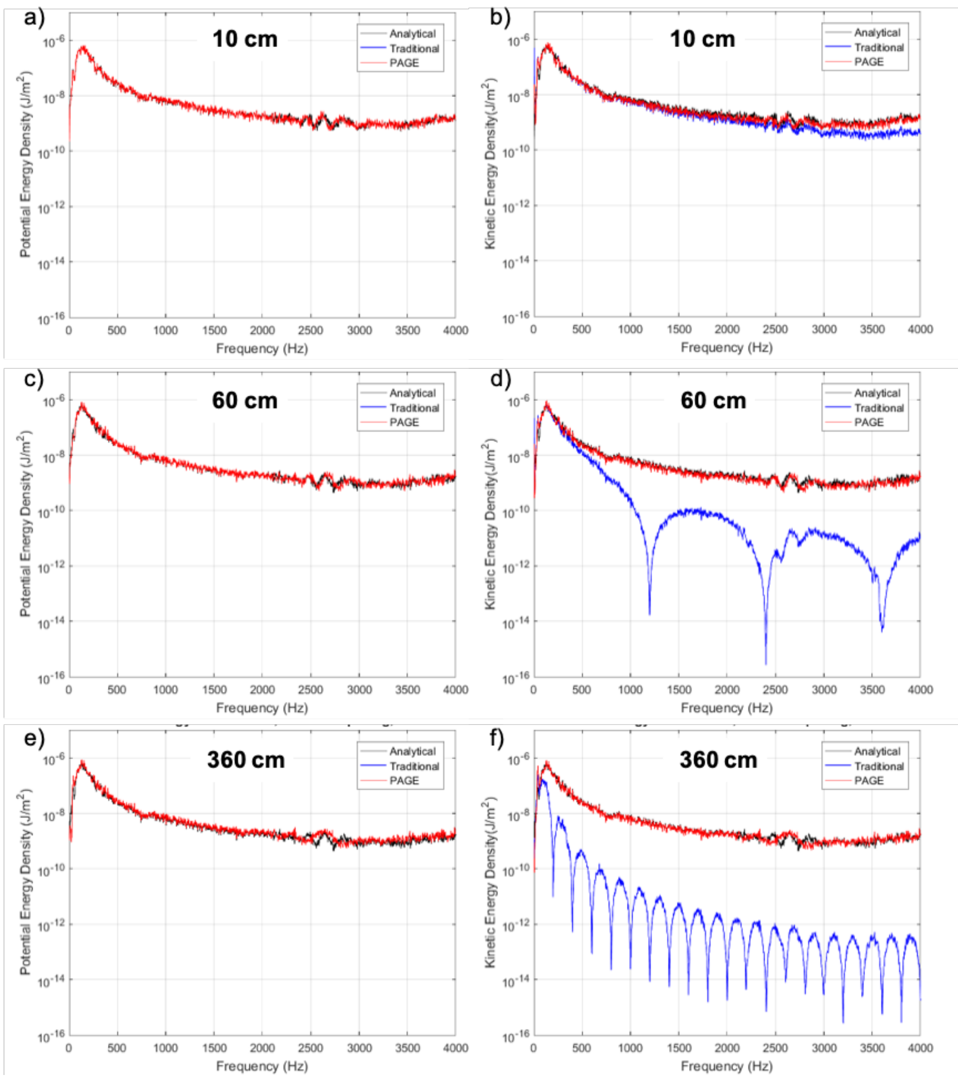

Figure 4. Potential (left) and kinetic (right) energy densities intensity using the analytical plane wave expression (black) compared to that estimated by the traditional method (blue), and the PAGE method (red) for microphone spacing, d=10, 60, and 360 cm.

C. IMPEDANCE

In the plane-wave environment, the expected magnitude of the specific acoustic impedance is $\rho_0 c$ and the expected phase is 0°. These analytical values and those estimated by the two processing methods are displayed in Figure 5. Below 500 Hz, the oscillations are likely due to limitations of the anechoic termination at these frequencies, and the rapid oscillations above 2000 Hz are related to the cross modes of the tube. Aside from these features, the general trends in the estimated impedance can be seen. As the frequency increases, the traditional method significantly overestimates the impedance magnitude. Likewise, the phase estimates by the traditional method change almost linearly with frequency, straying from the expected 0° phase. The PAGE method, however, obtains estimates near the expected values for both magnitude and phase of impedance. The offset in the impedance magnitude is likely due to the atmospheric conditions.

