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On the physical meaning of subsonic wavenumber truncation as applied to source identification on vibrating structures

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Wavenumber methods have been used to identify the supersonic portions of a vibrating structure that radiate to the farfield. To estimate the supersonic wave energy of a vibrating structure, the discrete Fourier transform can be used to determine the wavenumber spectrum which is then truncated above the acoustic wavenumber. The purely supersonic wavenumber spectrum can now be transformed back into the spatial domain to determine the vibration pattern associated only with the supersonic waves. Often, a cut-off coefficient associated with the acoustic wavenumber of the spatial radiation filter is used to reduce error. Equivalent spatial convolutions have also been formulated to obtain supersonic components of a vibrating pattern. This paper discusses the physical meaning of wavenumber truncation and its accuracy in identifying the surface areas of a vibrating structure that radiate sound. It is shown that truncating subsonic components of the vibration pattern will create non-physical vibration patterns which lie outside of the physical boundaries of the structure. Thus, subsonic wavenumber truncation methods may not be a reliable method for determining the radiated portions of a vibration pattern.

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1. INTRODUCTION

Wavenumber truncation has been used to determine the portions of a vibrating structure that contribute most significantly to the far field sound power radiated by the structure. Specifically, pressure and velocity on the surface of a structure have been filtered to eliminate the subsonic wavenumber components and then used to compute a quantity referred to as supersonic intensity.[1] While this technique was proposed for experimental measurements, it has also been used in numerical simulations of radiating structures.[2] The original work acknowledges without explanation, that when the structure is small with respect to the acoustic wavelength, source identification using supersonic intensity is not possible. Subsequent works, however, used supersonic intensity in this realm regardless.[2-3] This paper attempts to explain the physical meaning of wavenumber truncation and the small compared to wavelength assumption as it applies to the identification of radiating regions of vibrating structures. It is shown that the physical meaning of a velocity distribution on a structure is lost when performing supersonic filtering in the spatial domain if the bending wavenumber is near or above the acoustic wavenumber. Therefore supersonic intensity may not be an appropriate tool for determining the radiating portions of a vibrating structure.

2. ACOUSTIC INTENSITY

It is well known that a bending wave propagating in an infinite structure will not radiate to the farfield unless the wavespeed is greater than or equal to the wavespeed of the surrounding fluid. When the wave is subsonic, the velocity at the fluid-structure interface is not able to couple effectively into the fluid and only reactive pressure is created in the near-field of the structure. In finite structures, the supersonic wave components are able to effectively couple to the fluid and therefore are responsible for the acoustic power radiated to the far-field.

To determine the amount of power radiated by a structure, the acoustic intensity can be used. The complex acoustic intensity can be divided into real and imaginary components according to

$$\vec{I}(r) = \frac{1}{2}p(r)\vec{u}^*(r) = \vec{I}_a(r) + j\vec{I}_r(r), \tag{1}$$

where I_a is the active intensity, computed as the real portion of the complex intensity and I_r is the reactive intensity, defined as the imaginary portion of the complex intensity. In Eq. 1, p represents the acoustic pressure, u represents the particle velocity, r is the spatial location (with the vector symbol dropped), the arrow symbol signifies a vector quantity, and * indicates the complex conjugate. The intensity vector is a function of space with its direction dictated by the direction of the velocity vector. The active intensity is the time-averaged intensity that is radiated to the far-field while the reactive intensity accounts for the intensity that remains in the near-field. The radiated power is found by integrating the normal component of the active intensity over a surface enclosing the structure of interest.

3. WAVENUMBER TRUNCATION

A. WAVENUMBER ANALOGY

Wavenumber analysis is the spatial analogy to frequency analysis in the time domain where the wavenumber $k=2\pi/\lambda$ is the spatial equivalent of angular frequency $\omega=2\pi/T$. Here, λ is the structural wavelength and T is the period. Since the wavenumber is related to frequency and wavespeed by $k = \omega /c$, the acoustic wavenumber k_0 will be linear with frequency as long as the fluid medium is isotropic and at constant temperature.

