Case study: Noise reduction of a vacuum-assisted toilet

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Flushing a vacuum-assisted toilet generates noise levels that can be disturbing both to users and those nearby. Peak radiated noise levels correlate with the time when the valve opens and closes, while the noise levels when the valve is completely open are also relatively high. Significant noise ranges between 300 Hz and 3000 Hz. It was hypothesized that increasing the in-tube distance between the flush valve and the bowl in addition to increasing the bend radius of the tube would reduce radiated noise levels. These modifications resulted in a reduction of about 14 dB in the radiated noise during the valve opening and closing in addition to a reduction of about 5 dB while the valve is completely opened. Intermediate results of varying the tube length and bend radius are presented to show their effects on the radiated sound levels. Two tube inserts were designed to fit (1) underneath and (2) behind the toilet in a compact manner. They were tested to show that they maintain noise control performance without modifying any other part of the toilet.

Primary subject classification: 21.6.6; Secondary subject classification: 37.4

1 INTRODUCTION

1.1 Background and History

Vacuum-assisted toilets utilize a pressure difference, rather than gravity, to transport waste from the toilet to a septic tank, beneficially reducing the amount of water required per flush. Conventional gravity toilets require 5.68 L (1.5 gallons) of water per flush, while vacuum-assisted toilets require only 0.12 L (1/2 cup) of water per flush.[1] Water reduction is desirable because it reduces weight and, by extension, fuel costs for transportation vehicles such as airplanes, cruise ships, and trains. Cost savings, however, come at the expense of higher noise levels.

A number of past studies have focused on the subject of noise control on vacuum-assisted toilets. In 1994, Frank and Sparr [2] patented the idea to recycle grey water to increase rinse water levels in toilets above 0.12 L (1/2 cup) of water to reduce the radiated noise. Moore[3] in 1998 patented the idea of using a lid on vacuum-assisted toilets designed to provide significant noise transmission loss. Hufenbach et al.[4] in 2008 made an acoustical analysis of a vacuum-assisted toilet and showed that decreasing the vacuum strength, modifying the valve and outlet area, and using a lid are some ways to reduce radiated noise. From an intensity map of a vacuum-assisted toilet in Hufenbach's article, the sound levels near the tubes and sides of the bowl are significantly lower than those near the top of the bowl and right at the valve. It is clear that the fluid–structure coupling to the bowl and tubes is not a significant contributor to the radiated noise. In 2013, Boodaghians et al.[5] patented the idea to use a bypass valve to provide a secondary source of air, enabling the primary flush valve to close the acoustic transmission path to the user. In 2016, Seibt and Mueller[6] patented another noise control strategy, which includes an in-line expansion chamber with a tube between a sink and a vacuum tank to act as a muffler. A recent investigation of the authors used a structural damping noise control technique that reduced structural vibrations of the bowl by about 20 dB that only translated to a 3 dB noise reduction for just one part of the flush cycle.[7] Because of the relatively small reduction in radiated sound when structural vibrations were significantly reduced, it was concluded that fluid–structure coupling was not a dominant contributor to the noise level.

Other previous work suggests a need to reduce flow velocity to limit noise production. Davies et al.[8] show that

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there is a power law relationship between the flow velocity to the sixth power and the propagating sound for large- and small-scale turbulence in the plane-wave mode of a tube. Consequently, a small decrease in flow velocity can correspond to a large decrease in sound radiated. The relationship of in-tube radiation exceeds that of turbulence in free space by a factor of M^{-2}, where M is the Mach number, implying even more pronounced noise production for flow through tubes. However, high-frequency small-scale turbulence can excite higher-order modes which do not have the M^{-2} boost when nearly all modes are excited. Hufenbach et al.[4] show that with a 2/3 reduction in vacuum level (and thus flow velocity) overall sound pressure level of the flush can be reduced by 6 dB.

1.2 Motivation for This Work's Noise Reduction Techniques

Several previous works suggest increasing the tube length between sound sources and the receiver location to affect noise reduction. The Occupational Safety and Health Administration (OSHA)[9] recommends placing all bends and valves at least 10 pipe diameters away from each other. Hoff[10] showed a 2 to 3 dB/m attenuation from viscous losses in the 1 to 3 kHz frequency range for sound propagating in the upstream direction of a 90 m/s gas flow. This suggests that increasing tube length between aerodynamic noise sources in the tube and bowl can significantly reduce noise. According to Davies et al.[8] low-frequency small-scale turbulence radiates into modes that decay exponentially with distance. This suggests that noise from small-scale low-frequency turbulence can be reduced with sufficient tube distance from the source location to the bowl. Additional references that present a mathematical development of evanescence in tubes can be found in Refs. 11–13.