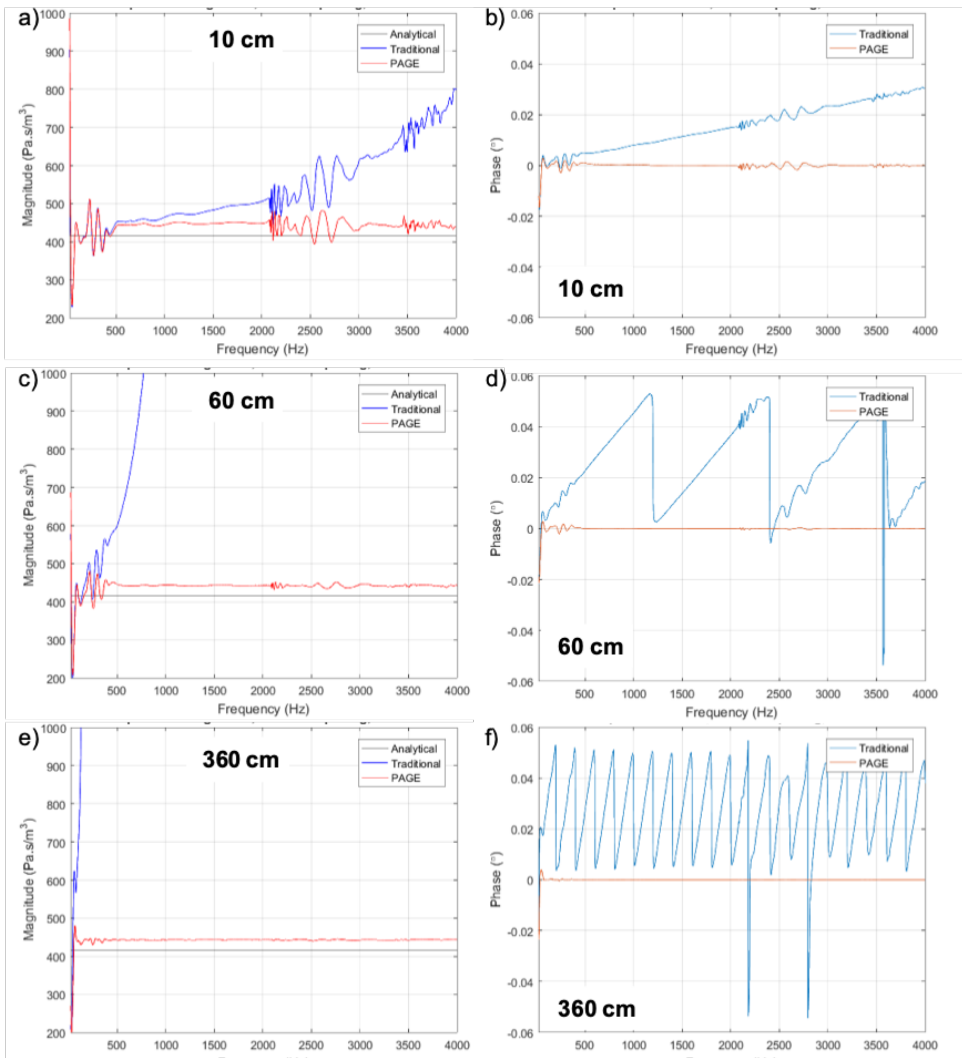

Figure 5. Impedance magnitude (left) and phase (right) using the analytical plane wave expression (black) compared to that estimated by the traditional method (blue), and the PAGE method (red) for microphone spacing, d=10, 60, and 360 cm.