Using the discrete Fourier transform, a velocity distribution on a structure can be transformed to the wavenumber domain, often referred to as k-space, to determine the wavenumber content. When performing wavenumber analysis, the convolution property allows a convolution in space to be performed as a multiplication in k-space. While spatial convolution may be more direct and compact, using multiplication in k-space is usually quicker and easier to visualize. Utilizing the well-known relationships for discrete time analysis with the time-space analogy, the maximum wavenumber is dictated by the distance between the known velocity points while the wavenumber resolution is set by the total length.[4]

$$k_{max} = \frac{\pi}{dx}, \qquad dk = \frac{2\pi}{L} \tag{2}$$

A common practice to extend the length L and obtain finer wavenumber resolution is to zeropad the known structural velocity out to an arbitrary distance. This is often necessary since the velocity distribution is on a finite structure, which is not periodic in space as is assumed in the DFT.

Working in k-space can be convenient when computing the sound power radiated by a vibrating structure. This has been illustrated analytically using a simply supported thin, rectangular plate. The primary velocity distributions (i.e. mode shapes) have an exact form in k-space[5] such that the power per unit length can be written as

$$P(\omega) = \frac{1}{4\pi}\Re\left\{\int_{-\infty}^{\infty} \pm\omega\rho|U(k_x)|^2/\sqrt{k_0^2 - k_x^2}\,dk_x\right\}, \tag{3}$$

where $\Re$ indicates taking the real portion, ρ is the mass density of the plate's material and $U(k_x)$ is the velocity wavenumber spectrum.[6] Eq. 3 can be reduced to an integration over k_x from $-k_0$ to k_0 since the denominator will be imaginary otherwise and discarded from the calculation. This illustrates that the acoustic power radiated by the plate is entirely from supersonic wavenumber components of the structural velocity.

B. SUMMARY OF PREVIOUS WORK

To identify radiating portions of a structure, Williams proposed filtering the pressure and velocity in a manner which eliminates subsonic wavenumbers.[1] This can be performed either in the spatial domain using a convolution with filter h^{k0} (Eq. 4) or in the wavenumber domain with multiplication by the transformed filter function H^{k0} (Eq. 5).

$$p_s(r) = p(r) *\ h_s^{k0}(r), \qquad \overrightarrow{u_s}(r) = \vec{u}(r) *\ h_s^{k0}(r) \tag{4}$$

$$P_s(k) = P(k)\,H_s^{k0}(k), \qquad \overrightarrow{U_s}(k) = \vec{U}(k)\,H_s^{k0}(k) \tag{5}$$

Here, capital letter indicate that the Fourier transform operation has been applied. If the k-space filter is a one-dimensional rectangular window from $-k_0$ to $+k_0$, then the spatial filter will be a sinc function. In two dimensions, the spatial filter will be a Bessel function in k-space. It should be noted that non-ideal filters such as the Lanczos filter have also been proposed to provide a smoother cut-off than the rectangular window.[3] Equations 4-5 have also been derived in terms of convolution integrals leading to compact solutions for the supersonic pressure and velocity for planar coordinates.[3,7] In this work, we will refer to this type of filtering as wavenumber truncation, regardless of the domain used to do the filtering, since certain wavenumbers are eliminated or truncated from the system.

The filtered pressure and velocity, referred to as supersonic pressure p_s and supersonic velocity v_s, can be multiplied to obtain the so-called supersonic intensity

$$\vec{I}_s(r) = \frac{1}{2}\Re\{\, p_s(r)\,\overrightarrow{u_s}^{\,*}(r)\,\}, \tag{6}$$

which has been proposed as a quantity that can identify radiating portions of a velocity distribution since the integrated supersonic intensity gives the radiated power of the structure. Without explanation, it is noted at the end of the initial paper that the technique does not work if the structure is small compared to wavelength.