Previous works also suggest reducing noise by increasing the bend radius of tube bends. Experiments done by Hufenbach et al.[4] suggest modifying the bowl outlet geometry such that the bend radius is as large as it can be. An 8 dB noise reduction occurs in a tube with an infinite bend radius, i.e., a straight tube, on a vacuum-assisted toilet. Aissaoui et al.[14] achieved a 4 dB noise reduction by performing a numerical optimization of an HVAC system in an automobile that considered geometric modifications to the tubes connecting the blowers to the outlets. Qiu et al.[15] achieved a 2.5 dB noise reduction for jet engine bypass flow noise by numerically optimizing the tube geometry near the outlet. Vizzini et al.[16] compared flow at 96 m/s through a straight tube to the same flow speed through a tube with a 90° curve having a 7.5 cm bend radius. In the frequency range of interest of this article, the straight pipe radiated 2 to 5 dB less than the tube with the 90° bend.

Our hypothesis is that the radiated noise from a vacuum-assisted toilet can be reduced by increasing the bend radius of tubes near the bowl and by increasing the length of the tube between the valve and bowl.

1.3 Paper Layout

The layout of the article is as follows: Section 2 describes the setup of the vacuum-assisted toilet, the tube materials and shapes, and the data collection and analysis techniques used for this investigation. Section 3 presents data regarding the radiated noise of a double spiral involving a long tube wrapped twice underneath the bowl, an investigation on the effect of reducing the bend radius and tube length from that of the double spiral, the acoustic equivalence of using different tube materials, and two tube inserts that use the bend radius and tube length constraints defined from the aforementioned investigation. In Sec. 4, conclusions are made that by increasing the bend radius and tube length between the bowl and valve, the radiated noise of a vacuum-assisted toilet can be reduced and that there is a critical bend radius and tube length required to maintain noise control performance relative to a tube with a large bend radius and long tube length.

2 METHODS

2.1 Experimental Setup

Noise associated with vacuum-assisted flushes was investigated in a hemi-anechoic chamber. The toilet setup is as follows: We connected a commercial vacuum-assisted toilet to a septic tank of volume 0.68 m^3 (180 gal.) and evacuated the air in the tank down to a gauge pressure of −68 kPa (−20 inHg or − 2/3 atm) for each flush. No water was added to the bowl prior to a flush. During each flush, 0.12 L (1/2 cup) of water was injected from a rinse ring near the rim of the bowl over a duration of 0.8 seconds at a gauge pressure of 276 kPa (40 psi). The tank and valve were connected via a 5-m-long, 5.04 cm (2 in) smooth inner diameter semi-flexible vacuum tube. The flush valve is a thin plate in-line with the tube coming from the toilet which separates the relative vacuum from atmospheric pressure. The plate has a circular cutout that starts outside the tube and sweeps down into the tube exposing the relative vacuum to the atmospheric pressure. At the end of a flush, the flush plate sweeps back to its original position outside the tube, closing off the vacuum tank from the open toilet. A programmed stepper motor controls the sweeping action of the flush plate.

The acoustic setup is as follows: A 1.27 cm (0.5 in) prepolarized GRAS 40AE free-field microphone was placed 1 m above the front edge of the bowl pointing downwards toward the toilet gathered the acoustical data. A NI

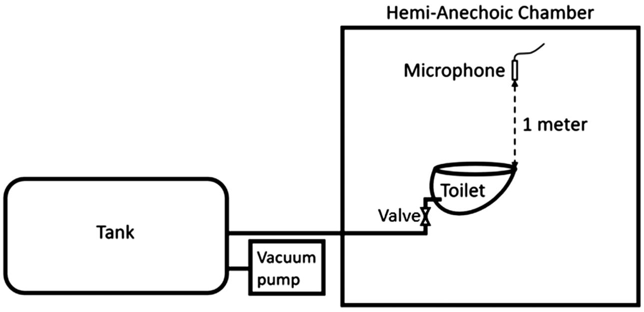

Fig. 1—Schematic of experimental setup.

cDAQ-9178 chassis and 9234 BNC module were used to acquire the data from the microphone. Figure 1 shows this in schematic form. Each flush cycle was repeated five times from which levels were averaged. An individual flush did not vary more than 1 dB from another flush of the same configuration.