D. REACTIVE INTENSITY

Theoretically, the reactive intensity should be zero for a plane wave. However, we see non-zero values in this experiment using both the traditional and the PAGE method. Nonzero values indicate a partially reactive sound field inside the tube. At low frequencies, this can be caused by limitations in the anechoic termination. Above 2000 Hz, the nonzero reactive intensity could be related to cross-modes being excited by the broadband noise. However, another possible reason—potentially affecting all frequencies—is that the tube contains additional noise generated outside the tube but entering in random directions. The effect of this background noise on the reactive intensity was evaluated using an ambient measurement: the broadband signal was off but the other measurement equipment (shown in Figure 1) was on. For this nondriven condition, the traditional and PAGE processing methods were applied with d = 10, 60, and 360 cm. The two processing results are similar, as shown in the upper row of Figure 6. The reactive intensity levels are approximately -20 dB for f= 500-2000 Hz. These levels for reactive intensity are reasonable given the background noise from the equipment. However, when the tube is driven by the loudspeaker generating broadband noise, the reactive intensity levels estimated by the traditional and PAGE methods are considerably higher, as shown on the lower row of Figure 6. In the 500-2000 Hz band, the reactive intensity levels estimated by the traditional method are similar to those obtained for active intensity, shown in Figure 2. The PAGE estimates for reactive intensity of the driven tube (lower row of Figure 6), while higher than expected, are approximately 25 dB less than those obtained for active intensity, shown in Figure 2. The reasons for these reactive intensity levels in the driven tube are investigated by Lawrence *et al.*[18]

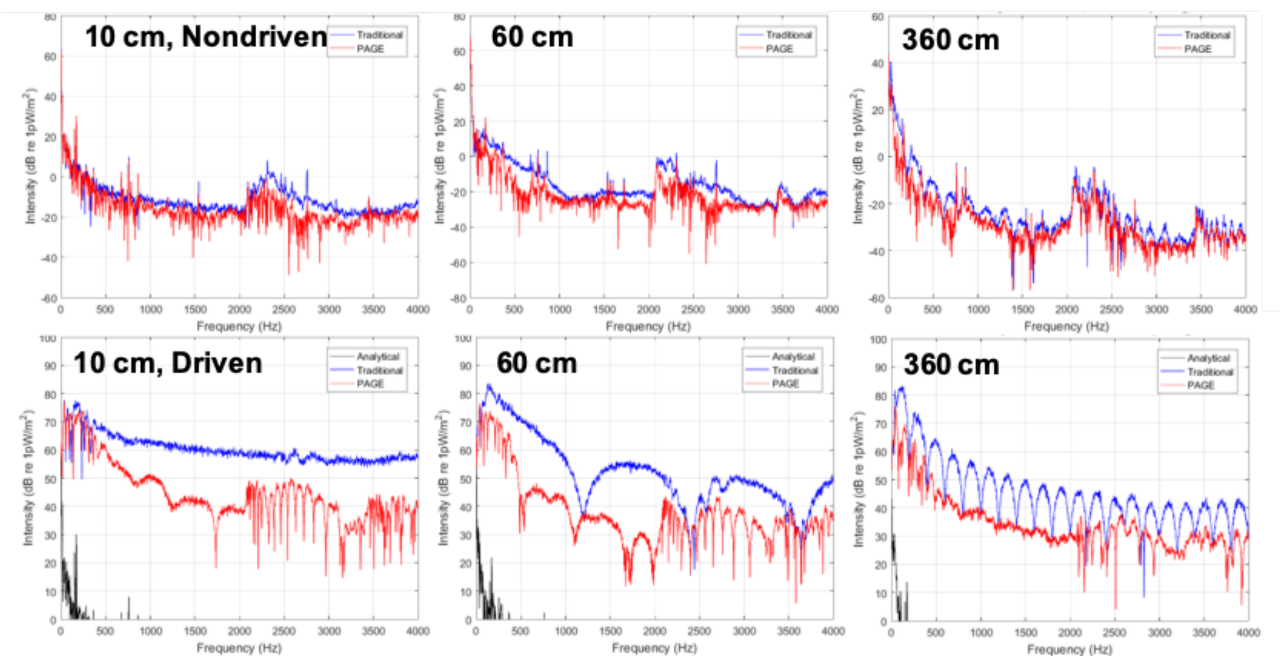

Figure 6. Reactive intensity estimated using the traditional (blue) and PAGE (red) methods for the nondriven tube (upper) and the tube driven with broadband noise (lower) for d =10 cm (left), 60 cm (center), and 360 cm (right). (The black lines are derived from the nondriven case, not the analytical value.)