Fernandez-Grande and Jacobsen later recommended applying a cut-off coefficient α to reduce the effect of wavenumber leakage created by using a finite-length aperture in a measurement, such that $k_0 = \alpha\, k_0$.[8] As seen in Eq. 2, the wavenumber resolution is large when L is small. Since typical measurement apertures are necessarily restricted to around the structure of interest, L is often small such that leakage effects are significant if zeropadding is not used. When $\alpha > 1$, the supersonic components which are leaked outside of the filter region are included and it is noted that the supersonic intensity results seemed to improve. In 2016, Liu *et al*. then examined the values of α using numerical simulations of a plate, cylinder and automobile part for conditions where the structure is both large and small compared to wavelength.[2]

Williams proposed another similar quantity about 15 years later which was referred to as the "hybrid bipolar intensity."[9] The definitions for these quantities are given as

$$\vec{I}_u(r) = \frac{1}{2}\vec{u}(r)\, p_s(r)^* \qquad and \qquad \vec{I}_p(r) = \frac{1}{2} p(r)\, \overrightarrow{u_s}^*(r). \tag{7}$$

For the hybrid intensities, the supersonic quantities are written entirely in terms of the non-filtered quantity (i.e. supersonic pressure is written in terms of velocity and supersonic velocity is written in terms of pressure).

C. PHYSICAL MEANING OF WAVENUMBER TRUNCATION

The use of a cut-off coefficient greater than unity is at odds with the very definition of supersonic intensity since it retains subsonic components in the calculation. Additionally, it is not clear if the "small compared to a wavelength" condition is required since no research has shown rigorously why it would be necessary. Therefore, a simple experiment was performed to more clearly understand the physical meaning of wavenumber truncation for identification of radiating portions of a structure along with its wavelength limitations.

A simply supported, rectangular plate in a rigid baffle was used as the example structure following the example presented by Liu *et al*.[2] The plate was 1m x 1m and the (2,2) mode was used such that the bending wavenumber in each dimension is 2π, as shown in Fig. 1. The DFT was used to transform the velocity distribution into k-space using 10x zeropadding, leading to dk=0.628 m^{-1} and k_{max}=314.1 m^{-1}. The wavenumber spectrum was truncated for three different conditions: $k_0 a = 2$, $k_0 a = 8.5$ and $k_0 a = 40$. In the first condition, the structural wavenumber is subsonic while for the second two conditions it is barely supersonic and very supersonic, respectively. The filtered wavenumber spectrum was then transformed back into the spatial domain to examine the effect of the truncation.

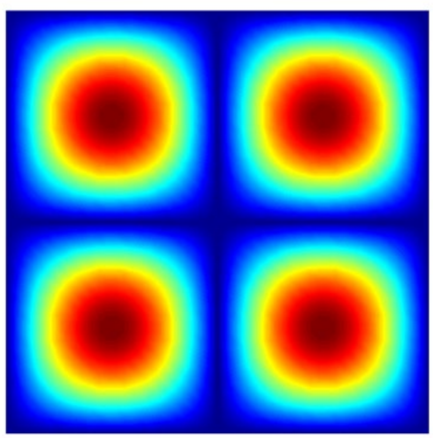

Figure 1. The (2,2) mode shape of a simply supported thin plate where red represents large deflection (positive or negative) while blue represents no deflection. The plate is located in a rigid baffle such that there is zero velocity everywhere off the plate. Since the amplitude of mode shapes are relative, the exact value of the deflection is arbitrary and the color scale is omitted.