2.2 Tube Materials and Geometries

Three tube materials were used throughout this investigation: first, a baseline tube of 4.45 cm (1.75 in) inner diameter (ID) made of hard plastic with a smooth inside used currently in vacuum-assisted toilets, shown in Fig. 2a; second, a 5.08 cm (2.0 in) inner diameter tube made of flexible plastic with 5 stiffening corrugations per 2.54 cm (1.0 in), shown in Fig. 2b; and third, a 5.08 cm (2.0 in) inner diameter tube 3D printed of ABS, shown in Fig. 2c. The flexible plastic tube (Fig. 2b) is the least identical to the current tubing used in vacuum-assisted toilets due to its corrugations and slightly larger inner diameter. The effect of corrugations is investigated in Sec. 3.6 by comparing an advantageous tube shape made with the flexible/corrugated tubing and a 3D-printed tube with

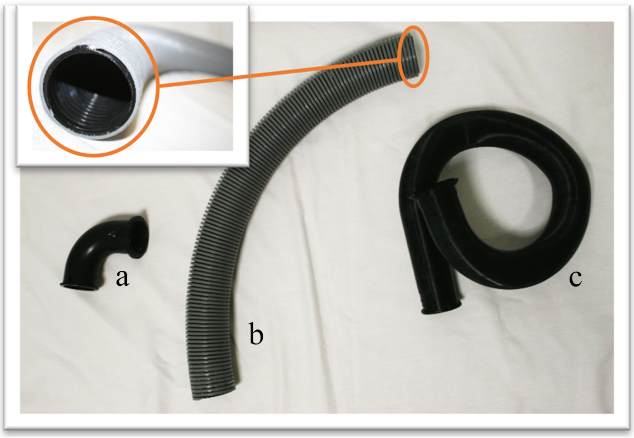

Fig. 2—Photo of tube materials used: (a) 90° elbow included with vacuum toilet, (b) corrugated flexible tube, and (c) 3D-printed tube.

Fig. 3—Notional diagram of baseline tube geometry in relation to the toilet.

comparable smoothness to the baseline tube. The tube flexibility allowed for quickly and cheaply testing many different tube configurations. Tube configurations using the flexible plastic tube included two adapters that vary the inner diameter linearly: a bowl/tube adapter that diffuses from 4.45 cm (1.75 in) ID to 5.08 cm (2.0 in) ID over 7.62 cm (3.0 in) and a tube/valve adapter that nozzles from 5.08 cm (2.0 in) ID to 4.45 cm (1.75 in) ID over 2.54 cm (1.0 in). For this article, the diffusion/nozzle effects are assumed negligible. Similar ID adapters were integrated into the 3D-printed tube configurations.

We investigated four tube geometries for connecting the bowl to the valve: first, the tube currently installed on vacuum-assisted toilets with a 90° bend and radius of curvature of 4.5 cm, shown in Figure 3; second, a flexible tube wrapped twice around the base of a toilet, shown in Figure 4; third, a flexible tube forming a straight connection between the bowl and valve with no bends, shown in Fig. 5; and fourth, a flexible tube in a spiral-esque shape with a pitch of 2.5 in per revolution that makes one

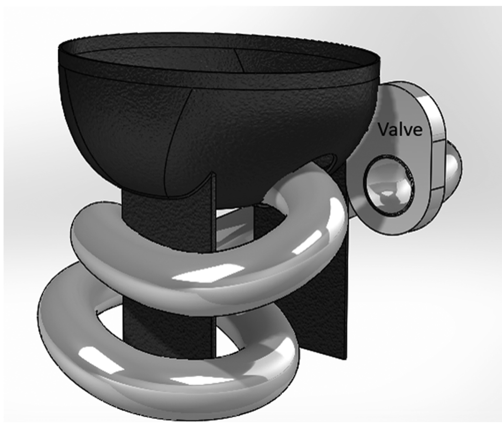

Fig. 4—Notional diagram of double-spiral tube geometry in relation to the toilet.

Fig. 5—Photo of straight-tube configuration.

revolution with variable bend radius, shown in Fig. 6. The combination of the smallest bend radius and tube length without significant loss in acoustic performance was 3D printed to evaluate the effect of a smooth versus corrugated tube.