5. CONCLUSIONS

The evaluation of the PAGE and traditional methods for calculating energy-based quantities has shown that the PAGE method substantially extends the usable frequency bandwidth for the active intensity vector, kinetic energy density, and specific acoustic impedance. A plane-wave tube experiment using broadband noise with anechoic termination has provided a means of further validating the PAGE processing method. Estimates for these energy-based quantities using a two-microphone probe were compared with analytical values expected for a plane wave for microphone spacings of 10 cm to 360 cm. The traditional p-p method underestimates the magnitude of active intensity and kinetic energy well below the spatial Nyquist frequency, f_{Nyq}. Meanwhile, the PAGE processing method obtains reliable estimates up to $f = f_{\mathrm{Nyq}}$, and above when phase unwrapping can be applied. The direction of the intensity vector is correct up to $f = f_{\mathrm{Nyq}}$ using both methods. Above this frequency, the traditional estimate of the direction becomes unstable while phase unwrapping allows the PAGE estimate to remain correct. The specific acoustic impedance magnitude and phase are considerably more consistent using the PAGE method than the traditional method. The levels of the reactive intensity, while not zero, are considerably lower using the PAGE method than the traditional method. Thus, the PAGE processing method has the potential to improve estimation of energy-density quantities with a two-microphone probe.

Several recent studies have explored the applications of the PAGE method. While the PAGE method extends the usable bandwidth for active intensity and kinetic energy up to $f = f_{\mathrm{Nyq}}$, reliable estimates for higher frequencies can be obtained only when phase unwrapping can be applied. Conditions under which phase unwrapping is likely to be successful have been explored; phase unwrapping tends to be successful for broadband signals when there is some degree of coherence between the microphone signals.[4] The effect of background noise on the PAGE method has been studied,[17] along with the possibility of using background noise to apply the PAGE method to narrowband signals.[16] The PAGE method has been applied to extend bandwidth of intensity-based sound power calculations[19] and in a preliminary real-time source localization device.[20] The approach of PAGE processing has been shown by Goates *et al.*[21] to extend the bandwidth of beamforming for a jet noise application by reducing the grating lobes. The PAGE method has the potential to extend the bandwidth of calculations typically limited by a spatial Nyquist frequency.

ACKNOWLEDGMENTS

This work was supported by National Science Foundation Grant No. 1538550, "Developing New Methods for Obtaining Energy-based Acoustic Quantities" and by funding for D. Hawks that came from the National Science Foundation REU program.