Figure 2 shows the squared wavenumber spectrum with the cut-off locations of the three filters. The negative wavenumber components are complex conjugate pairs with the positive wavenumber and are therefore not shown beyond -12 m^{-1} since the magnitude is the same. Examining the positive wavenumber spectrum, there is a single main lobe of energy with much smaller lobes beyond k = 12. For the $k_0 a = 2$ case, the filter eliminates a majority of the main lobe as well as all the smaller lobes. For the $k_0 a = 8.5$ case, the peak of the first lobe is maintained but a portion of the lobe is still truncated (as are the smaller lobes). In the last condition, all of the main lobe is maintained as well as the first four smaller lobes.

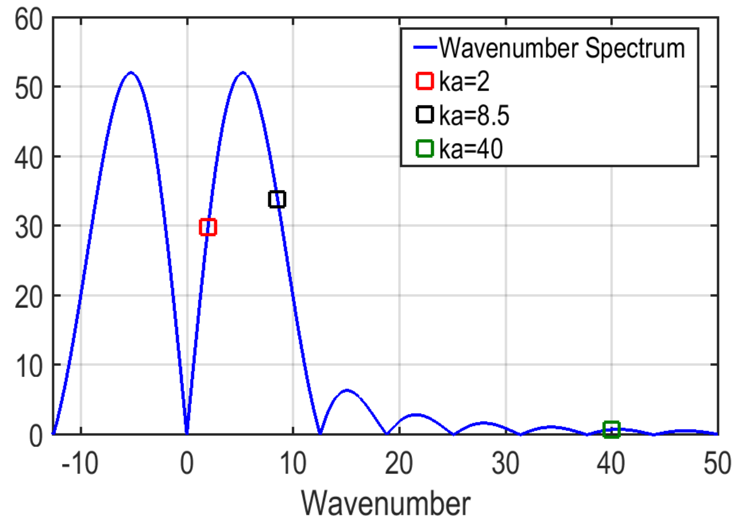

Figure 2. The squared wavenumber spectrum of a one-dimensional slice of the mode shape. The cut-off locations of the three conditions are shown as colored squares for the positive wavenumbers only. Although the magnitude squared is shown here, the filter was applied to the linear wavenumber spectrum.

The truncated waveforms were then transformed back to the spatial domain. The full spatial domain extends out to 10 m due to zeropadding even though the physical space of nonzero velocity only spans from 0 to 1 m. The region past 1 m represents the location of the rigid baffle which has, by definition, zero velocity. The results for the three cases are shown in Fig. 3 with the spatial domain shown to 5 m for ease of visualization and the same color scheme used in Fig. 2. The $k_0 a = 2$ case, shown in red, reveals a velocity distribution which extends over the entire range shown, most noticeably beyond the border of plate where the velocity distribution is physically located. The $k_0 a = 8.5$ in black also shows significant velocity oscillation beyond the physical boundary of the plate. The green $k_0 a = 40$ curve, on the other hand, looks very similar to the original velocity distribution, which is shown in blue. While a smoother window could have been used here, the existence of non-zero velocity in the baffle region would not be eliminated.

The presence of non-zero intensity outside the physical region of the structure indicates that when the truncated wavenumber spectra is transformed back to the spatial domain, the existence of the physical plate boundaries has been lost. Therefore, the velocity distribution for a truncated wavenumber spectrum is only mathematically equivalent when transformed back into the spatial domain and should not be used to infer the radiating portions of the velocity distribution. The $k_0 a = 40$ case is mostly confined to the boundaries of the plate. However, close inspection reveals small non-zero velocity content outside of the plate which, because the truncated portion of the wavenumber energy is nearly zero, is very small and basically trivial. As a note, this non-zero content outside the physical region of the plate is apparent in other works but is not noted or explained.[1,3,7,9]

The same conclusion can be drawn when filtering the wavenumber in both dimensions as shown in Fig. 4. Here, the active and supersonic intensity are shown for $k_0 a = 1$, with a white box outlining the physical boundaries of the plate. The active intensity is only non-zero where the plate physically exists while the supersonic intensity is non-zero for the entire spatial domain. The supersonic intensity actually reaches its maximum value in the corners of the domain which clearly do not contribute to the radiated power of the structure since the velocity is zero. Although not shown here, the active intensity and supersonic intensity are equivalent at high values of $k_0 a$.