3 RESULTS AND DISCUSSION

3.1 Characterization of the Noise, Baseline Tube

Current noise levels for vacuum-assisted toilets are time dependent. Three stages of the flush cycle correspond to distinct radiated sound levels as shown in Fig. 7 by the OASPL-A. A spectrogram of the noise levels is shown in Fig. 8. The first stage of the flush cycle is the valve opening event. While the valve opens and vacuum pressure is introduced to the toilet, the highest noise level is measured. The initial noise contains significant energy from about 300 Hz to 3 kHz with some insignificant frequency banding. The second stage occurs while the valve is completely open, which we refer to as the "steady-vacuum" stage. During this stage, the noise level plateaus about 7 dB lower than the opening peak. The steady-vacuum noise contains significant energy from about 300 Hz to about 8 kHz with evidence of tonal noise apparent by the horizontal banding in the spectrogram. The difference in noise level for the steady vacuum state and the opening event indicates that the valve being in the flow has a large impact and suggests that modifications to the valve may reduce noise levels. As will be seen later in this article, the bowl-to-valve tube distance plays a significant role in noise levels. The third noise

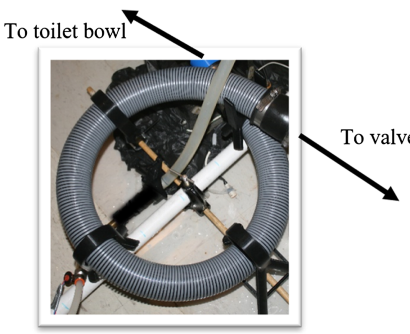

Fig. 6—Photo of varying bend radius investigation.

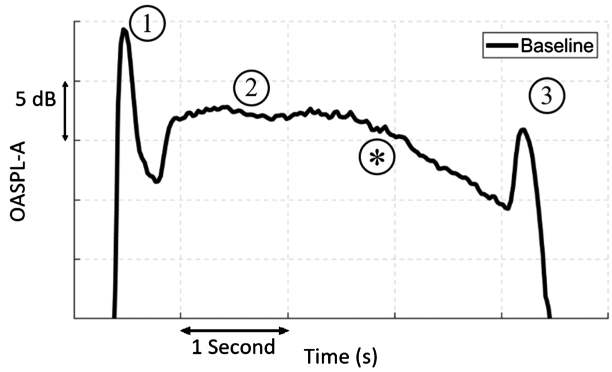

Fig. 7—Running OASPL-A for the baseline tube geometry averaged over 5 flushes. The encircled "1" indicates the valve opening sound level, "2" indicates the steady vacuum phase, "" indicates the time during the flush that vacuum suction is no longer sufficient to replicate a normal flush, and "3" indicates the valve closing sound level.*

event occurs as the valve closes and is manifest by another peak in the overall noise level. In our experimental apparatus, we are able to replicate the running OASPL of a flush for the first 2 seconds of a 4-second flush cycle. The closing peak noise level at 4 seconds in Figs. 7 and 8 is lower than what it would be in practice.

3.2 Double-spiral tube

In order to prove whether increasing the bend radius and tube length between the bowl and valve reduces the radiated noise, a 1.7 m flexible tube was inserted which connected the bowl to the valve and wrapped twice around the base of the toilet with an approximate 16.5 cm bend radius, similar to the drawing in Fig. 4. Note that the volume of the tube between the bowl and the valve does not need to be

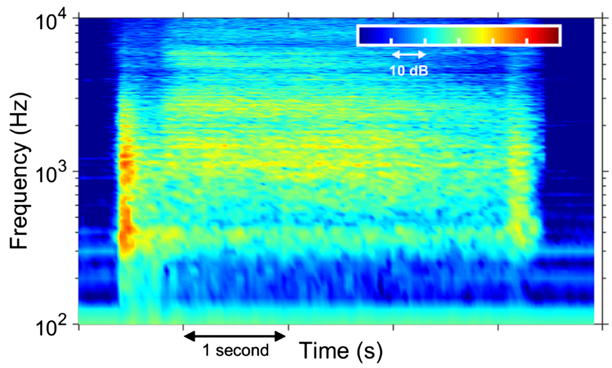

Fig. 8—Spectrogram of an individual baseline flush.