REFERENCES

[1] Thomas, Derek C., Benjamin Y. Christensen, and Kent L. Gee. "Phase and amplitude gradient method for the estimation of acoustic vector quantities," J. Acoust. Soc. Am.**137**, 3366-3376 (2015).
[2] K. L. Gee, E. B. Whiting, T. B. Neilsen, M. M. James, and A. R. Salton, "Development of a Near-field Intensity Measurement Capability for Static Rocket Firings," Trans. JSASS Aerospace Tech. Japan **14**, No. Po_2_9-Po_2_15, (2016).
[3] T. A. Stout, K. L. Gee, T. B. Neilsen, A. T. Wall, and M. M. James, "Source characterization of full-scale jet noise using acoustic intensity," Noise Control Eng. J. **63**, 522-536 (2015).
[4]K. L. Gee, T. B. Neilsen, S. D. Sommerfeldt, M. Akamine, and K. Okamoto, "Experimental validation of acoustic intensity bandwidth extension by phase unwrapping," J. Acoust. Soc. Am., **141**(4), EL357-EL362 (2017).
[5] M. J. Crocker, "Measurement of sound intensity," in Handbook of Acoustical Measurements and Noise Control, edited by C. M. Harris (McGraw-Hill, New York, 1991), pp. 14.1–14.17.
[6] J. A. Mann III and J. Tichy, "Near-field identification of vibration sources, resonant cavities, and diffraction using acoustic intensity measurements," J. Acoust. Soc. Am. **90**, 720–729 (1991).
[7] C 1043:1993, "Electroacoustics—Instruments for the measurement of sound intensity—Measurement with pairs of pressure sensing microphones" (International Electrotechnical Commission, Geneva, Switzerland, 1993).
[8]J. Y. Chung, "Cross-spectral method of measuring acoustic intensity without error caused by instrument phase mismatch," J. Acoust. Soc. Am. **64,** 1613-1616 (1978).
[9] C. P. Wiederhold, K. L. Gee, J. D. Blotter, S. D. Sommerfeldt, & J. H. Giraud, "Comparison of multimicrophone probe design and processing methods in measuring acoustic intensity," J. Acoust. Soc. Am. **135,** 2797-2807 (2014).
[10] ANSI/ASA S1.9-1996, Instruments for the Measurement of Sound Intensity (Acoustic. Soc. Am., Melville, NY, 1996).
[11] IEC 1043:1993, "Electroacoustics—Instruments for the measurement of sound intensity—Measurement with pairs of pressure sensing microphones" (International Electrotechnical Commission, Geneva, Switzerland, 1993).
[12] E. B. Whiting, J. S. Lawrence, K. L. Gee, T. B. Neilsen, and S. D. Sommerfeldt, "Bias error analysis for phase and amplitude gradient estimation of acoustic intensity and specific acoustic impedance," J. Acoust. Soc. Am, **142**, 2208-2218 (2017).

[13] J. S. Lawrence, E. B. Whiting, K. L. Gee, R. D. Rasband, T. B. Neilsen, and S. D. Sommerfeldt, "Three-microphone probe bias errors for acoustic intensity and specific acoustic impedance," J. Acoust. Soc. Am. **143**, EL81-EL86 (2018).
[14] M. J. Crocker, "Measurement of sound intensity," in Handbook of Acoustical Measurements and Noise Control, edited by C. M. Harris (McGraw-Hill, New York, 1991), pp. 14.1–14.17.
[15] M. R. Cook, K. L. Gee, S. D. Sommerfeldt, and T. B. Neilsen, "Coherence-based phase unwrapping for broadband acoustic signals," Proc. Mtgs. Acoust. **30**, 055005 (2017).
[16] M. R. Cook, K.F. Succo, K.L. Gee, S. D. Sommerfeldt, and T. B. Neilsen, "Bandwidth extension of narrowband-signal intensity calculation using additive, low-level broadband noise," J. Acoust. Soc. Am. **145**, 3146-3153 (2019).
[17] M. R. Cook, K. L. Gee, T. B. Neilsen, S. D. Sommerfeldt, "The effects of contaminating noise on the calculation of active acoustic intensity for pressure gradient methods," J. Acoust. Soc. Am. **145**, 173-184 (2019);
[18] J. S. Lawrence, K. L. Gee, T. B. Neilsen, S. D. Sommerfeldt, "Higher-order estimation of active and reactive acoustic intensity," Proc. Mtgs. Acoust. **30**, 055004 (2017).
[19] M. Mortenson, S. Gilbert, T. B. Neilsen, K. L. Gee, S. D. Sommerfeldt, "Extending bandwidth for sound power measurements," J. Acoust. Soc. Am. **145** 1884 (2019).
[20] J. G. Young, J. S. Lawrence, K. L. Gee, "Real-time localization of sources using the phase and amplitude gradient estimator for acoustic intensity," Proc. Mtgs. Acoust. **33**, 030001 (2018).
[21] C. B. Goates, B. M. Harker, T. B. Neilsen, and K. L. Gee, "Extending the bandwidth of an acoustic beamforming array using phase unwrapping and array interpolation," J. Acoust. Soc. Am., **141**(4), EL407-EL412 (2017).