The hybrid intensity quantities from Eq. 7 are shown in Fig. 5. The hybrid-v intensity is only non-zero on the plate while the hybrid-p intensity is non-zero over the entire domain. While these results are interesting, it is still unclear what the physical meaning of the hybrid intensity quantities are.

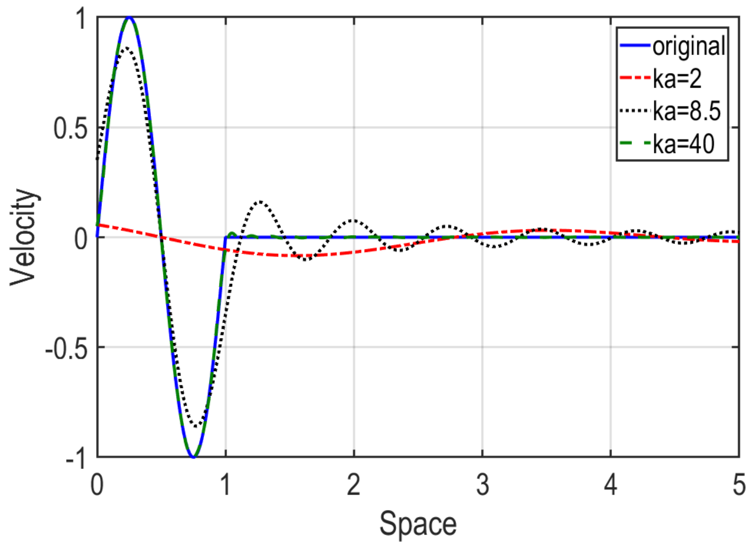

Figure 3. The 1-D velocity distributions of the three truncated wavenumber spectra when transformed back to the spatial domain compared to the original velocity distribution. The existence of a physical boundary of the plate between 0 – 1 m has been lost.

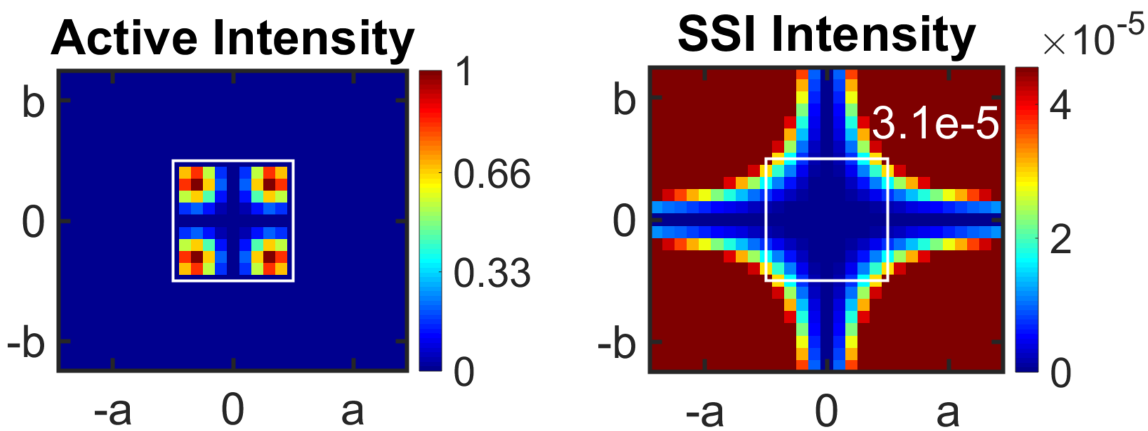

Figure 4. The two-dimensional active intensity (left) and supersonic intensity (right) for $k_0 a = 1$. The physical boundary of the plate is shown by the white box.