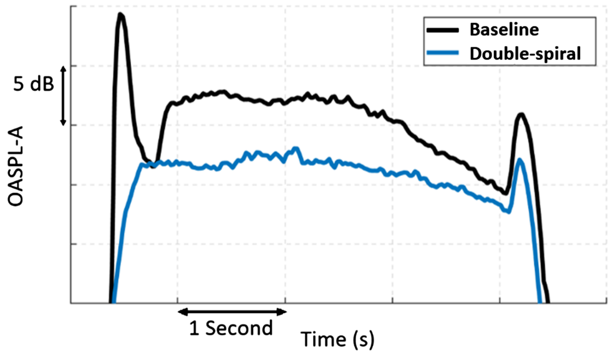

Fig. 9—Running OASPL-A for baseline flush and double spiral flush, each averaged over five flushes.

filled with water in order to transport waste like a traditional gravity toilet. The same rinse volume of 0.12 L (1/2 cup) for the baseline flush is used for the double-spiral tube configuration as well. The double-spiral tube reduced the initial peak by 14 dB, the steady-vacuum level by 4 dB and the closing peak by 4 dB as shown in Fig. 9. Note that the closing peak is subject to reduced vacuum strength, a limitation of the experimental setup. The double-spiral tube reduced the noise significantly over the broad frequency range of 300 Hz to 10 kHz throughout the whole flush cycle as shown by comparing Fig. 8 to Fig. 10.

The success in reducing the opening peak noise level can be attributed to the points described in Sec. 1.2, which includes references to corroborating experiments. As discussed, the noise reduction principles in play are source-receiver separation, viscous losses, evanescence, and reduced turbulence by not obstructing streamlines with tight bends which are achieved by simply increasing tube length and bend radius between the bowl and valve. Since the double-spiral tube was relatively large, the rest of this article investigates how short the tube length and

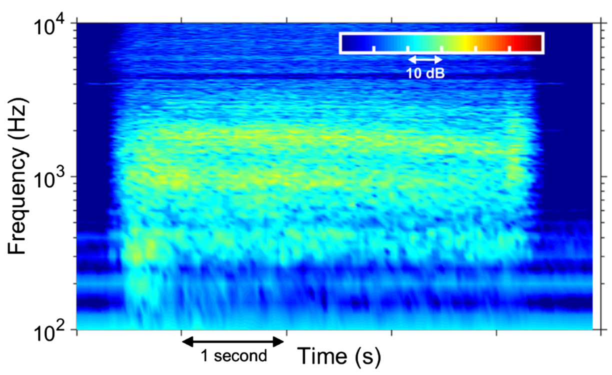

Fig. 10—Spectrogram of an individual double-spiral tube flush.

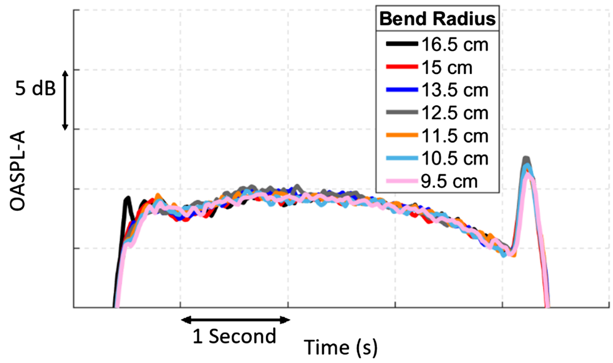

Fig. 11—Running OASPL-A for the varying bend radius investigation. The bend radius is varied from 16.5 cm to 9.5 cm while the tube length is kept constant. All curves are averaged over 5 flushes.

how small the bend radius can be while preserving noise control performance.

3.3 Reduced Bend Radius with Constant Length Tube

In our first configuration, the bend radius was decreased without modifying the tube length. Flexible tube was coiled into a spiral with one revolution behind the toilet. The valve was also moved behind the toilet and spiral for the feasibility of this study and not for practical application. Starting at 16.5 cm, the bend radius was progressively reduced to 15, 13.5, 12.5, 11.5, 10.5, and 9.5 cm. As the bend radius was decreased, more of the flexible tube continued tangentially after the spiral. No further decrease to the bend radius was possible because the flexible tube would not bend tighter than 9.5 cm. Figure 11 shows the OASPL-A for each configuration. Minimal variation occurred in the running OASPL-A curves from one configuration to the next as the bend radius was decreased. The levels for the initial peak vary by 4 dB, but the maximum level is within 1 dB of the steady-vacuum levels (as opposed to the 7 dB difference with the baseline tube). The steady-vacuum level varies by about 1 dB, and the closing peak varies by slightly more than 1 dB.

3.4 Reduced Tube Length with Constant Bend Radius

In the second configuration, only the tube length was decreased. The flexible 1.7 m tube was attached to the bowl and connected to the valve behind the toilet, keeping the tube completely straight. The tube was progressively shortened from 1.7 to 1.3, 1.22, 1.10, and 1.04 m. Figure 12 shows the OASPL-A for each configuration. The 1.04 m

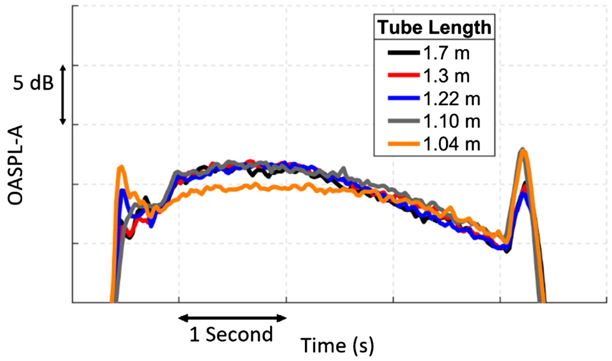

Fig. 12—Running OASPL-A for the varying tube length investigation. The tube length was varied from 1.7 m to 1.04 m, while the bend radius was kept constant. All curves are averaged over 5 flushes.

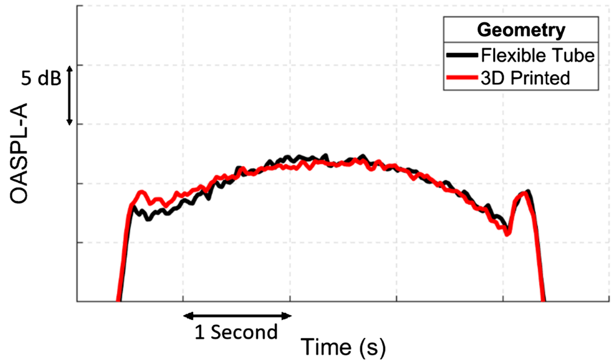

Fig. 14—Comparison of the flexible tube geometry with 9.5 cm bend radius to the 3D printed version. Levels are nearly the same. Both curves are averaged over 5 flushes.

length seems to be an outlier during the steady vacuum for the straight configuration. This effect is not present when varying both the bend radius and tube length simultaneously. Shorter than 1.3 m, the initial noise level appears to increase with the 1.04 m tube being 5 dB higher than the 1.7 m tube. The steady-vacuum level varies by less than 1 dB (besides the outlier 1.04 m tube). The 1.10 m and 1.04 m tubes increase in level by 4 dB during the closing peak. These results suggest that performance begins to be affected when shortening the tube below 1.3 m. The significance of this result is diminished by the improved performance when the shorter tubes are coiled in a spiral.

3.5 Reduced Bend Radius with Reduced Tube Length

In the final configuration, both bend radius and tube length were decreased simultaneously by removing excess tubing after each contraction of the bend radius. Figure 13 shows the OASPL-A curves, while the combinations of bend radii and tube length are reported in the legend. The initial peak varied by less than 1 dB. The steady-vacuum levels varied by about 1 dB for tube lengths 1.04 m and greater, while the tube with 0.77 m length increased by about 2 dB. The closing peak varied by 3 dB, except for the 0.77 m tube which was 2 dB lower than the next lowest configuration. There seems to be a tradeoff between the steady-vacuum level and the closing peak level with this tube size, i.e., a 2 dB increase during the steady-vacuum phase for 2 dB decrease in the closing peak level compared to the 1.7 m length and 16.5 cm bend radius tube. Larger tubes show no significant variation in the initial peak and steady-vacuum phase from the largest tube, while there is a 3 dB spread in the closing peak level. We chose the 9.5 cm bend radius and 0.77 m tube as our smallest version that preserves a significant amount of the reduction performance.

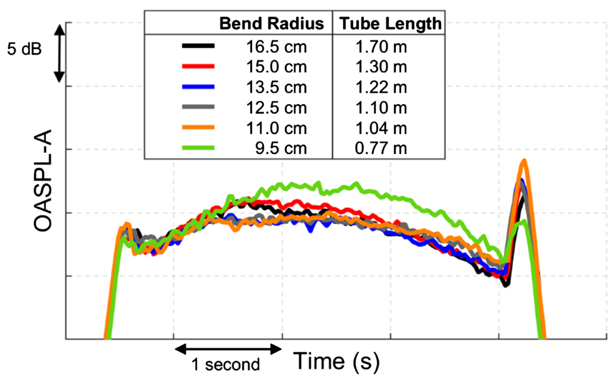

Fig. 13—Running OASPL-A for the changing bend radius and tube length investigation. All curves are averaged over 5 flushes.