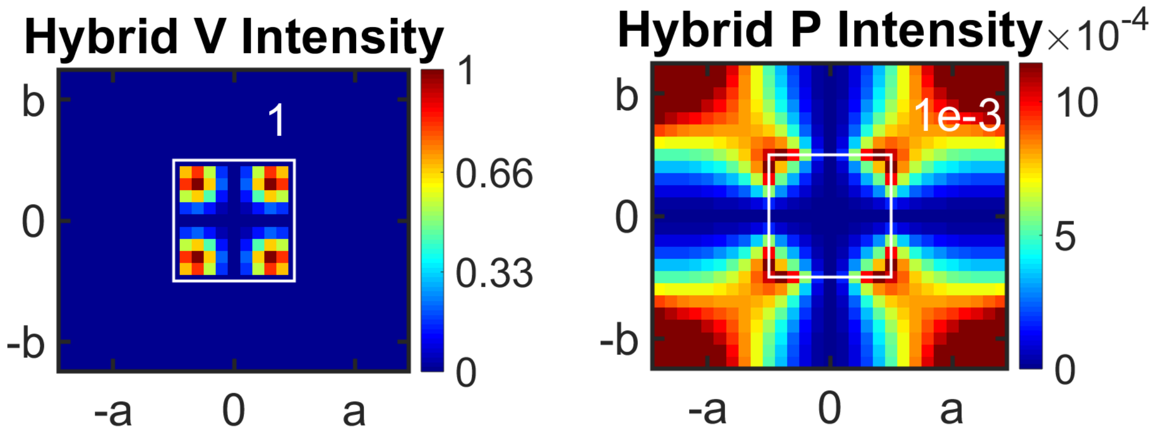

Figure 5. The two-dimensional hybrid-v intensity (left) and hybrid-p intensity (right) for $k_0 a = 1$. The physical boundary of the plate is shown by the white box.

D. DISCUSSION

A number of interesting outcomes from this basic simulation merit discussion. First, while finite measurement apertures can lead to wavenumber leakage (without sufficient zeropadding), numerical simulations should not have significant leakage since explicit aperture constraints, which are ever present in measurements, do not exist but is determined by the user. Naturally, the spatial domain can easily be defined to effectively eliminate leakage. Therefore, a cut-off coefficient should not be necessary for numerical simulations of supersonic intensity.

Second, a wavenumber-truncated velocity distribution does not maintain the physical meaning in the spatial domain since the supersonic velocity distribution is not confined to the physical dimensions of the structure. This explains why supersonic intensity does not work if the structure is small compared to wavelength. In fact, the notion of identifying radiating portions of a structure using wavenumber truncation seems to be entirely non-physical, regardless of size of the structure compared to the acoustic wavelength since large values of $k_0 a$ will lead to supersonic intensity being an approximation of the active intensity.

Finally, the hybrid definitions of intensity which includes either just supersonic velocity or just supersonic pressure still lack a clear explanation of their meaning and how they could be used. However, it is worth noting that the particle velocity is not a smooth function of space while the pressure is. This may be why the hybrid velocity intensity method gives the active intensity while the hybrid pressure intensity method does not. Further investigation would be required to confirm this.

4. CONCLUSIONS

Wavenumber truncation has been examined for a simply supported square plate to determine its physical meaning. It is shown that quantities supersonic pressure and supersonic velocity lose their physical meaning if transformed back into the spatial domain and are only mathematically equivalent expressions. The use of a cut-off coefficient effectively includes more energy in wavenumber space thus causing the supersonic intensity to approach the active intensity.

Further research is required to determine appropriate techniques for determining which portions of a vibrating structure contribute to the far field radiated sound power. Investigations of other quantities such as the complex intensity, hybrid intensity or non-negative intensity could be performed on baffled and unbaffled structures to determine their physical meaning and practical effectiveness.

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