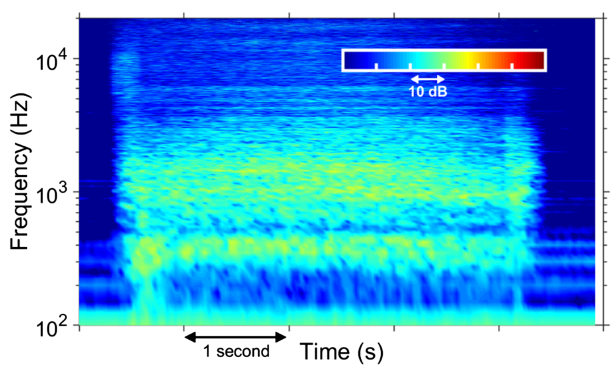

Fig. 16—Spectrogram of an individual flush with a 3D-printed 9.5 cm bend radius tube.

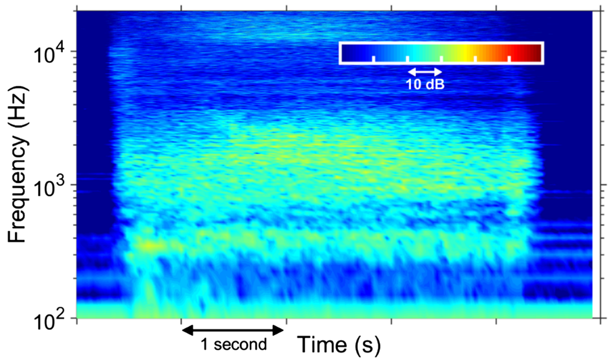

Fig. 15—Spectrogram of an individual flush with a 9.5 cm bend radius flexible tube.

3.6 3D Printed Version

A spiral tube matching the geometry of the flexible tube was 3D printed and installed on the vacuum-assisted toilet. Figure 14 shows that the agreement of the OASPL-A measured with a flexible tube and a 3D-printed tube is within 1 dB throughout the flush cycle. Figures 15 and 16 show spectrograms of the flushes with a flexible tube and 3D-printed tube. The flexible tube has more energy in the 2 to 3 kHz and 10 to 20 kHz range, while the 3D printed version has more energy in the frequency range from 3 to 10 kHz than the flexible tube. Consequently, the choice of material may be important for sound quality but not for overall sound level.

There may not be enough space for a full spiral behind the toilet in most practical applications, but a tube designed to fit into the space allotted could give similar noise reductions. Other form factors besides the previously selected spiral can be printed and tested with similar bend radii and tube lengths.

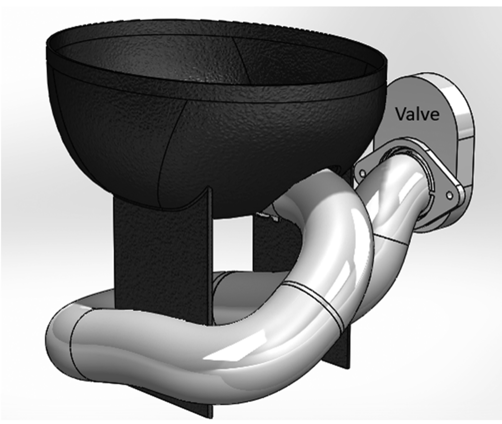

Fig. 17—Notional diagram of how the wrapped tube interfaces with the toilet.

Fig. 18—Notional diagram of how an insert may fit behind the toilet, i.e., back-only insert.

3.7 Wrapped and Back-Only Inserts

A replacement for the 90° tube was designed to fit underneath the bowl without modifying any other toilet component. With the above constraints, we designed a replacement insert which wraps around the left or right half of the base which effectively increases both the

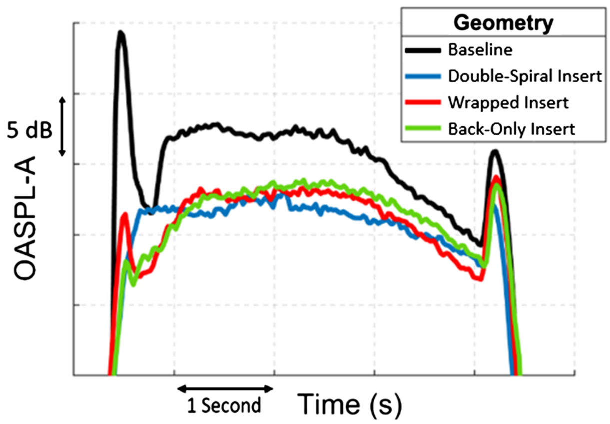

Fig. 19—Running OASPL-A comparing the baseline setup to the double-spiral insert, wrapped insert, and the back-only insert. The wrapped insert brings the initial peak level down 14 dB, while the back only insert brings it down 17 dB and the initial peak is not visible for the double-spiral tube. The wrapped and back-only inserts are within 1 dB of the double-spiral insert during the steady-vacuum phase and 2 dB during the valve closing. All curves are averaged over 5 flushes.

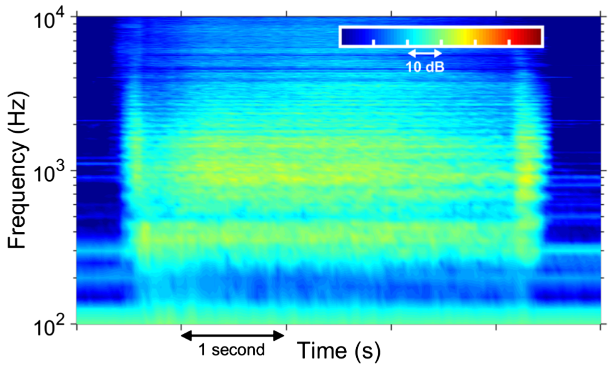

Fig. 20—Spectrogram of an individual flush with the wrapped tube.

tube length and bend radius between the bowl and valve as shown in Fig. 17. Another insert was designed to fit completely behind the back of the toilet while still following the above constraints as shown in Fig. 18. Figure 19 shows the running overall sound pressure level resulting from the baseline, double spiral, wrapped, and back-only tubes. Importantly, the initial peak level of the wrapped tube is the same as the double spiral, while the back-only insert is 1.5 dB lower. The wrapped and back-only inserts produced a steady-vacuum level within 1.5 dB of the levels produced with the double-spiral insert. The closing peak levels of the wrapped and back-only inserts are within 2 dB of the levels with the double-spiral insert. The sound of the valve opening is still present with the wrapped and back-only inserts although 14 to 16 dB quieter than the baseline. However, with the double-spiral tube, the sound of the valve opening is not present but seems to only have the steady-vacuum noise. What is meant by "the sound of the valve opening" is the characteristic rising OASPL-A followed by a drop in level and then the steady-vacuum level is reached. The spectrograms of the baseline, back-only, and wrapped tubes show a broadband pulse at the beginning of the flush cycle as

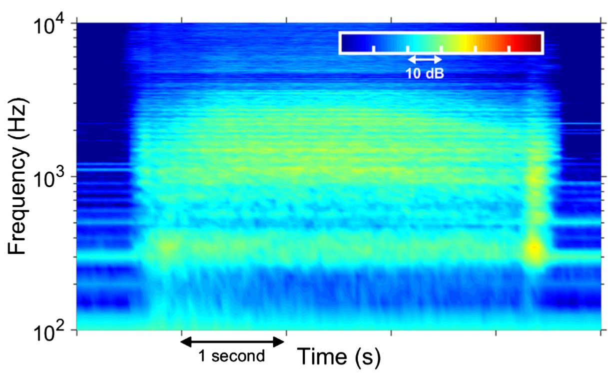

Fig. 21—Spectrogram of an individual flush with the back-only insert.

shown in Figures 8, 20, and 21, respectively, while the double-spiral tube does not in Figure 10. Instead, the double spiral begins with a raise in OASPL-A directly to the steady-vacuum phase.

4 CONCLUSIONS

This investigation of radiated noise from a vacuum-assisted toilet indicates that a 1.7 m tube with a bend radius of 16.5 cm wrapped 1.5 times underneath the toilet can reduce the radiated noise of a vacuum-assisted toilet by 14 dB during the valve opening and 4 dB during the "steady-vacuum" phase. After an investigation of placing a spiral tube behind the toilet, a similar noise reduction was achieved with a tube length of 0.77 m and bend radius of 9.5 cm. Using a tube with either a smooth or corrugated inside surface did not affect the overall levels but did have some impact on the spectral content which is linked to sound quality. We designed a tube to fit underneath the toilet bowl or fit completely behind the bowl in a compact manner while applying these tube length and bend radius constraints. Noise reduction performance was maintained with both smaller configurations. These advances may help provide an improved experience for transport vehicle lavatory users and passengers. Ongoing and future investigations may use this tube design in concert with other noise control strategies.

5 ACKNOWLEDGEMENTS

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